Recent Results on Acoustic Metamaterials and Sonic Crystals

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Outline

1. Introduction
2. Acoustic metamaterials with negative parameters
3. Visco-thermal effects in acoustic metamaterials with double-negative parameters
4. Sound absorption and redirection with sonic crystals based on metamaterials units

METAgentierie2017, 2-7 July.
Recent Results on Acoustic metamaterials

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METAgenierie2017, 2-7 July.
Acoustic metamaterials are artificial structures made of subwavelength units such that their acoustic properties are NEW in comparison with that of the building units.
Acoustic metamaterials

\[ c = \sqrt{\frac{\kappa}{\rho}} \]

\( \kappa \): bulk modulus
\( \rho \): density
\( c \): speed of sound

(Monopolar +dipolar Resonances)

(Dipolar resonances)

(No resonances)

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Li and Chan, PRE (2004)

Acoustic metamaterials / Metafluids

\( \rho_{\text{eff}} < 0 \)

(No resonances)

\( \rho_{\text{eff}} > 0 \)

\( B_{\text{eff}} > 0 \)

\( \rho_{\text{eff}} < 0 \)

\( B_{\text{eff}} < 0 \)

Z. Yang et al. PRL (2008)

S.H. Lee et al. PRL (2010)

N. Fang et al. Nat. Mat (2006)
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3. Visco-thermal effects in double-negative acoustic metamaterials

4. Sound redirection and splitting based on metamaterial units
INTRODUCTION: metamaterials with negative bulk modulus

N. Fang et al., *Ultrasonic metamaterials with negative bulk modulus*, Nat. Mat. 5, 452 (2006)

1D waterfilled waveguide

Effective modulus:

\[
E^{-1}_{\text{eff}}(\omega) = E_0^{-1} \left[ 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\Gamma\omega} \right],
\]

where \(\Gamma\) is the dissipation loss in the HR:

\[\Gamma = 2\pi \times 400\text{Hz}\]

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The quasi-2D metafluid

2D Waveguide \((h) +\) cylindrical holes \((R, L)\)

hexagonal lattice with parameter \(a\)

holes with \(R=1\ cm, L=9\ cm, a=3cm\)

2D waveguide with \(h=5\ cm\)

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From $R(\omega)$ and $T(\omega)$

\[ B_m^{-1} = B_0^{-1} \left[ 1 - \frac{F \omega_0^2}{\omega^2 - \omega_0^2 + i \Gamma \omega} \right] \]

\[ \omega_0 < \omega < \omega_0 \sqrt{1 + F} \]

Parameters:
\[ \omega_0 = 2\pi \times 874 \text{ Hz} \]
\[ \Gamma = 2\pi \times 3.4 \text{ Hz} \]

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Double negative MtM: building unit

- **Pressure Field In the Region I**

\[ P^I(r, \theta, z; w) = \sum_{q,n} \left[ A_{qn} J_q(K_n^I r) + B_{qn} H_q(K_n^I r) \right] \Phi_n^I(z) \exp(iq\theta) \]

\[ K_n^I = \sqrt{\left( \frac{w}{C_b} \right)^2 - \left( \frac{n\pi}{h} \right)^2} \]

- **Pressure Field in the Region II**

\[ P^{II}(r, \theta, z; w) = \sum_{q,m} \left[ C_{qm} J_q(K_m^{II} r) - \frac{j_q(K_m^{II} R_a)}{\dot{Y}_q(K_m^{II} R_a)} Y_q(K_m^{II} r) \right] \Phi_m^{II}(z) \exp(iq\theta) \]

\[ K_m^{II} = \sqrt{\left( \frac{w}{C_b} \right)^2 - \left( \frac{m\pi}{h+L} \right)^2} \]

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Double negative metamaterial: T matrix approach


Building unit: Metafluid cylinder with $c < c_{\text{air}}$

Bulk modulus: $B_a(\omega)$

$$\frac{B_a}{B_b} = \frac{k_b^2 R_b^2}{2 \ln k_b R_b - \frac{1}{2} k_b R_b \chi_0}$$

Mass density: $\rho_a(\omega)$

$$\frac{\rho_a}{\rho_b} = \frac{\chi_1}{k_b R_b}$$

h=R_b R_a=0.5R_b

$\rho_s = 2\rho_{\text{air}}$, $c_s = 0.3c_{\text{air}}$

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MtM with negative parameters: resonant behavior of the building units

