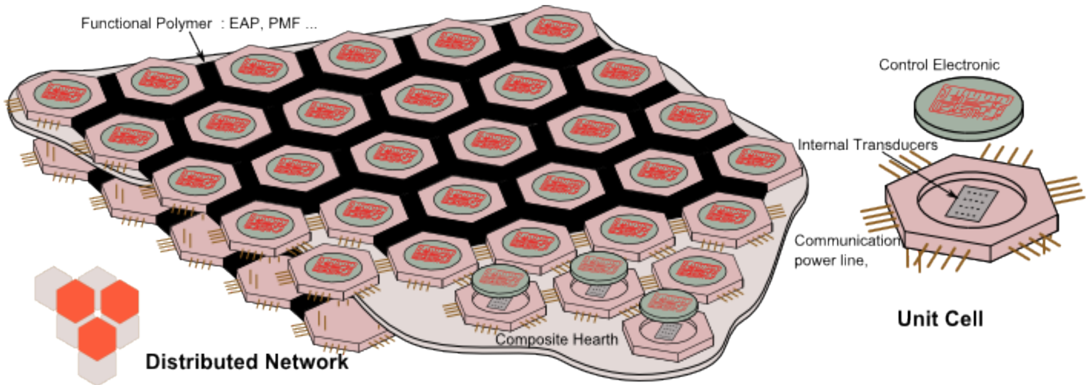
Laboratoire de Tribologie et Dynamique des Systèmes

Les Métamatériaux Mécaniques : Architecturer pour fonctionnaliser…



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cnrs · ÉCOLE CENTRALE LYON · ENISE · ENTPE · Membres de Université de Lyon

Context and Issues

« Green » technologies– structural weight reduction

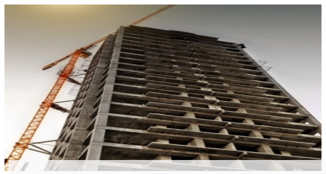
(decrease CO_2 emission (5-15%), noise control....)

- Intensified dynamical environnement
- Fatigue and damage : security
- Stability problem
- Adapted design methodologies

FR & EC research strategies, Clean Sky , DREAM EU Project s, AIAA's Emerging Technologies Committee (ETC) …

Transports

Nuclear



Civil Engineering

Aerospace

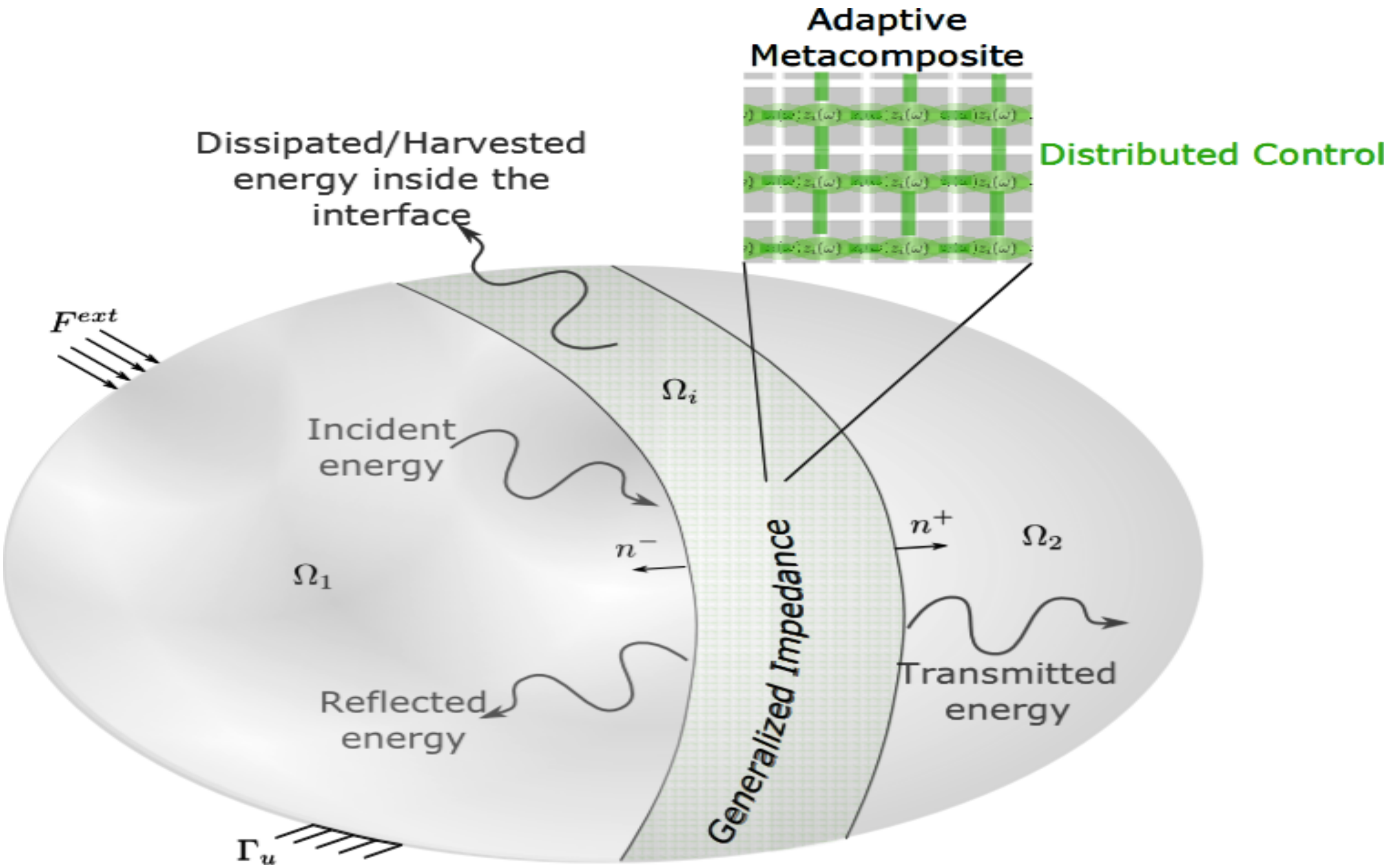
New Integrated functionalities

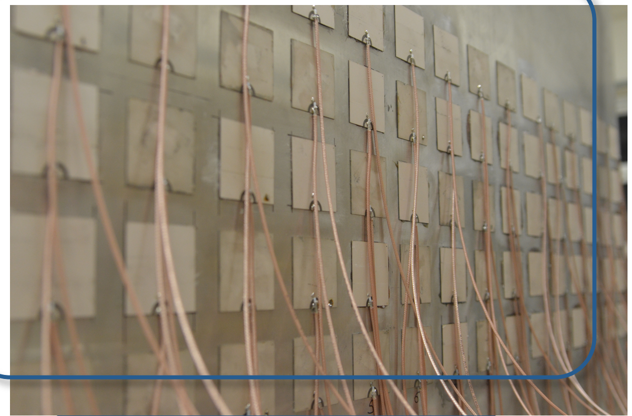
- **Vibration Control**
- **Noise Control**
- **Structural Health Monitoring -SHM-**
- **NDE, PHM**
- Waves trapping
- And more….

Meyer et al.: Advanced Microsystems for Automotive Applications 2009 - Smart Systems for Safety, Sustainability and Comfort, Springer 2009

Material Architecture for Structural Optimization

Synthesis of **generalized Impedance operator** using **distributed** (low cost, low energy) **individual** (communicating) **cells**



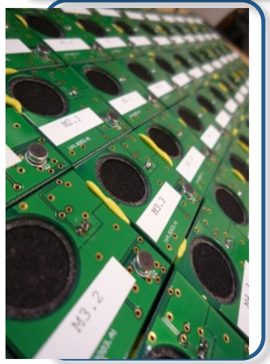


Metacomposites:

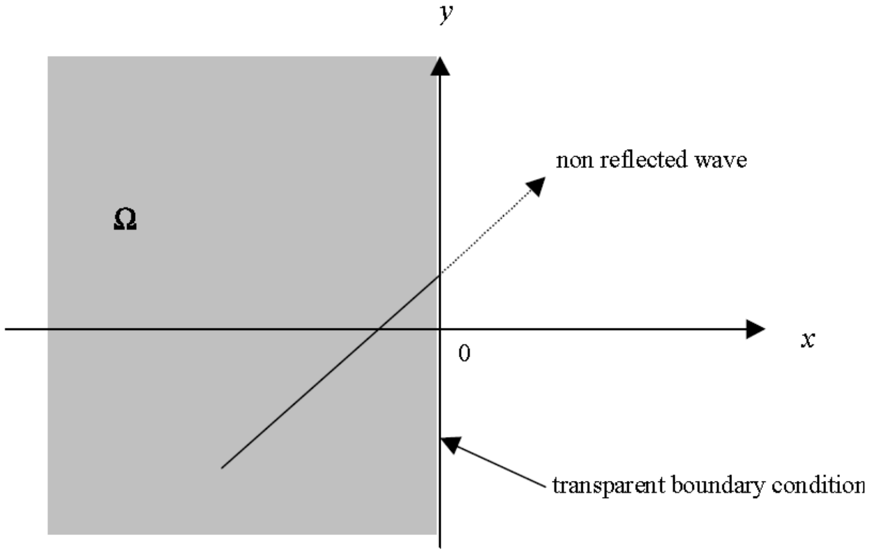
Synthesis of functional constitutive laws inside **hybrid composite material** by using **distributed sets of smart cells**

Scale of interest:

mm -> few cm



Example :Generalized Impedance operator for acoustics



$$\begin{cases} \partial_t^2\theta - \Delta\theta = 0 \text{ on } \Omega_\theta \times \mathbb{R}_t^{+*} \\ \theta(0,y,t) := v(y,t) \\ w(y,t) := \partial_x\theta(0,y,t), \end{cases}$$

Acoustic System

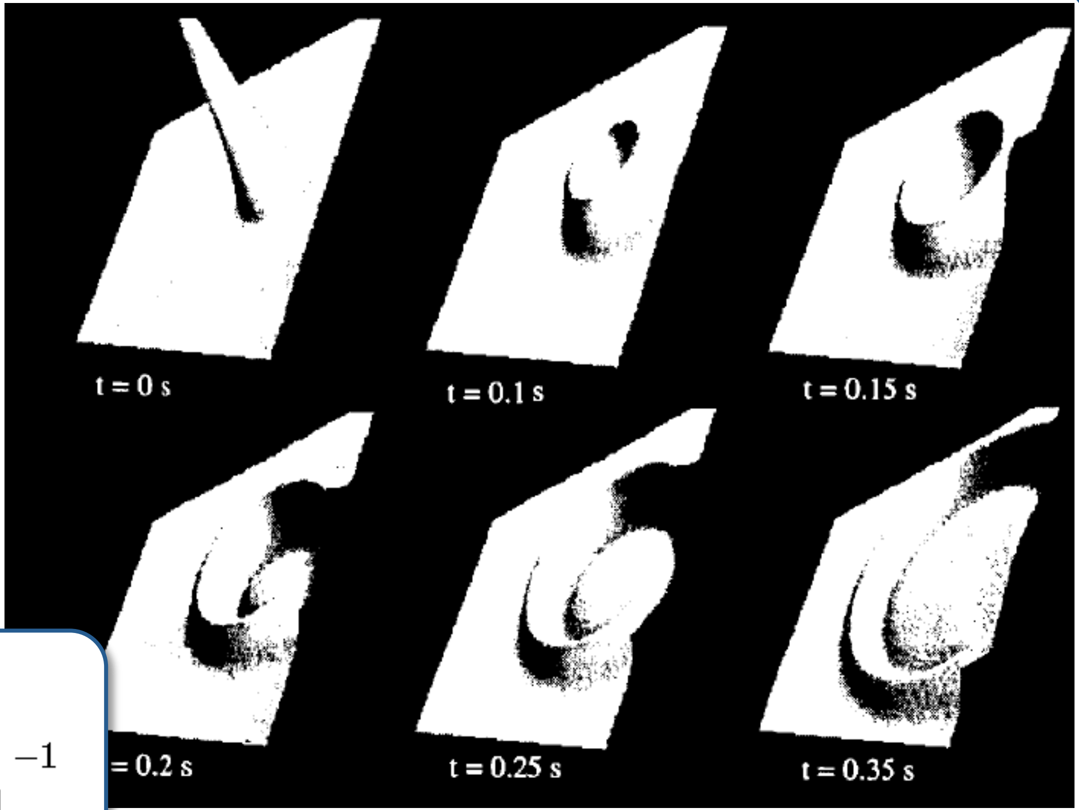
$$v := \mathcal{K} w$$

$$\mathcal{K} = -\left(\sqrt{\partial_t^2 - \partial_y^2}\right)^{-1}$$

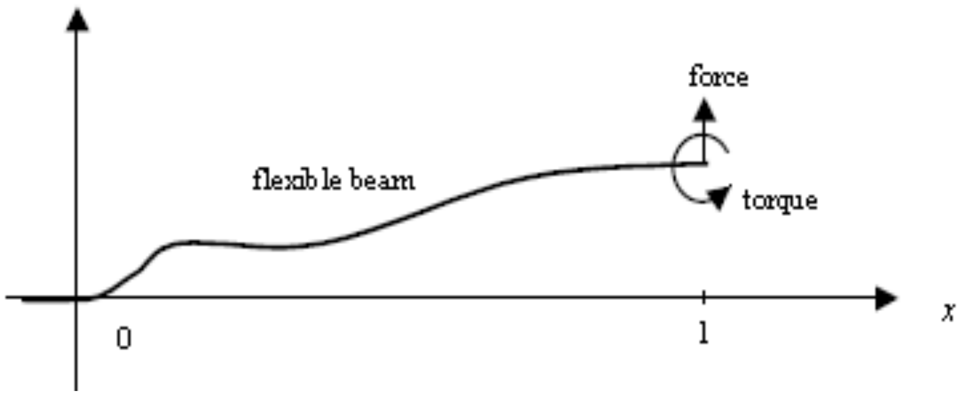
Controlled System :

Pseudo derivative operator

G. Monseny, LAAS report, Toulouse, 2002



Example : Generalized Impedance operator for beams



G. Monseny, LAAS report, Toulouse, 2002

$$\begin{cases} \partial_t^2\theta + \partial_x^4\theta = 0 \\ \theta(x,0) = \theta_0(x) \\ \partial_t\theta(x,0) = \theta_1(x) \\ \theta(0,t) = 0 \\ \partial_x\theta(0,t) = 0 \\ \partial_x^2\theta(1,t) = u(t) \\ \partial_x^3\theta(1,t) = v(t), \end{cases}$$

Mechanical System

$$\begin{cases} \partial_t^2\theta + \partial_x^4\theta = 0, \quad x \in (0,1) \\ \partial_x^2\theta(1,t) = -\partial_t\theta(1,t) - \sqrt{2}\partial_t^{1/2}\partial_x\theta(1,t) \\ \partial_x^3\theta(1,t) = \sqrt{2}\partial_t^{3/2}\theta(1,t) + \partial_t\partial_x\theta(1,t), \end{cases}$$

Controlled Mechanical System :

Pseudo derivative operator

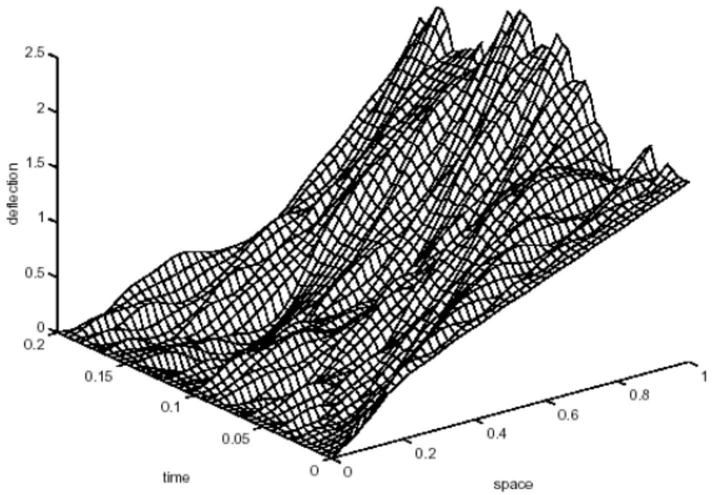


Figure 2: Autonomous beam

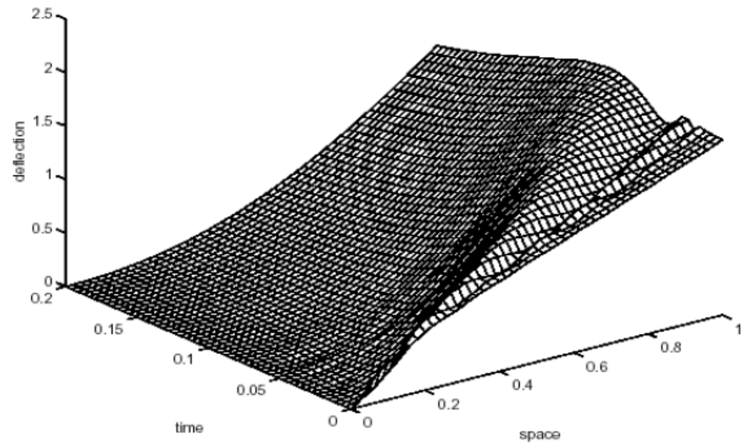
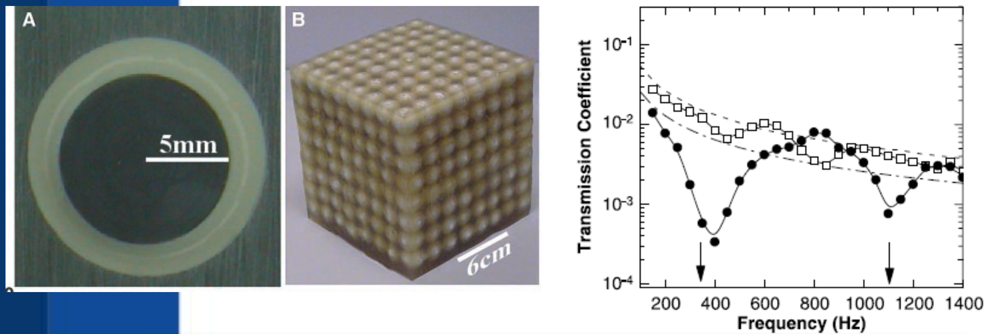
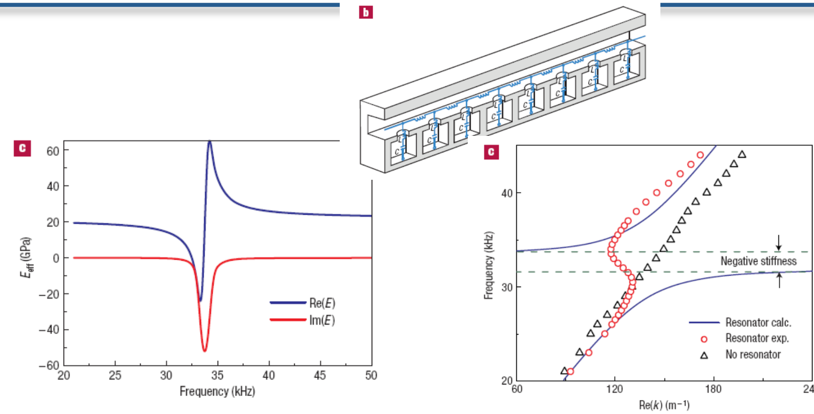


Figure 3: Beam with feedback Z_0

Material Architecture for Structural Optimization

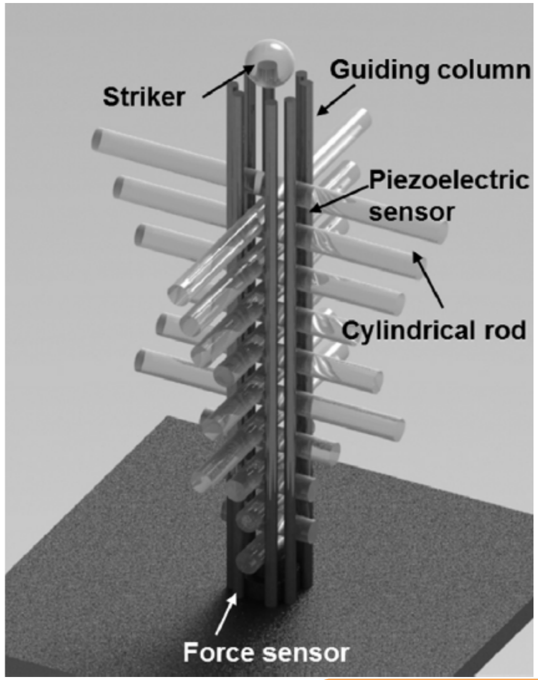


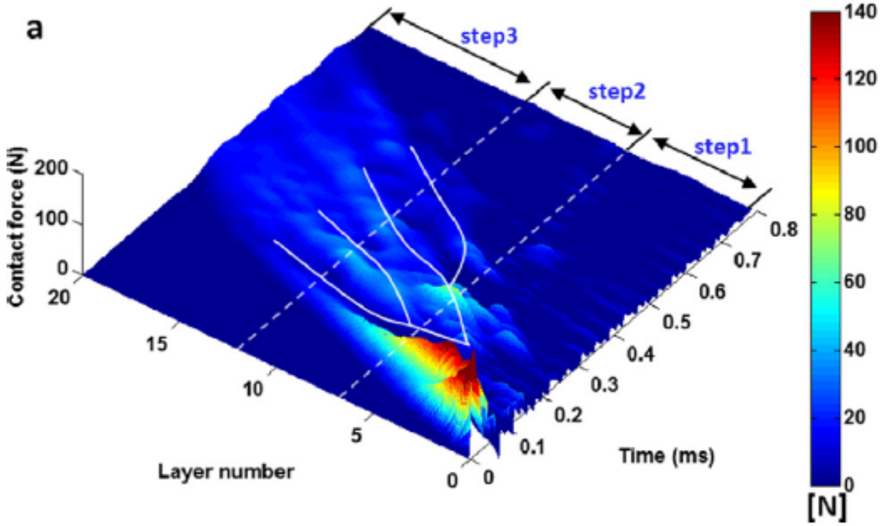
Liu Z. et al. "Locally Resonant Sonic Materials" Science, Vol. 289, September 2000, pp. 1734-1736.



N. Fang et al., "Ultrasonic Metamaterials With Negative Modulus" *Nature Materials*, Vol. 5, June 2006, pp. 452-456

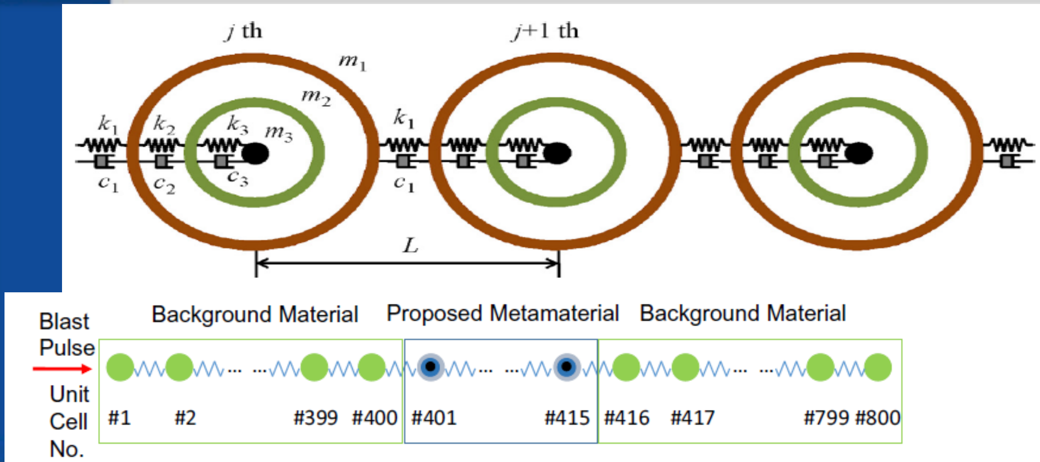
Classical Sonic Métamatérials





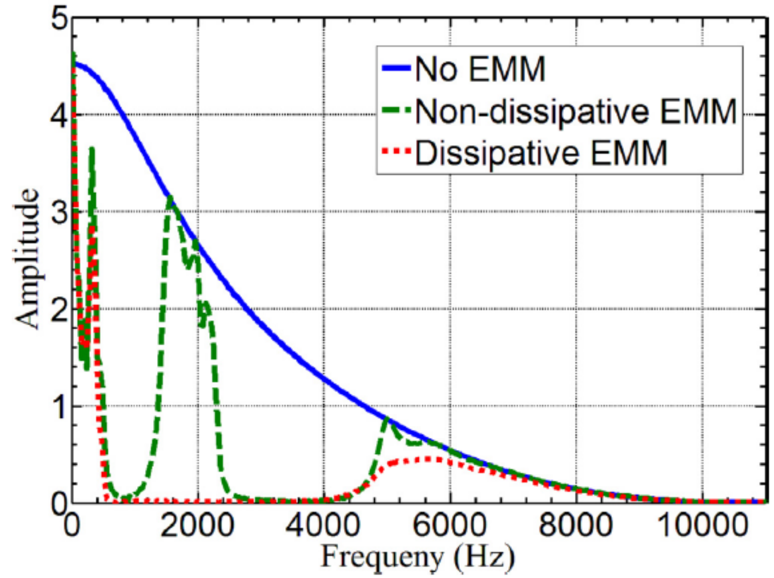
Impact and blast mitigation using locally resonant woodpile metamaterials
Eunho Kim, Jinkyu Yang, HuiYun Hwang, Chang Won Shul

Material Architecture for Structural Optimization

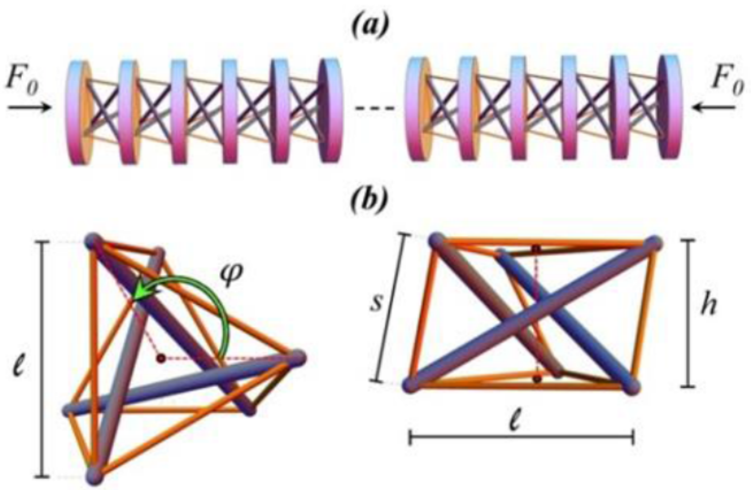


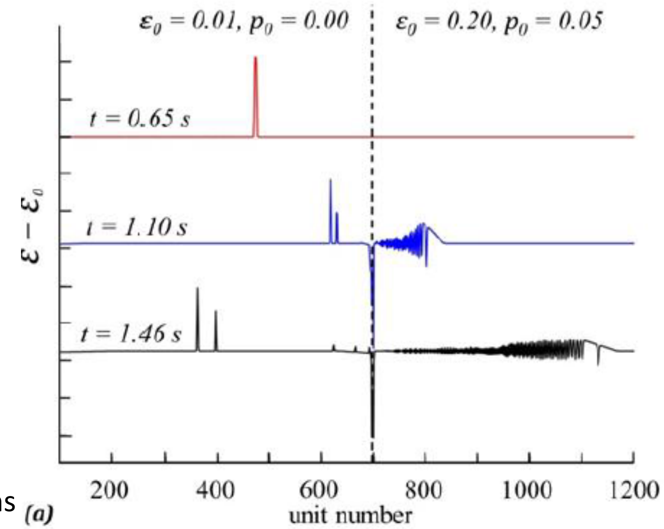
Dissipative elastic metamaterials for broadband wave mitigation at subwavelength scale