Scatter Pressure Field

\[ B_\alpha(\omega)/B_b \]

- Monopolar Resonances

\[ \nu = 0.06u_r \]

Graciá-Salgado, Torrent and JSD, NJP 14, 103052 (2011)

\[ d_\alpha(\omega)/d_b \]

- Dipolar Resonances

\[ \nu = 0.12u_r \]

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Quasi-2D structure for double negative and $\rho \approx 0$ (DNZ) behavior

Scheme of the artificial structure

Building unit

Transversal section

$\rho_s = \frac{d_1 + d_2}{d_2} \rho_b = 2\rho_b$

Spioucas et al., APL (2011)

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Quasi-2D structure for double negative and DNZ behavior

**ω-L Phase diagram**

- $\rho_m < 0$
- $B_m < 0$

$L = 3.5 \ h; \ R_a = 0.5 \ R_b$

**ω-R\_h Phase diagram**

- $B_m < 0$
- $\rho_m < 0$

$L = 3.5 \ h$


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\[ \rho_m < 0 \approx 0; \quad B_m < 0 \]

\[ c_m = \sqrt{\frac{B_m}{\rho_m}} \rightarrow \infty \]

\[ n_m \approx 0 \]

\[ |Z_m|^2 = \rho_m B_m \approx \rho_b B_b = |Z_b|^2 \]

Transmission through narrow channels \( \lambda >> a \)

**EM counterpart:** Edwards *et al.*, PRL 100, 033903 (2008)

Liu *et al.*, PRL 100, 023903 (2008)
Applications of DNZ metamaterials: control of the radiation pattern

Scattering by a rigid cylinder + MtM slab (both embedded in a 2D waveguide)

Scattering at the frequency where the MtM behaves as a ρ-near-zero material

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Applications of DNZ metamaterials:

- Power splitter
- Perfect transmission through waveguides with sharp corners
Quasi-2D acoustic metamaterials: Practical realization

Sample A
$a=21\text{ mm}$
$R_b=9.2\text{mm}$
$h=9\text{mm}$
$L=3.5h$

Sample B
$a=21\text{ mm}$
$R_b=7\text{mm}$
$h=9\text{mm}$
$L=2.5h$

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Quasi-2D acoustic metamaterials: Practical realization

Experimental characterization

Double negative

$\rho_m < 0$

Sample A

Sample B

Double negative

Model

Acoustic metamaterials

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1. Introduction
2. Acoustic metamaterials with negative mass density and density near zero
3. Visco-thermal in acoustic metamaterials with double-negative parameters
4. Sound redirection and splitting with sonic crystals based on metamaterial units
**Viscous boundary layer**

\[ V_t = 0 \text{ at the boundary} \]

**Thermal boundary layer**

\[ \approx 2.5 \times 10^{-3} (f)^{-1/2} \text{ m} \]

**Viscous losses**

\[ \approx \text{constant temperature} \]

**Thermal losses**

\[ \approx \text{constant temperature} \]

---

**Table**

<table>
<thead>
<tr>
<th>f (Hz)</th>
<th>viscous layer (μm)</th>
<th>thermal layer (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>210</td>
<td>250</td>
</tr>
<tr>
<td>1000</td>
<td>66</td>
<td>79</td>
</tr>
<tr>
<td>10000</td>
<td>21</td>
<td>25</td>
</tr>
<tr>
<td>20000</td>
<td>15</td>
<td>16</td>
</tr>
</tbody>
</table>

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Linearized Navier-Stokes model:

- Conservation of mass:
  \[
  \frac{\partial \rho}{\partial t} + \rho \nabla \vec{v} = 0
  \]

- Conservation of energy:
  \[
  \rho_o T_o \frac{\partial s}{\partial t} = \lambda \Delta T
  \]

- Conservation of momentum (Navier-Stokes equation):
  \[
  \rho_o \frac{\partial \vec{v}}{\partial t} = -\nabla p + \left( \eta + \frac{4}{3} \mu \right) \nabla \Phi \vec{v} - \mu \nabla \times \nabla \times \vec{v}
  \]

- Thermodynamic equations:
  \[
  s = \frac{C_p}{T_o} \left( T - \frac{\gamma - 1}{\beta \gamma} p \right)
  \]

  \[
  \rho = \frac{\gamma}{c^2} \Phi - \beta T
  \]
Numerical methods for Visco-Thermal losses

Low reduced frequency model (LRFM):
- Neglects pressure variation and velocity component in the direction normal to the boundary. Restricted to some geometries

Finite Element Method (FEM) implementation:
- Direct implementation of the linearized Navier-Stokes equations. No restricting assumptions.

Boundary Element Method (BEM) implementation:
- Uses Kirchhoff’s decomposition of the linearized N-S equations. No restricting assumptions.

<table>
<thead>
<tr>
<th>Time/Load</th>
<th>SW package</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast/Low</td>
<td>ACTRAN, ANSYS, COMSOL</td>
</tr>
<tr>
<td>Slow/High</td>
<td>COMSOL</td>
</tr>
<tr>
<td>Slow/High</td>
<td>OpenBEM (non-commercial)</td>
</tr>
</tbody>
</table>


Metamaterial: transmission and reflection

Three-point method: Reflection and transmission coefficients:

\[ R(\omega) = \frac{p_2 e^{-i k x_2}}{p_1 e^{-i k x_2}} - \frac{p_1 e^{-i k x_2}}{p_2 e^{-i k x_1}} \]
\[ T(\omega) = \frac{p_3}{p_2} \frac{e^{-i k x_2} + R(\omega) e^{i k x_2}}{e^{-i k x_3}} e^{-i k x_2 - x_1} \]

- Transmittance (fraction of transmitted power): \(|T(\omega)|^2\)
- Reflectance (fraction of reflected power): \(|R(\omega)|^2\)
- Absorbance (fraction of absorbed power): \(1 - |T(\omega)|^2 - |R(\omega)|^2\)
Double-negative metamaterial

BEM: mesh with 4810 quadratic elements and 9616 nodes


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FEM versus BEM

![Graph comparing FEM and BEM transmittance vs frequency.](image-url)
Acoustic band structure (no losses)
Parameter extraction (with losses)

\[ B_m' < 0 \]

\[ \rho_m' < 0 \]

**Frequency (kHz)**

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Extracted from R and T, following the method of Fokin et al., PRB 77, 144302 (2007)
Absorption in the first passband increases with decreasing $v_g$.

In the DN band there is a huge reflectance and almost a 100% of the transmitted energy is absorbed.
No losses

With losses

\[ f_{FP} = 1675 \text{ Hz} \]
Double-negative metamaterial

Visco-thermal effects on the double-negative band

$f_{DN}=2380 \text{ Hz}$

No losses

With losses

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Scaling of the metamaterial

- As the size grows the frequency is scaled down: the mesh can be reused
- The behavior should be the same, except for viscous and thermal losses (as $f^{1/2}$):

$$
\delta_v \approx \sqrt{\frac{2v}{\rho_0 \omega}} \\
\delta_\kappa \approx \sqrt{\frac{2\kappa}{c_p \rho_0 \omega}}
$$

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Double-negative metamaterial

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Double-negative metamaterial


- Transmittance
- Reflectance
- Absorbance
Visco-thermal effects:

• Visco-thermal losses **should be considered** in order to obtain a realistic design of single- and double-negative metamaterials.

• The double-negative phenomena might be **suppressed by losses**.

• High loss is **persistent** at the double-negative band, even when the structure is **scaled up**.

• Double-negative metamaterials might be a **good alternative to conventional absorbers** for specific situations, e.g., when dealing low frequencies or when the excitation is narrow banded.

• Some properties of metamaterials may survive losses, with the proper design:
  – Use less rows of units.
  – Find resonators with less losses (e.g. with optimization)

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Sonic crystals / Phononic Crystals

Eusebio Sempere (Spanish artist)

Transmission properties of sonic crystals

\[ f = 0.4 \]

\( \omega(k) \)

Complete bandgap

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What is the minimum number of rows?

Multiple scattering simulations

3 rows
h=3 m

α=22 cm

d=16 cm

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Noise barriers made of porous materials (rubber crumb)

300 million used tires are removed annually in the 27 EU Member states

- Perforated shells with mm-size holes are (almost) acoustically transparent.
- They are used as containers of absorbing materials (rubber crumb, fiber glass, etc.).