Y.Y. Chen [a], M.V. Barnhart [a], J.K. Chen [a], G.K. Hu [b], C.T. Sun [c], G.L. Huang [a,*]



Classical Sonic Métamatérials



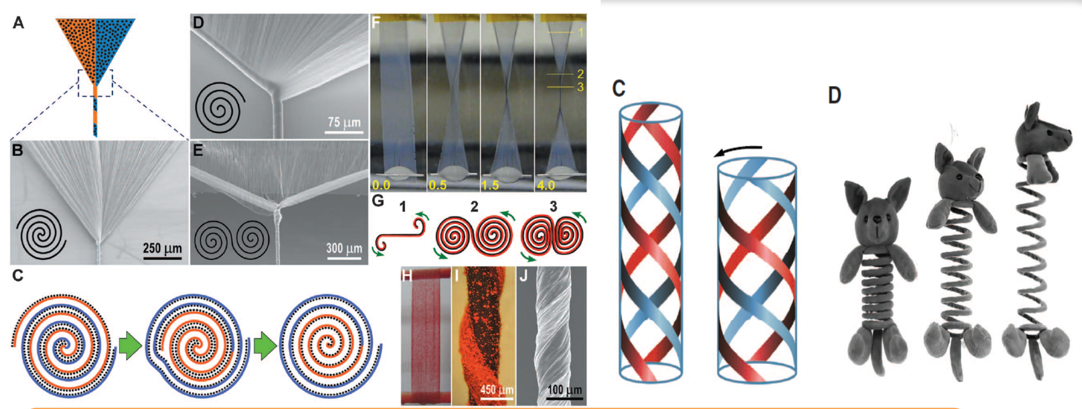


Strongly nonlinear discrete systems with unprecedented tenability: chain of tensegrity prisms and lumped masses

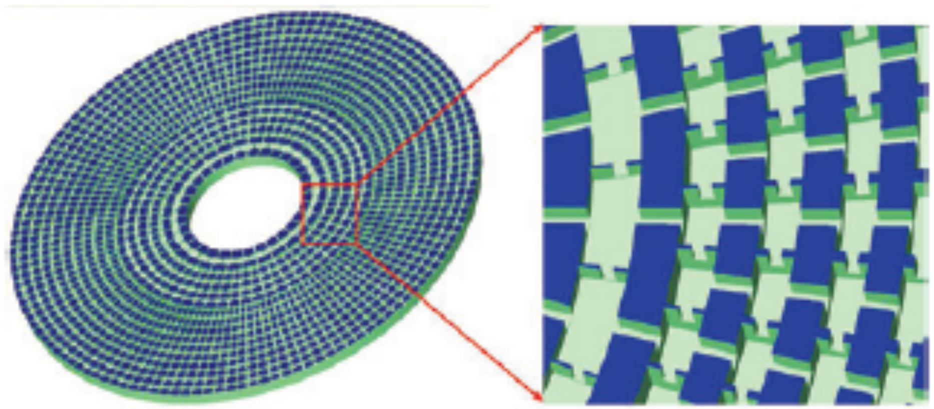
F. Fraternali, G. Carpentieri, A. Amendola, R. E. Skelton, and V. F. Nesterenko, Multiscale tunability of solitary wave dynamics in tensegrity metamaterials, arXiv 1409.7097

Material Architecture for Structural Optimization

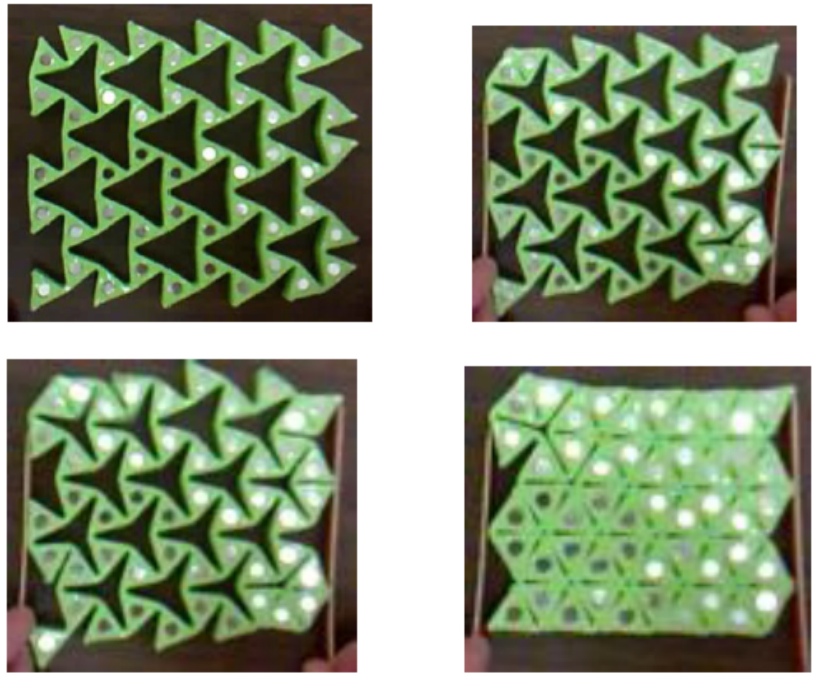
A sequence of images demonstrating the self-folding of a 4D printed multi-material single strand into the world of MIT



M.D Limaet al Biscrolling Nanotube Sheets and Functional Guests into Yarns, Science 331, 51, 2011



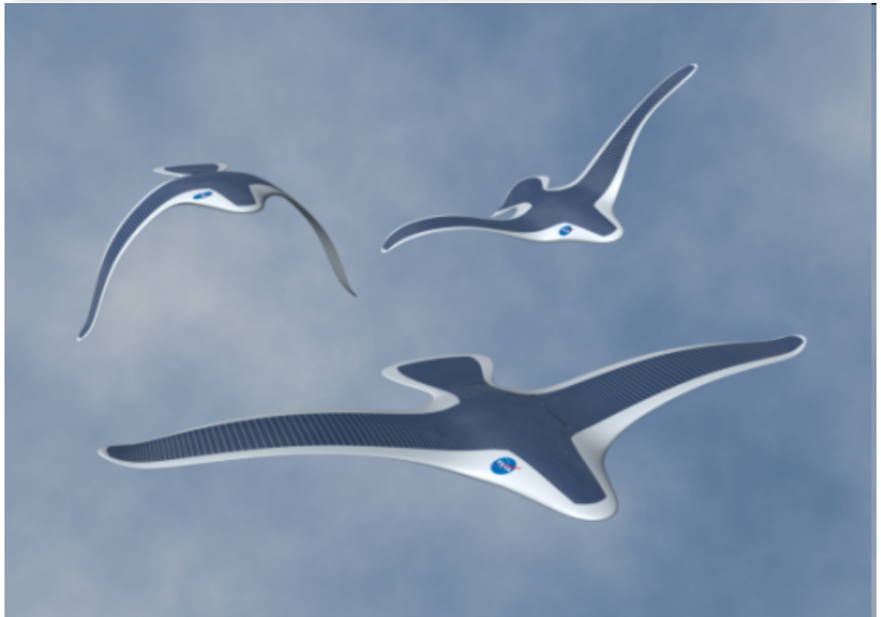
S. Zhang, C. Xia and N. Fang,
“Broadband acoustic cloak for ultrasound waves”, PRL, 106,2,2 4301, 2011



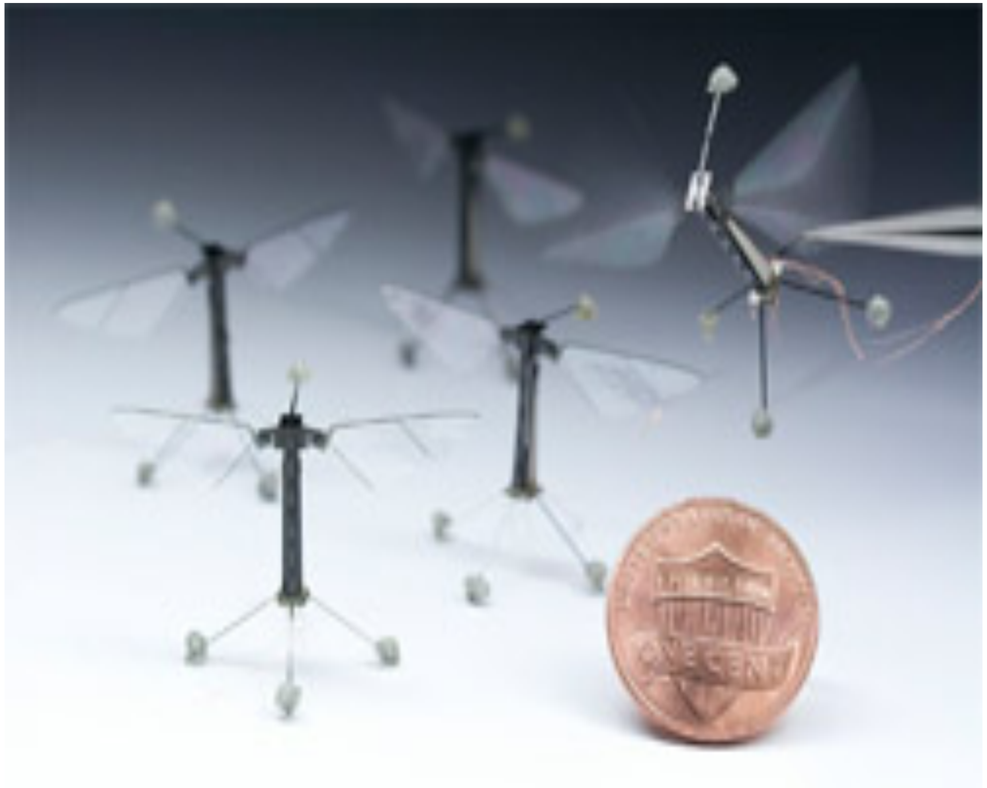
M. D. Schaeffer; M. Ruzzene, “Wave propagation in 2D magneto-elastic kagome lattices”, Proc. SPIE 9064, SMS 2014

Material Architecture for Structural Optimization

Research in Programmable Matter Directed by Carnegie Mellon and Intel



Morpheus Lab Univ Maryland

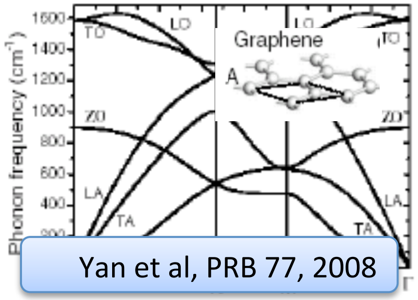


Harvard Microrobotics Lab

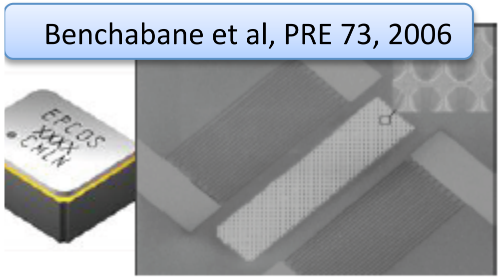
Physics of periodic structures

Structure	Physical properties	Waves support	Gap
Crystalline solids	Periodic arrangement of atoms ~ 5 Å	Electrons (Ψ) **Schrödinger** eq.	Absence of **electron** states
Photonic crystal	Periodic modulation of ε, μ (macro scale)	EM (E,B) **Maxwell** eqs.	Absence of states of the **EM** field
Phononic crystal	Periodic modulation of ρ, E, ν (macro scale)	Elastic (u) **Elasticity** eqs.	Absence of states of the **elastic** field