Rubber crumb: 0-7mm
Objetive function: index of isolation for airborne noise

$$DL_R = -10 \log \left( \sum_{i=1}^{18} 10^{0.1L_i} 10^{-0.1R_i} \right)$$

\( (UNE-EN 1793) \)

\( L_i \) is the normalized spectrum of traffic noise (defined in 18 thirds of octave band between 100 Hz and 5kHz)

\( R_i \) is the transmission loss by the barrier

**Class B_1:** \( DL_R < 15 \text{ dB} \)

**Class B_2:** \( DL_R = 15 \text{ dB to } 24 \text{ dB} \)

**Class B_3:** \( DL_R > 24 \text{ dB} \)
Barriers for traffic noise based on rubber crumb

Table 1
Barrier parameters (see Fig. 2) obtained from the optimization algorithm. Length dimensions are in cm. Last row contains the airborne insulation index $DL_R$. Note that the highest quality barriers, class $B_3$, according to the European normative is achieved when $DL_R > 24$ dB [16].

<table>
<thead>
<tr>
<th></th>
<th>$T$</th>
<th>$T_{eff}$</th>
<th>$T'_{eff}$</th>
<th>$T''_{eff}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Box$</td>
<td>$\Delta$</td>
<td>$\Box$</td>
<td>$\Delta$</td>
</tr>
<tr>
<td>$r_1$</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>$r_2$</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>$r_3$</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>$r_{i1}$</td>
<td>4.6</td>
<td>10.0</td>
<td>3.4</td>
<td>5.1</td>
</tr>
<tr>
<td>$r_{i2}$</td>
<td>4.3</td>
<td>5.0</td>
<td>4.0</td>
<td>4.3</td>
</tr>
<tr>
<td>$r_{i3}$</td>
<td>4.7</td>
<td>9.5</td>
<td>4.5</td>
<td>10.0</td>
</tr>
<tr>
<td>$d_1$</td>
<td>32.1</td>
<td>18.2</td>
<td>31.2</td>
<td>18.2</td>
</tr>
<tr>
<td>$d_2$</td>
<td>48.9</td>
<td>18.2</td>
<td>49.8</td>
<td>18.2</td>
</tr>
<tr>
<td>$D$</td>
<td>40.0</td>
<td>21.0</td>
<td>40.0</td>
<td>21.0</td>
</tr>
<tr>
<td>$DL_R$ (dB)</td>
<td>7.2</td>
<td>18.6</td>
<td>6.7</td>
<td>16.6</td>
</tr>
</tbody>
</table>

Class $B_3$: $DL_R > 24$ dB
Noise barriers based on $\mu$-perforated shells

Properties of flat perforated panels

- Impedance of a flat perforated panel \cite{Ingard, Allard, Atalla, Åbom, etc.}:

$$Z_p = \frac{i\omega \rho_0 t}{\sigma} \left[ 1 - \frac{2}{s - i} \frac{J_1(s\sqrt{-i})}{J_1(s\sqrt{-i})} \right]^{-1} + \frac{4}{\sigma} \sqrt{2\eta_0 \omega \rho_0} + \frac{i\omega \rho_0}{\sigma} \frac{16r}{3\pi} \left( 1 - 2.5 \sqrt{\frac{\sigma}{\pi}} \right)$$

For large holes $r \gg \delta = \sqrt{\frac{2\eta_0}{\rho_0 \omega}}$

and moderate filling fractions $\sigma$, the panel has low $Z_p$.

Small holes ($r \approx 1 \mu$m) lead to absorbing panels.

Perforated cylindrical shells
multiple scattering approach

The T-matrix of a perforated shell is obtained from an impedance based model:

\[ T_q = -\frac{\rho_q J'_q \left( t_0 R^+ \right) - J_q \left( t_0 R^+ \right)}{\rho_q H'_q \left( t_0 R^+ \right) - H_q \left( t_0 R^+ \right)} \]

\[ \rho_q = \frac{J_q \left( t_0 R^- \right)}{J'_q \left( t_0 R^- \right)} \frac{iZ_p k_0}{\omega \rho_0} \]

Transmission through a lattice of perforated shells has been calculated using Multiple Scattering Theory.

García-Chocano, Cabrera and JSD., APL 101, 184101 (2012)
μ-perforated shells are interesting due to the absorptive properties of μ-perforations.