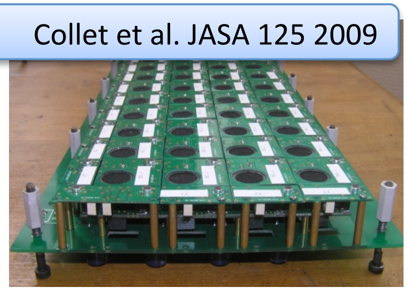
nm → µm → mm → cm → m

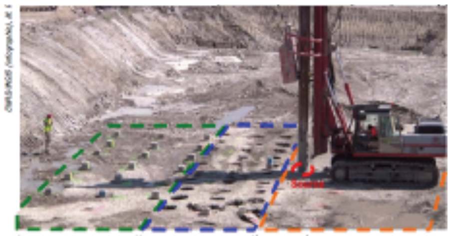


Yan et al, PRB 77, 2008


Benchabane et al, PRE 73, 2006


NDT, NDE

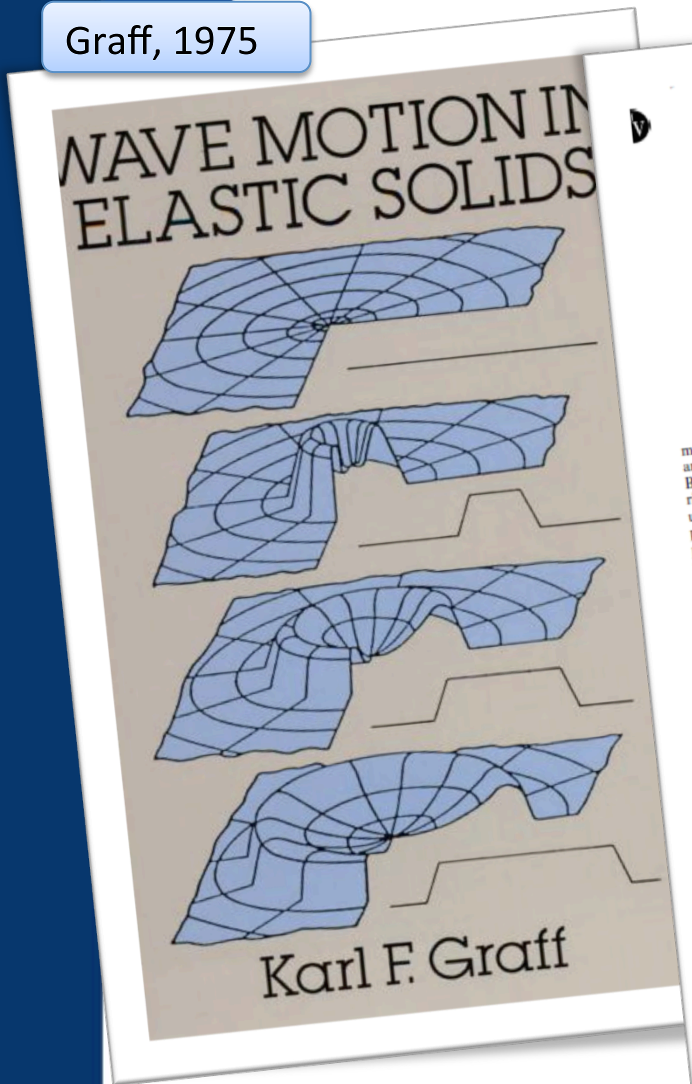

Collet et al. JASA 125 2009


Brûlé et al. PRL 112, 2014

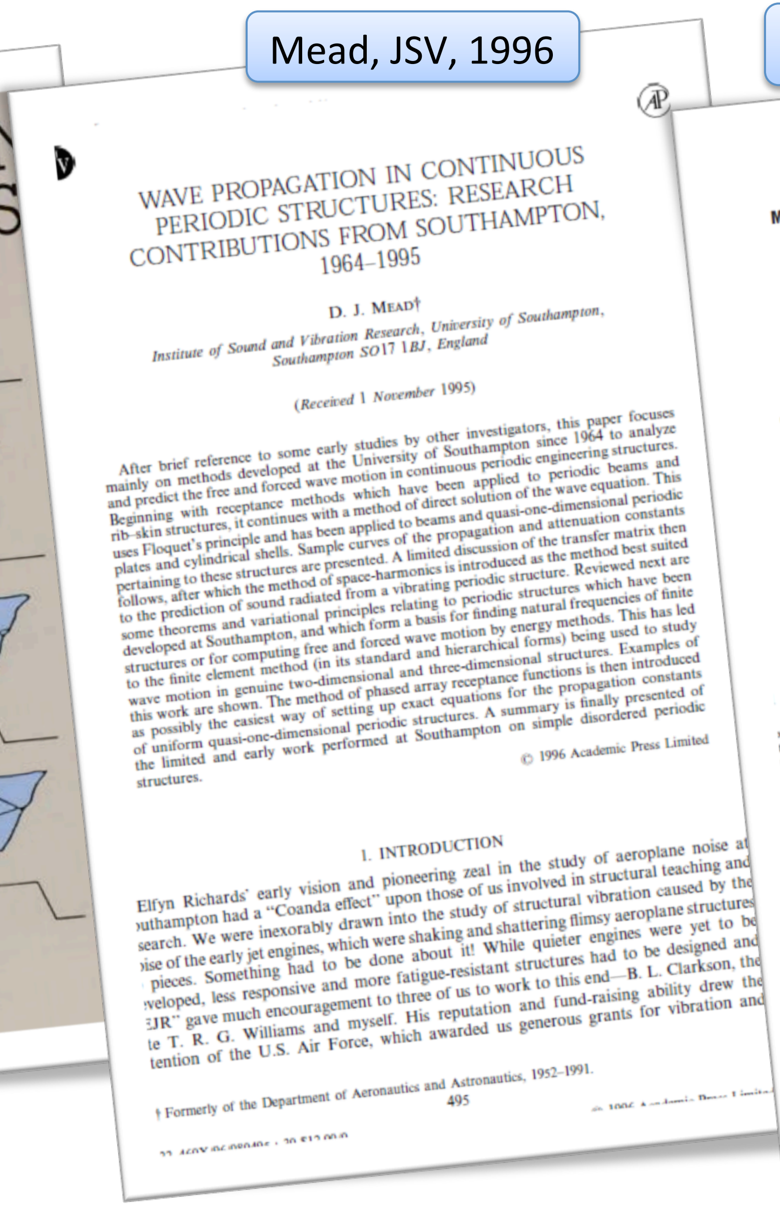
THz ← GHz ← MHz ← kHz ← Hz

An arbitrary choice of 3 top-level references

Graff, 1975



Mead, JSV, 1996



WAVE PROPAGATION IN CONTINUOUS PERIODIC STRUCTURES: RESEARCH CONTRIBUTIONS FROM SOUTHAMPTON, 1964–1995

D. J. Mead†

Institute of Sound and Vibration Research, University of Southampton, Southampton SO17 1BJ, England

(*Received* 1 *November* 1995)

After brief reference to some early studies by other investigators, this paper focuses mainly on methods developed at the University of Southampton since 1964 to analyze and predict the free and forced wave motion in continuous periodic engineering structures. Beginning with receptance methods which have been applied to periodic beams and rib-skin structures, it continues with a method of direct solution of the wave equation. This uses Floquet's principle and has been applied to beams and quasi-one-dimensional periodic plates and cylindrical shells. Sample curves of the propagation and attenuation constants pertaining to these structures are presented. A limited discussion of the transfer matrix then follows, after which the method of space-harmonics is introduced as the method best suited to the prediction of sound radiated from a vibrating periodic structure. Reviewed next are some theorems and variational principles relating to periodic structures which have been developed at Southampton, and which form a basis for finding natural frequencies of finite structures or for computing free and forced wave motion by energy methods. This has led to the finite element method (in its standard and hierarchical forms) being used to study wave motion in genuine two-dimensional and three-dimensional structures. Examples of this work are shown. The method of phased array receptance functions is then introduced as possibly the easiest way of setting up exact equations for the propagation constants of uniform quasi-one-dimensional periodic structures. A summary is finally presented of the limited and early work performed at Southampton on simple disordered periodic structures.

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1. INTRODUCTION

† Formerly of the Department of Aeronautics and Astronautics, 1952–1991.

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Hussein, Learny, Ruzzene, ASME Applied Mechanics Review, 2014

Dynamics of Phononic Materials and Structures: Historical Origins, Recent Progress, and Future Outlook

Mahmoud I. Hussein

Michael J. Leamy

Massimo Ruzzene[1]

Introduction

Course Outlines

1. Wave propagation – back to basics

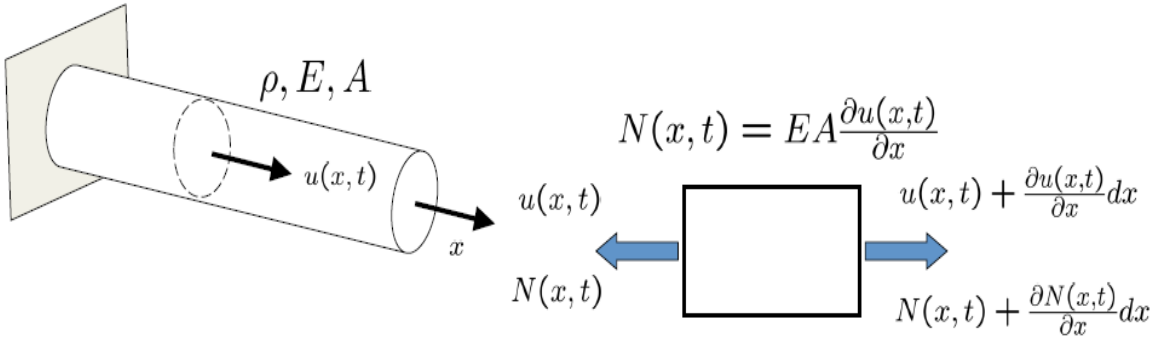
2. Modeling aspects
   a) Wave Finite Elements WFE
   b) Shift cell operator for multiphysic coupled metamaterial
   c) Plane Wave Expansion PWE

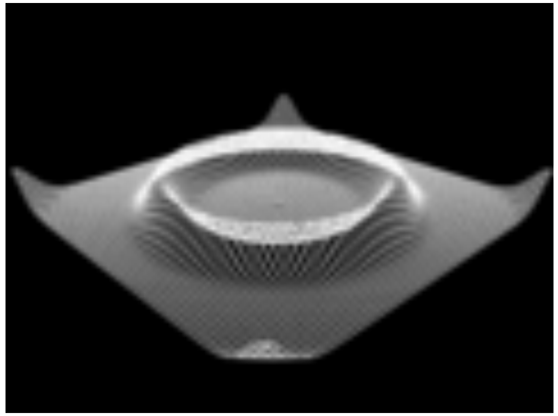
3. Design functional structures using band gap properties
   a) Waves diffusion : Reflection and absorption
   b) The boundary : a limit for the band gap efficiency

4. Beyond the Band –Gap
   a) Lensing
   b) Reciprocity breaking and diode
   c) …

Wave propagation – back to basics

1D Rod wave :





Wave Equation :

$$E\frac{\partial^2 u(x,t)}{\partial x^2} = \rho\frac{\partial^2 u(x,t)}{\partial t^2}$$

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2}$$

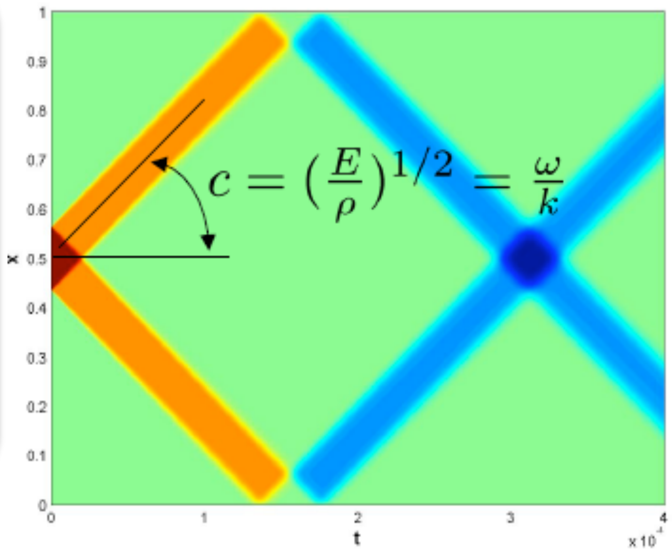
Wave velocity :

$$c = \left(\frac{E}{\rho}\right)^{1/2}$$

Initial Value:

$$u(x,0) = U(x)$$
$$\dot{u}(x,0) = 0$$

$$u(x,t) = \frac{1}{2}[U(x-ct) + U(x+ct)]$$



From M Ruzzene Courses

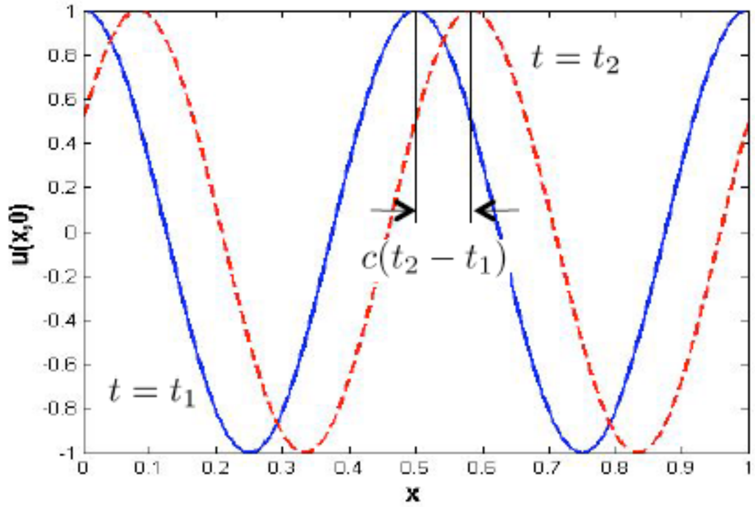
Wave propagation – back to basics

If initial condition : $u(x,0) = U_0 \cos(kx)$

k : wavenumber - $\lambda = \frac{2\pi}{k}$: wavelength

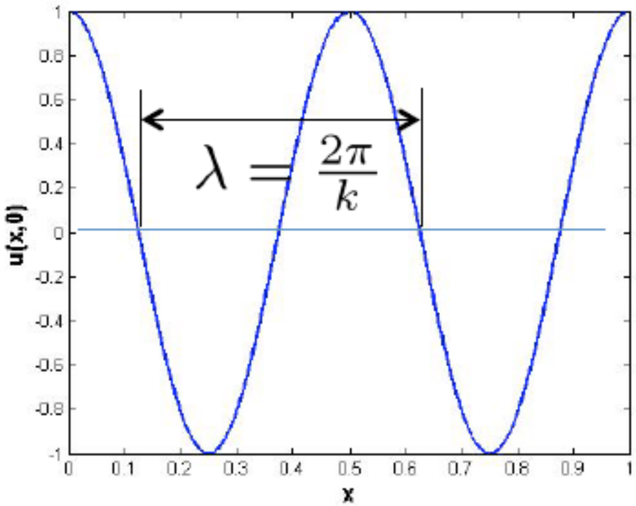
Solution is :

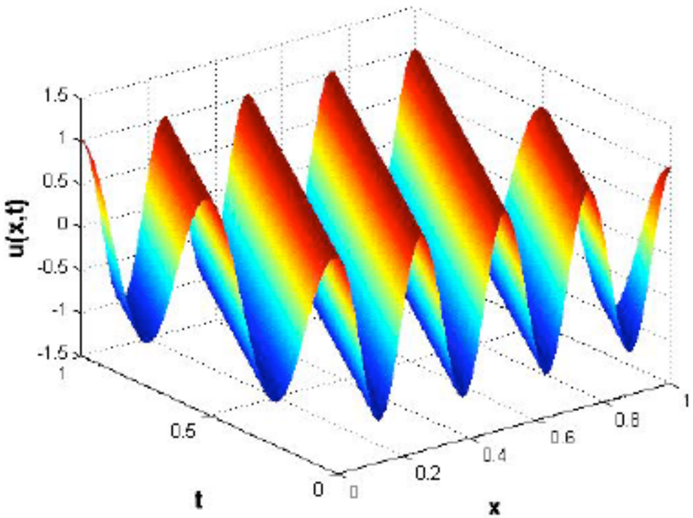
$$u(x,t) = \frac{U_0}{2}[\cos(kx - \omega t) + \cos(kx + \omega t)]$$



From M Ruzzene Courses

Dispersion relation: $c = \frac{\omega}{k} = (\frac{E}{\rho})^{1/2}$





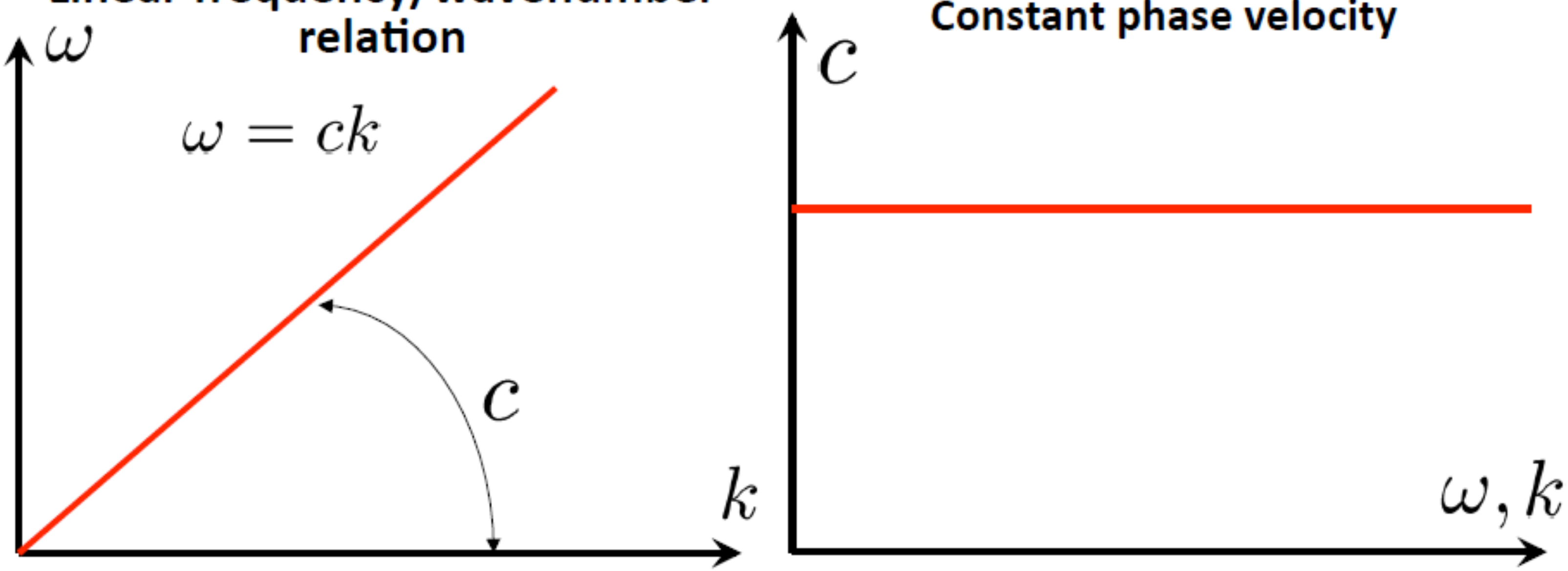
Dispersion relation: $\omega = \omega(k)$

Representation of the physical PDE operator

Non dispersive Media :

Linear frequency/wavenumber relation

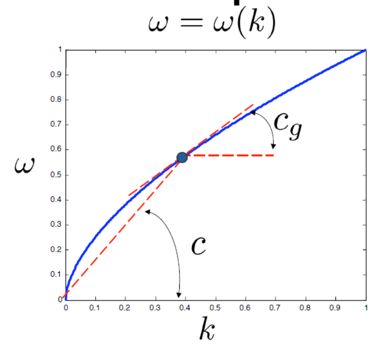
Constant phase velocity



Let's consider 2 waves, of equal amplitude and at two frequencies :

$$u(x,t) = U_0[\sin(k_1 x - \omega_1 t) + \sin(k_2 x - \omega_2 t)]$$

$$u(x,t) = 2U_0 \underbrace{\cos(\Delta k x - \Delta\omega t)}_{\text{Modulation}} \underbrace{\sin(k_0 x - \omega_0 t)}_{\text{Carrier wave}}$$



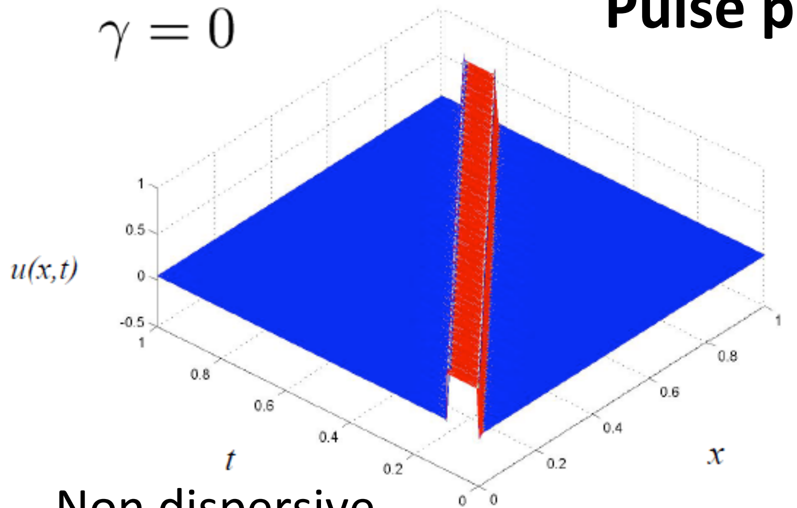
Speed of wave packet velocity : $x = \frac{\Delta\omega}{\Delta k} t + Const$ $c_g = \frac{\Delta\omega}{\Delta k}$

At the limit one obtains the Group Velocity : $c_g = \frac{d\omega}{dk}$

From M Ruzzene Courses

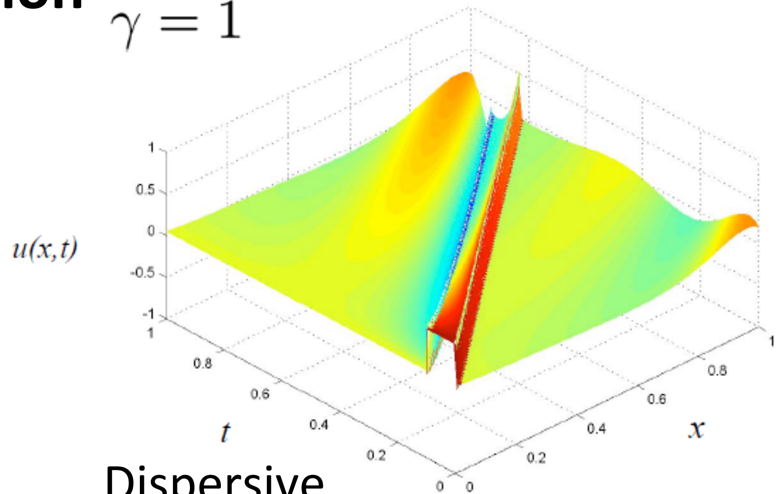
Pulse propagation

$\gamma = 0$



Non dispersive

$\gamma = 1$



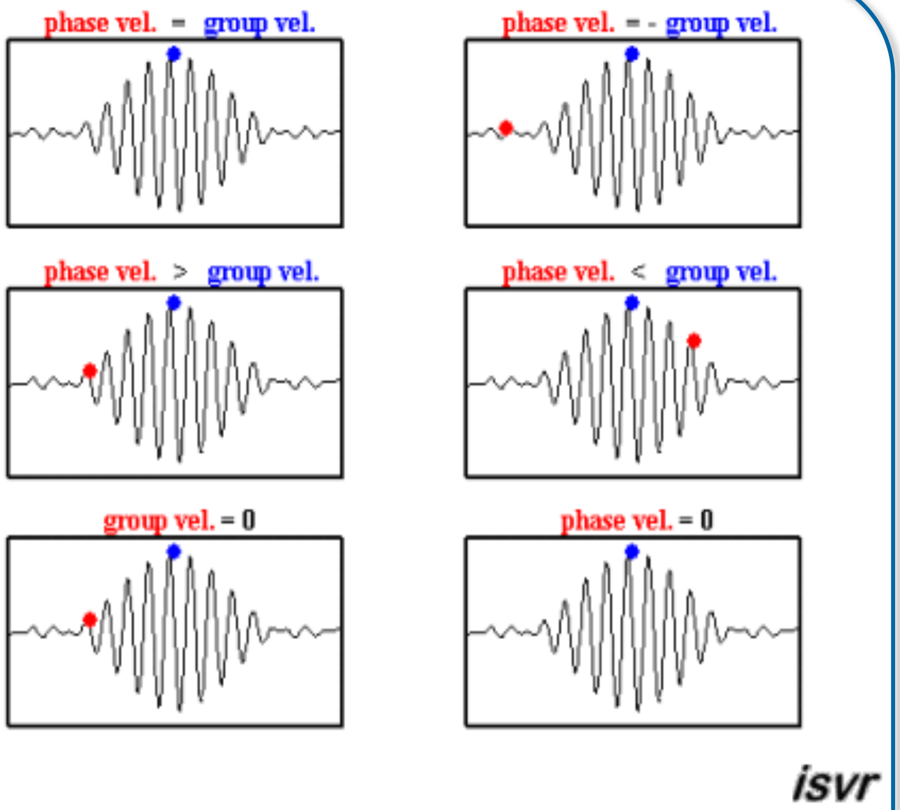
Dispersive

- Phase velocity
  - speed of the individual waves.: $c_\phi = \omega/k$
- Group velocity
  - speed of the wavepacket: $c_g = \partial\omega/\partial k$


Non-dispersive wave $c_\phi = c_g$


Dispersive wave $c_\phi \neq c_g$

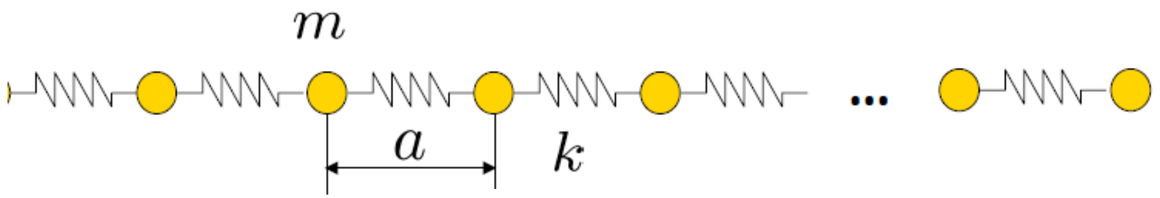
(undamped systems…)

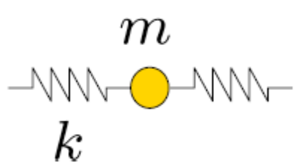


Taken from http://resource.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial_files/Web-further-dispersive.htm