3 rows of 30 cylinders (R=16cm, h=3m) and a=22cm.
Acoustic barriers based on lattices of $\mu$-perforated shells

PROS:

• No foundations are needed
• The flow of wind passes through the barrier
• Lightweight and robust
• Great aesthetic

CONS:

• Expensive
(The cost can be substantially reduced by using massive manufacturing methods)

2011

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Redirection of sound with a sonic crystal made of perforated thin shells
Fano-like resonance phenomena by flexural shell modes in sound transmission through two-dimensional periodic arrays of thin-walled hollow cylinders

Yuriy A. Kosevich,¹,* Cecile Goffaux,²,† and Jose Sánchez-Dehesa¹,‡

\[ \nu_n = \frac{n(n^2 - 1)}{d \sqrt{1 + n^2}} \frac{C_{ls}}{r^2 4\pi}, \]

4 layers Sonic Cristal

\[ a = 11 \text{ cm} \]

Thickness

- d = 1.2 mm
- d = 1.8 mm
- d = 1.9 mm
- d = 2.0 mm
if $\lambda$ is on the order of the spacing, the scattered quadrupoles are in phase and coherently redirect acoustic energy by 90 degrees.
Absorption enhancement occurs at the frequencies with minimum transmittance.

\[ A = 1 - R - T \]

- Reflectance
- Transmittance
- Absorption

\[ R = 4\text{cm} \]
\[ r = 0.25\text{mm} \]
\[ t = 0.50\text{mm} \]
\[ \sigma = 14.5\% \]
\[ a = 11\text{cm} \]

\[ \lambda \approx a \]

3 rows of cylindrical shells

Numerical simulations (\( \eta_0 \neq 0 \))

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Non-dissipative case ($\eta_0=0$)

- Full transmission occurs except at around 3kHz.
- Total reflection is found at $\lambda \approx a$ even for one row.
- T minima show Fano-like profiles indicating the excitation of resonant Wood anomalies.

Frequency

- 3080+i4
- 3042+i14
- 3105+i5
- 3034+i21
- 3059+i1
- 3103+i2
Energy redirection due to Wood anomalies

- The resonant anomaly involves modes guided along the slab.
- Propagating modes are observed when exciting the slab with a Gaussian beam.

Pressure at the far field (a. u.)

3037 Hz

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Energy splitting with a linear chain of perforated thin shells
The case of a linear chain

**Incident** plane wave:

\[ p(r) = p_0 \exp(ik \cdot r) \]

**Acoustic field inside each shell:**

\[ p_{in}(r_l, \varphi_l) = \sum_{n=-\infty}^{\infty} C_{ln} J_n(kr_l) e^{in\varphi_l} \]

**Scattered** acoustic field:

\[ p_{sc}(r, \varphi) = \sum_{l'} \sum_{n=-\infty}^{+\infty} B_{l'n} H_n(kr_{l'}) e^{in\varphi_{l'}} \]

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Boundary conditions

The **impedance approach** is used to match the acoustic fields *inside* and *outside* each individual microperforated shell.

\[ v_r|_{r=a} = v_r|_{r=b} = \frac{p|_{r=b} - p|_{r=a}}{Z_p} \]

**continuity of normal velocity**

**normal velocity is due to the discontinuous jump of pressure**

Effective acoustic impedance of the flat plate:

\[ Z_p = -\frac{i\omega \rho_0}{\sigma} \left[ h + \frac{16s}{3\pi} \left( 1 - 2.5 \sqrt{\frac{\sigma}{\pi}} \right) \right] \]


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Acoustic band structure: $\omega(q)$

After some algebra...

\[ \det \left| S_n \delta_{nn'} + F(n' - n) \right| = 0. \]

$n = 0, \pm 1, \pm 2, \ldots$

The eigenmode is excited if asymmetric bandgap eigenmodes are leaky (even w/o viscosity), so they can be excited.

infinite chain

\[ k_y = (2\pi f/c) \sin \theta = q(f) \]

\(\theta = 0\) symmetric

\(\theta = 5\)

asymmetric bandgap

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Eigenmode excitation

\[ f_1 = 2625 \text{ Hz} \]

\[ f_2 = 3715 \text{ Hz} \]

“normal” mode

“anomalous” mode

\[ \theta = 10 \]

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Splitting of a bi-frequency signal

25 perforated shells

inviscid air
8% (low f. component)
10% (high f. component)

viscous air
5% (low f. component)
7% (high f. component)

Mixture of the two sound waves

Low frequency component (in red) propagates against the natural direction!!

Splitting of a bi-frequency signal

Numerical experiments (FEM simulations)

\[ f_1 = 2520 \text{ Hz} \quad \text{METAgenierie2017, 2-7 July.} \]

\[ f_1 = 3520 \text{ Hz} \]
Thanks for your attention!