Wave propagation – back to basics

Let's consider a monoatomic lattice :





$$m\ddot{u}_n + 2ku_n - k(u_{n+1} + u_{n-1}) = 0$$

$$u(x_n, t) = u_n(t) = u_n(\omega)e^{-j\omega t}$$

$$u_n(\omega) = u_0[\kappa(\omega)]e^{j\kappa x_n}$$

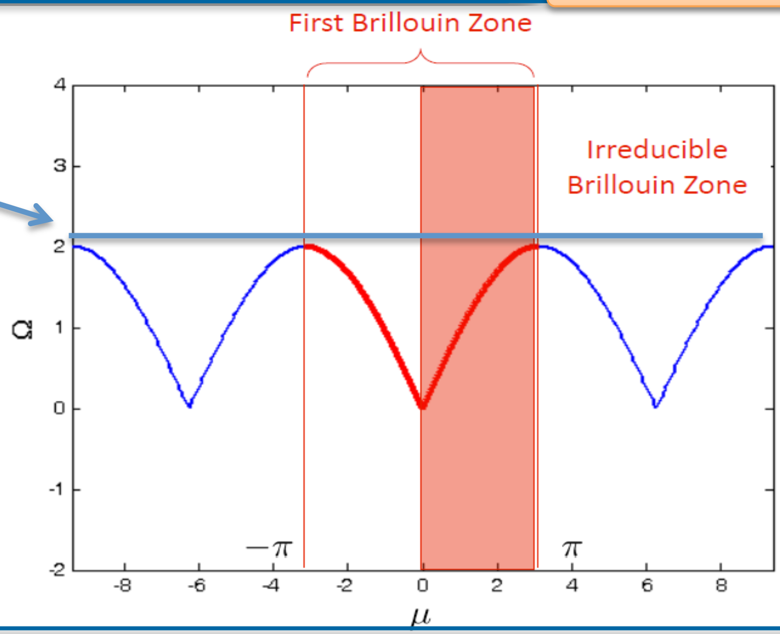
$$u_n(\omega) = u_0[\mu(\omega)]e^{j\mu n}$$

From M Ruzzene Courses

Dispersion – Bragg Scattering:

$$-\omega^2 m + 2k(1 - \cos\mu) = 0 \qquad \omega_0^2 = \frac{k}{m}$$

$$\Omega^2 = 2(1 - \cos\mu) \qquad \Omega = \frac{\omega}{\omega_0}$$

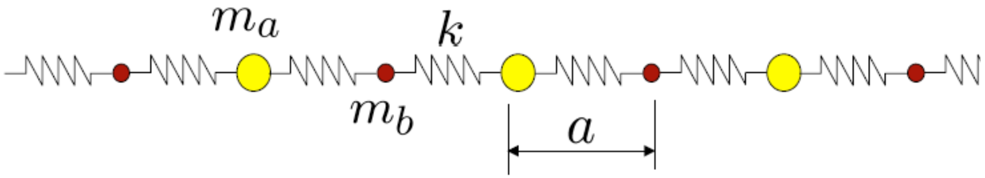


Wave propagation – back to basics

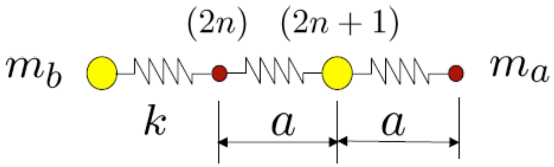
Let's consider a biatomic lattice :

$$-\omega^2 m_a u_{2n} + 2k u_{2n} - k(u_{2n+1} + u_{2n-1}) = 0$$

$$-\omega^2 m_b u_{2n+1} + 2k u_{2n+1} - k(u_{2n+2} + u_{2n}) = 0$$

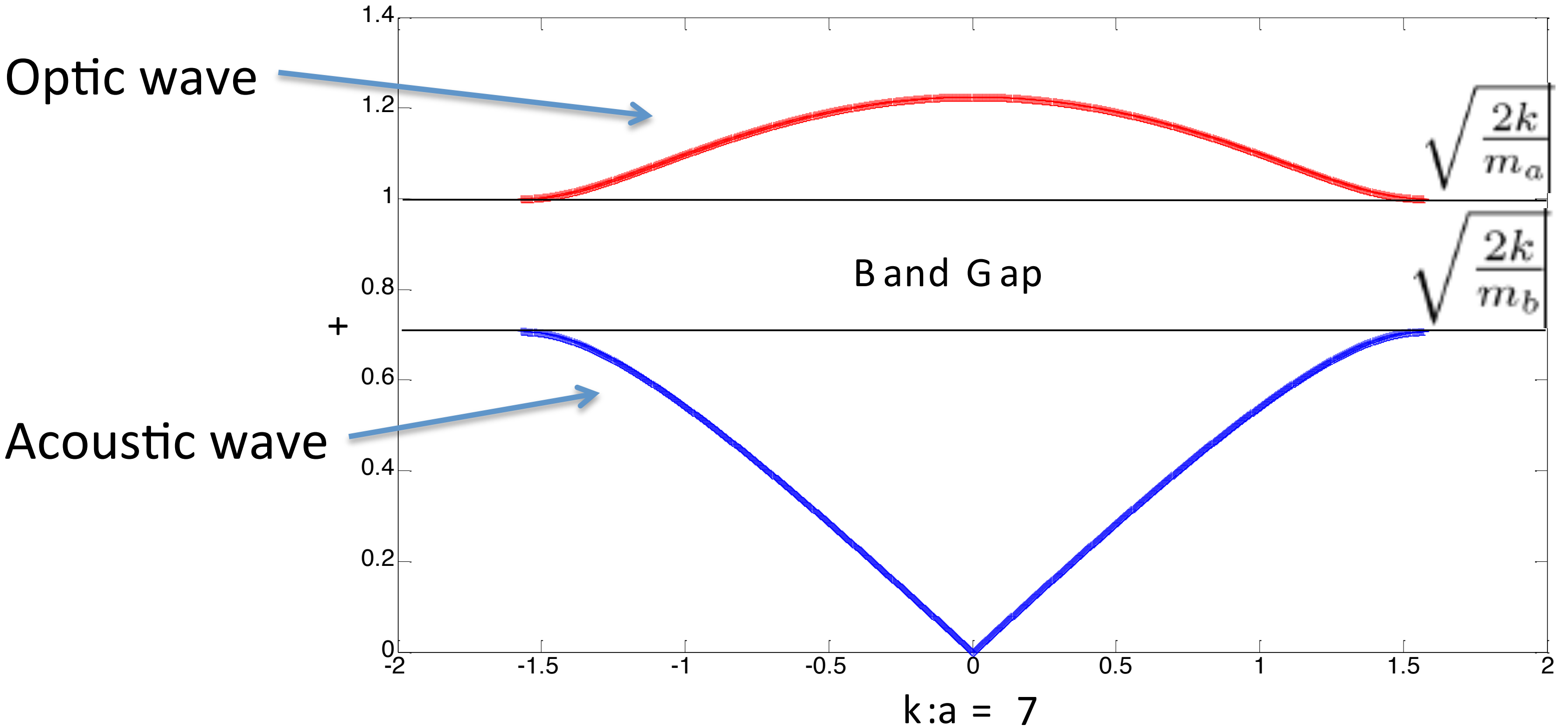


$$u_{2n}(\omega) = u_a(\omega) e^{j\kappa x_{2n}} = u_a(\omega) e^{j2a\kappa n}$$



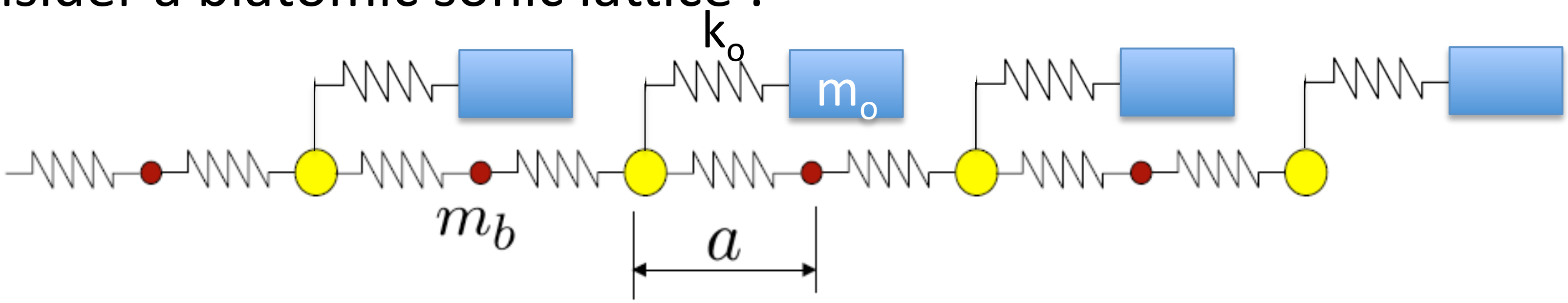
$$u_{2n+1}(\omega) = u_b(\omega) e^{j\kappa x_{2n+1}} = u_b(\omega) e^{j(2n+1)a\kappa}$$

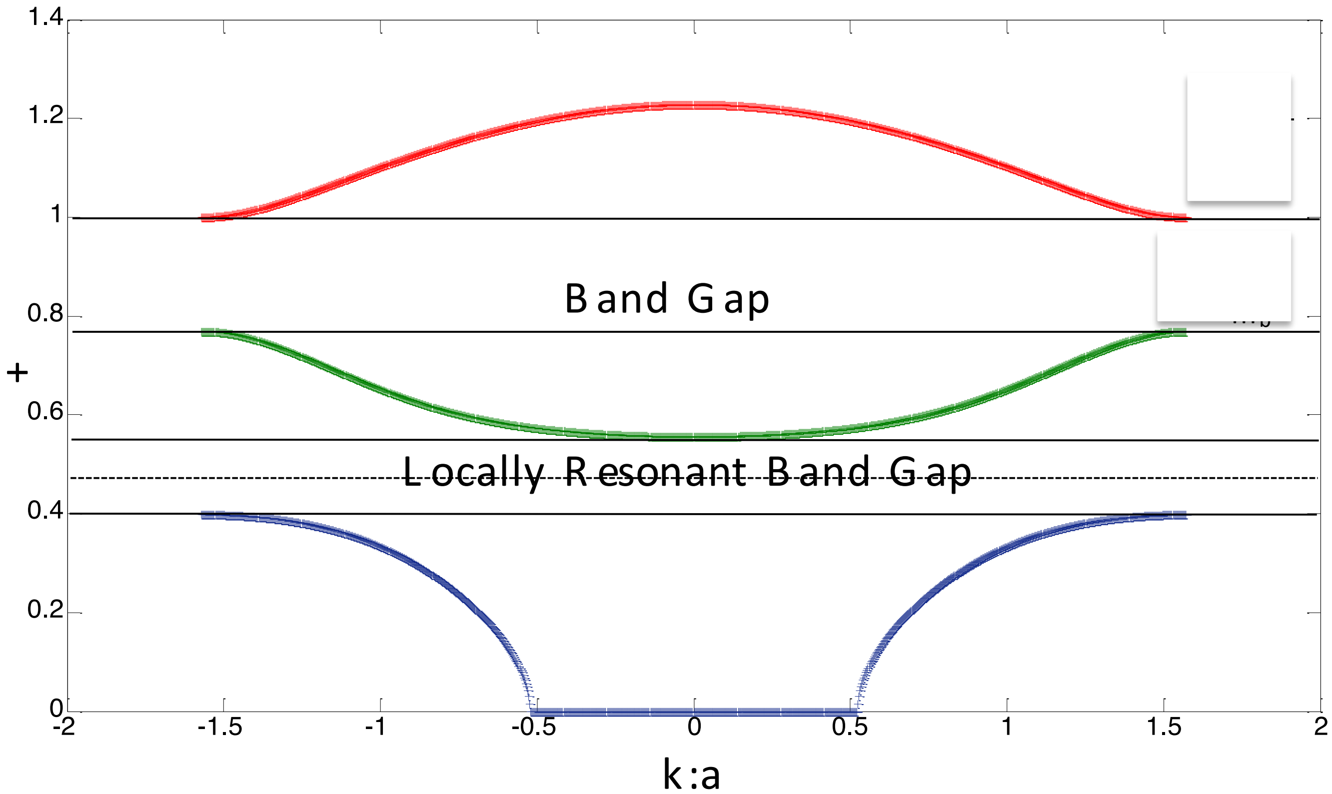
From M Ruzzene Courses



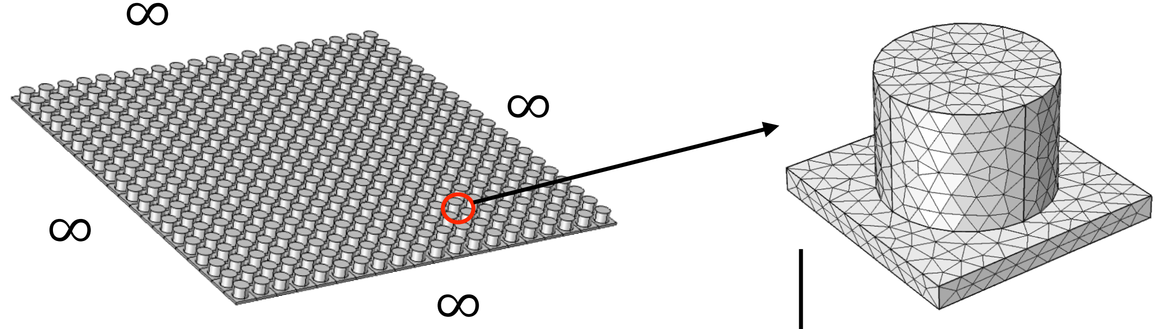
Band Gap and other effects – back to basics

Let's consider a biatomic sonic lattice :

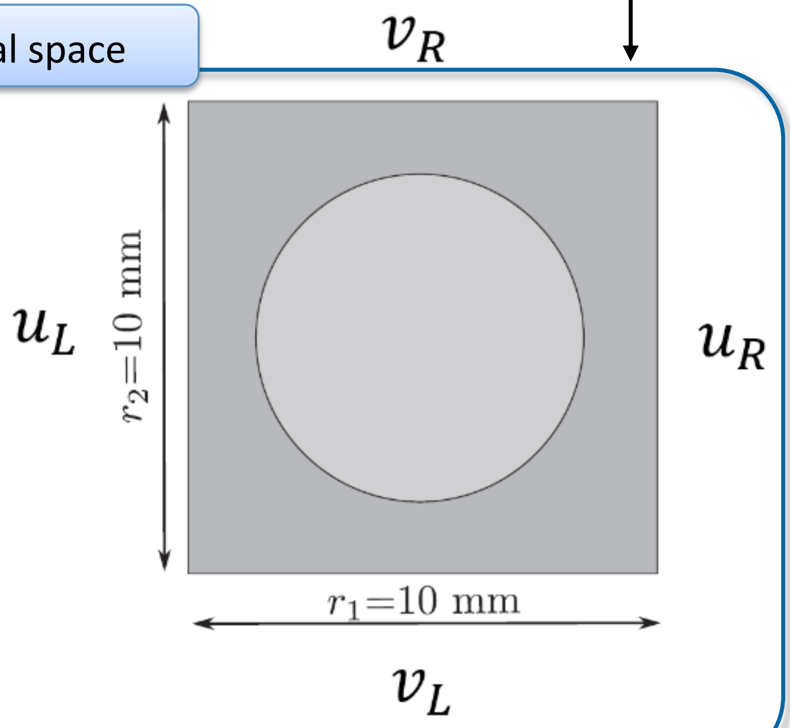




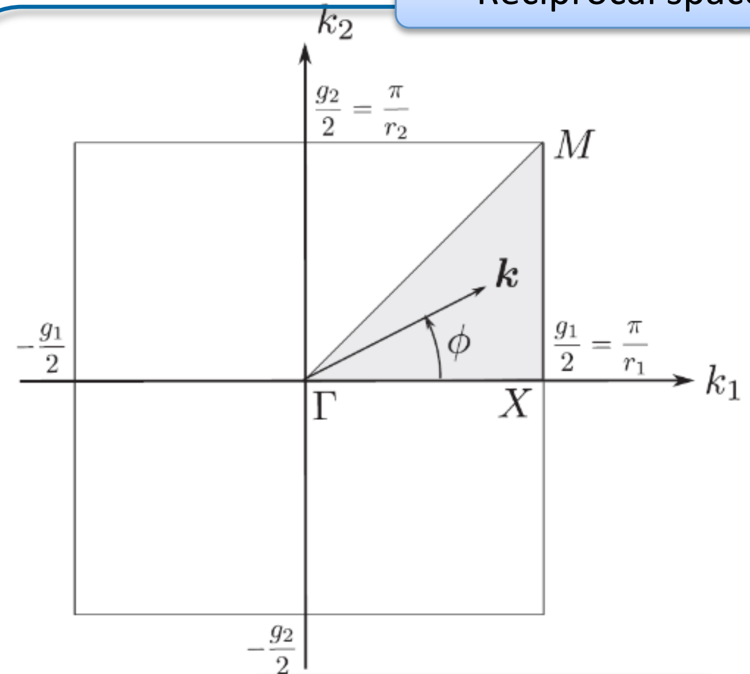
Wave propagation – back to basics



Physical space



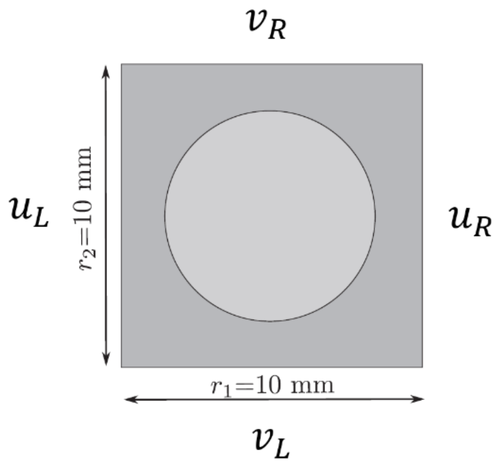
Reciprocal space



Irreducible Brillouin zone

Wave propagation – back to basics

<u>Wave propagation characteristics in the (infinite) periodic structure</u>

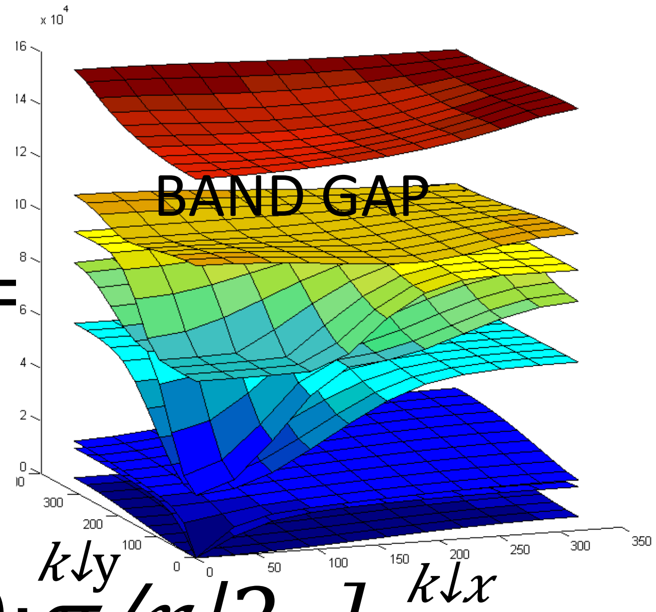


=> Solve the parametric eigenvalue problem

$$\begin{cases} -\rho\omega^2 u + \nabla\sigma = 0 \\ \sigma = C:\varepsilon \end{cases}$$

with « Floquet-Bloch » periodic conditions

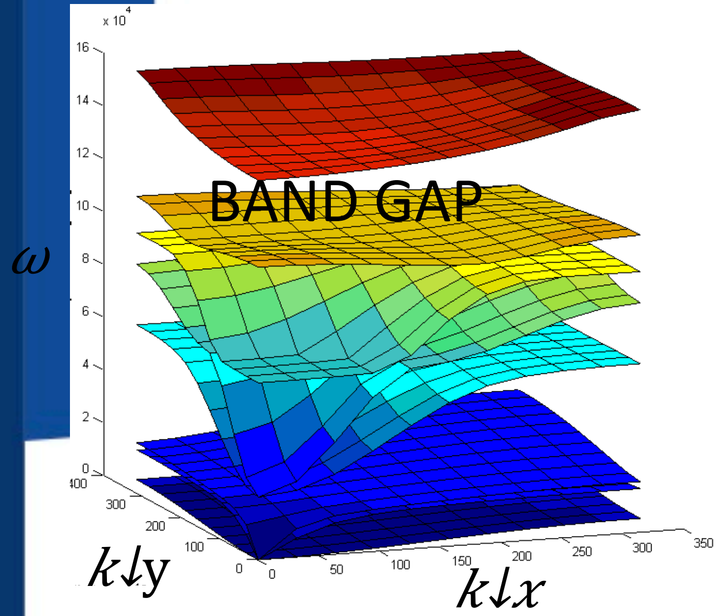
$$u_R = e^{-k_x r_1} u_L \quad \text{and} \quad v_R = e^{-k_y r_2} v_L$$

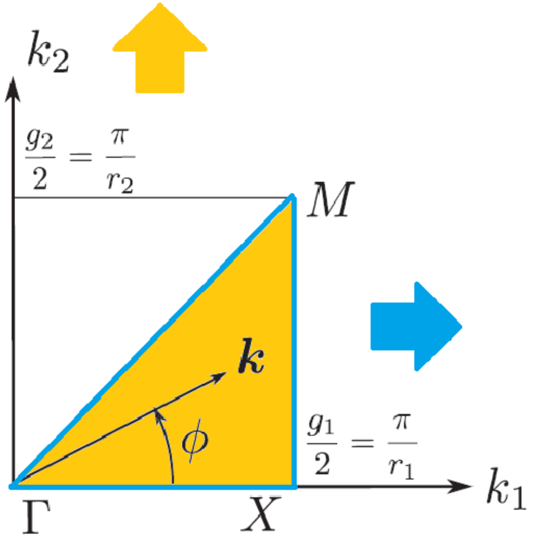


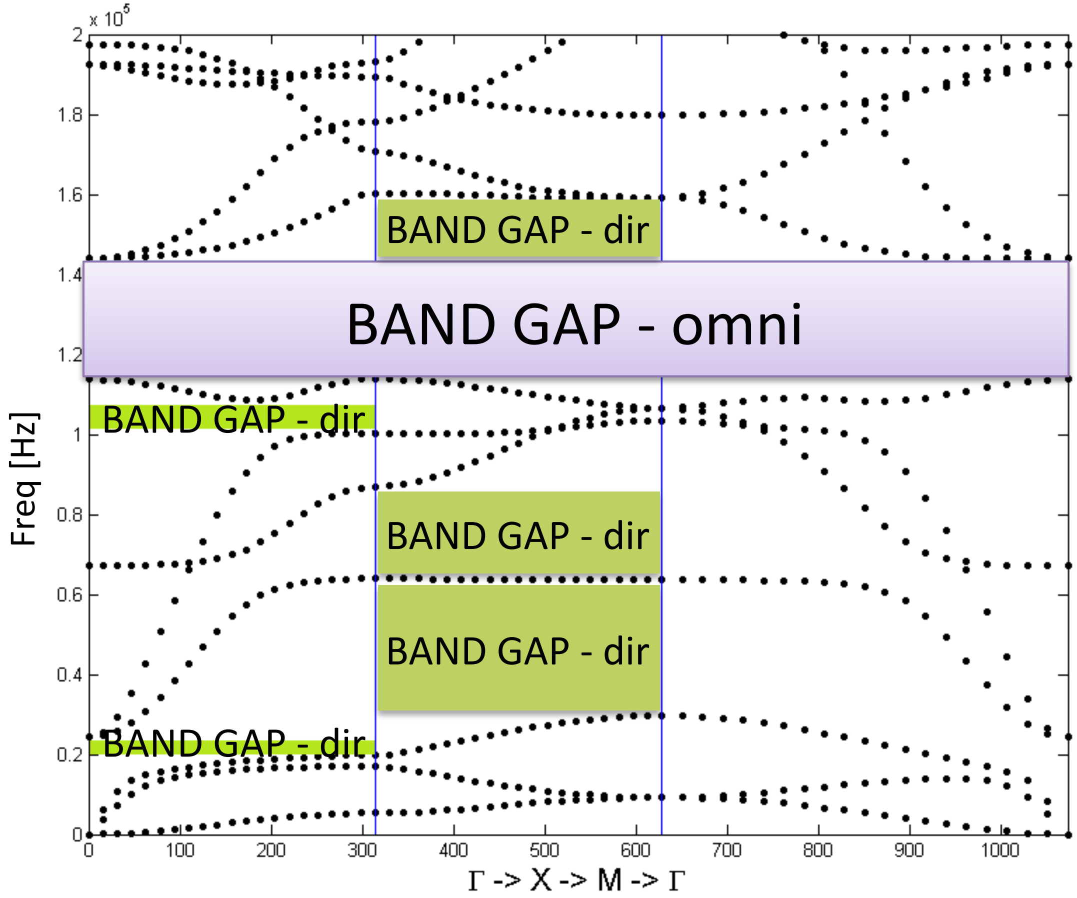
Parameters: $k_x = [0; \pi/r_1]$ and $k_y = [0; \pi/r_2]$.

*Nota Bene. If structural properties are **frequency-dependent**, this formulation yields a **nonlinear eigenvalue** problem (with eigenvalues at boundaries)…*

Wave propagation – back to basics

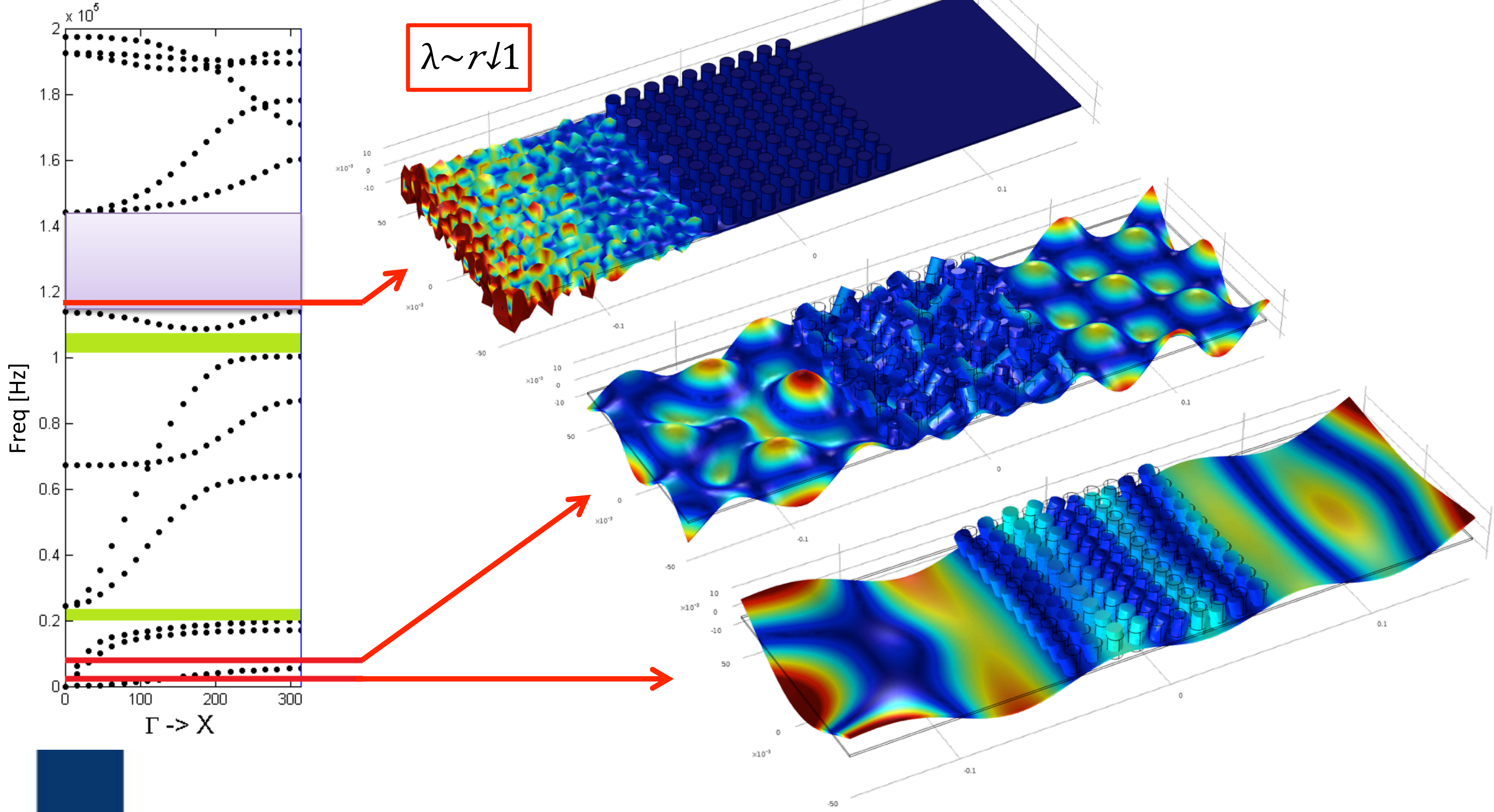






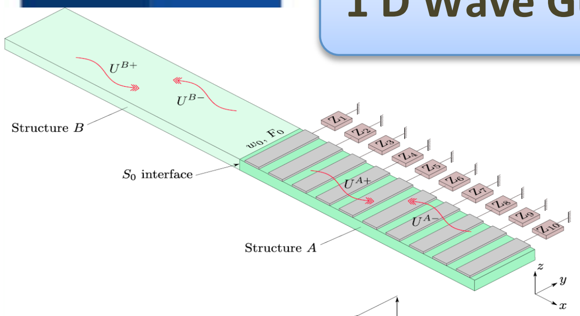
Band diagram: only for band gap analyses

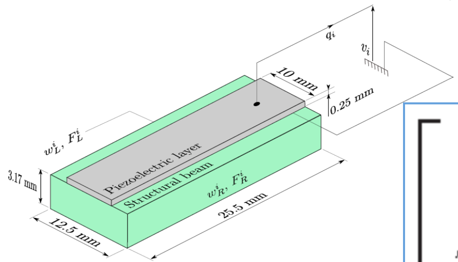
Wave propagation – back to basics

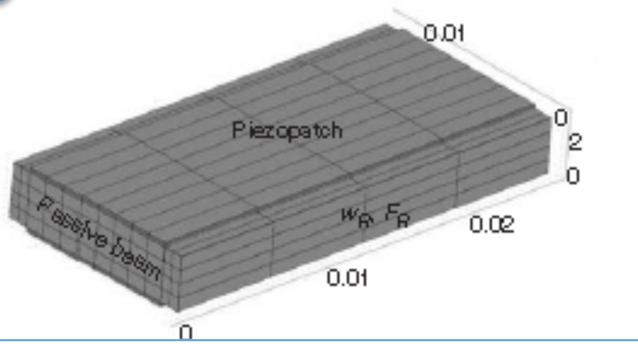


Modeling aspects : Wave Finite Elements WFE

1 D Wave Guide







$$w(x,y,z,i\omega) = W(y,z)e^{-ikx}e^{i\omega t}$$

$$\lambda = e^{-ikd}$$

$$W_R(y,z) = \lambda W_L(y,z)$$

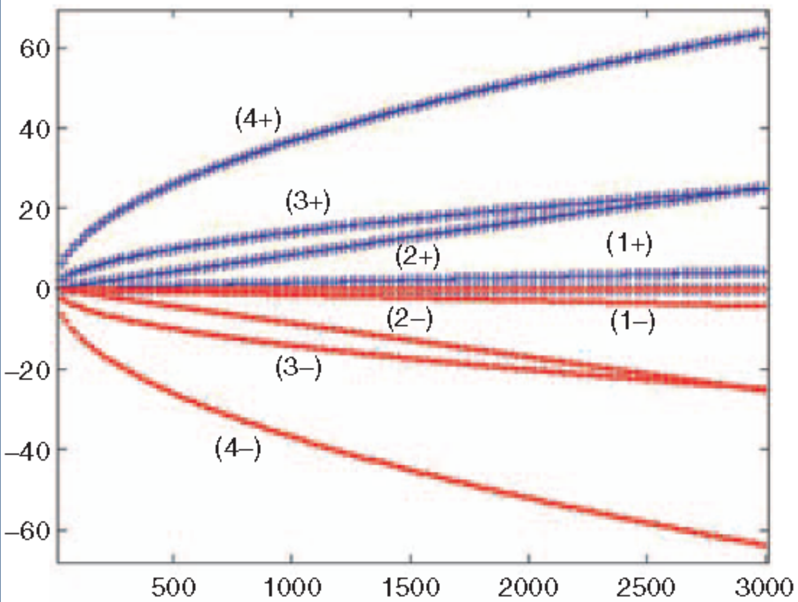
$$F_R^{(I)} = -\lambda F_L^{(I)}$$

$$\begin{bmatrix} \bar{S}_{ll} & \bar{S}_{lr} & -B_{LI} \\ \bar{S}_{lr}^T & \bar{S}_{rr} & -B_{RI} \\ Z(i\omega)B_{LI}^T & Z(i\omega)B_{RI}^T & 1+Z(i\omega)D_I \end{bmatrix} \begin{bmatrix} w_L \\ w_R \\ V_I \end{bmatrix} = \begin{bmatrix} F_L \\ F_R \\ 0 \end{bmatrix}$$

Floquet Bloch Conditions

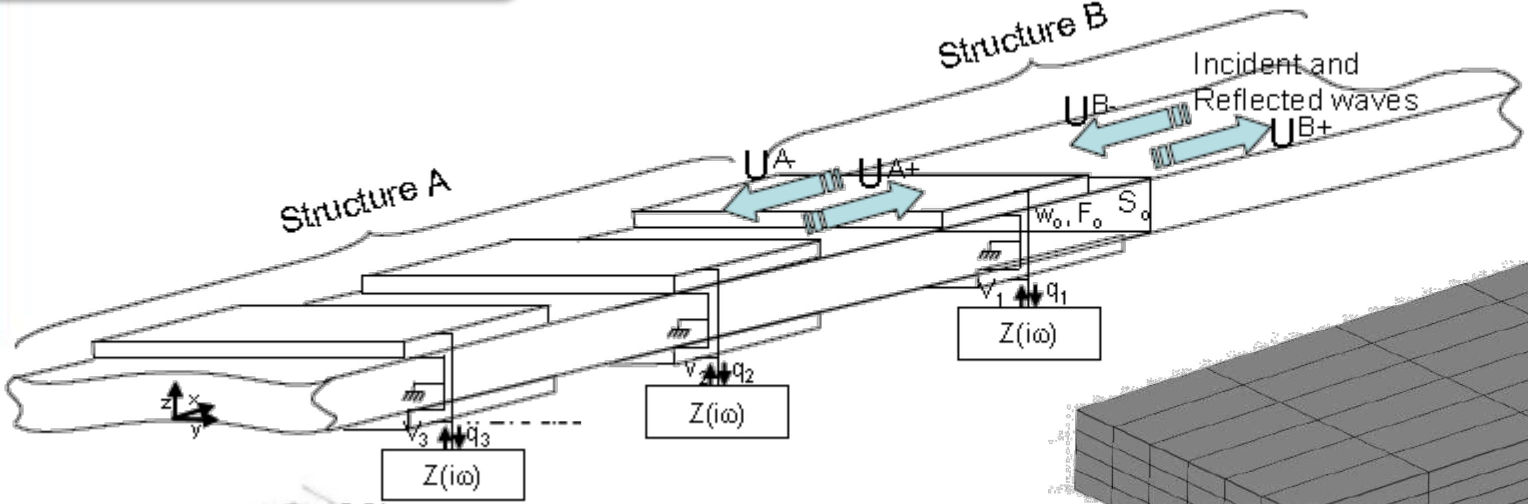
$$\begin{bmatrix} -S_{ll}^a & -S_{lr}^a \\ S_{lr}^{aT} & S_{rr}^a \end{bmatrix} \begin{bmatrix} w_L \\ \lambda w_L \end{bmatrix} = \begin{bmatrix} F'_L \\ \lambda F'_L \end{bmatrix}$$

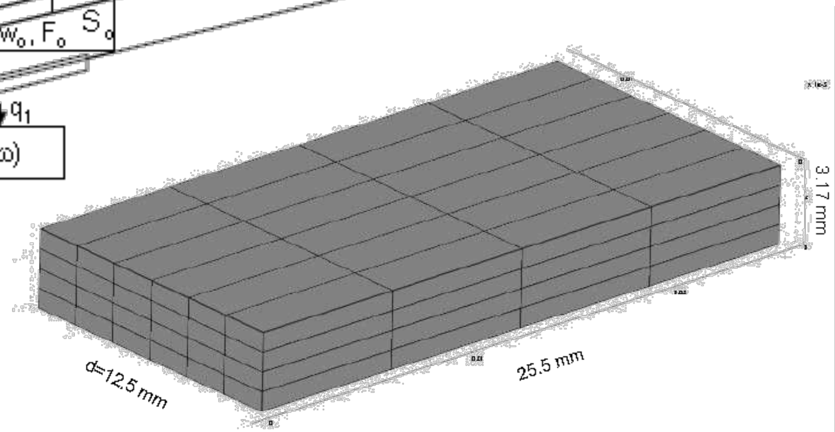
Sympletic Eigenvalue problem

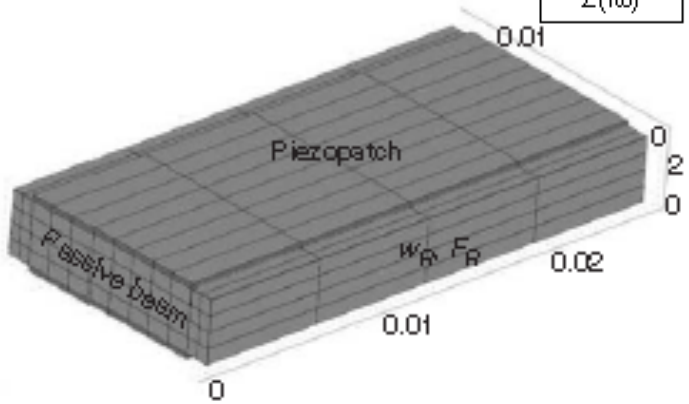


Modeling aspects : Wave Finite Elements WFE

1D Wave Guide : Diffusion operator





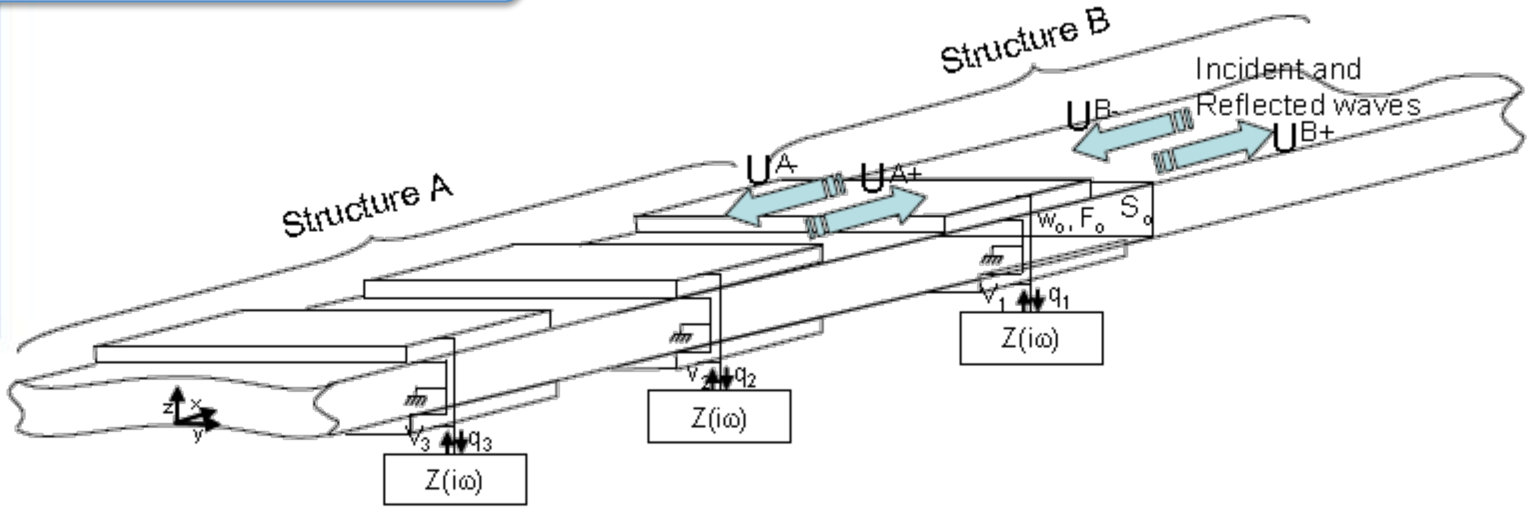


$$U_A^R = \sum_{i=1}^{n} \Phi_{A_i}^+ \alpha_{A_i}^+ + \Phi_{A_i}^- \alpha_{A_i}^-$$

$$U_B^L = \sum_{i=1}^{n} \Phi_{B_i}^+ \alpha_{B_i}^+ + \Phi_{B_i}^- \alpha_{B_i}^-$$

Modeling aspects : Wave Finite Elements WFE

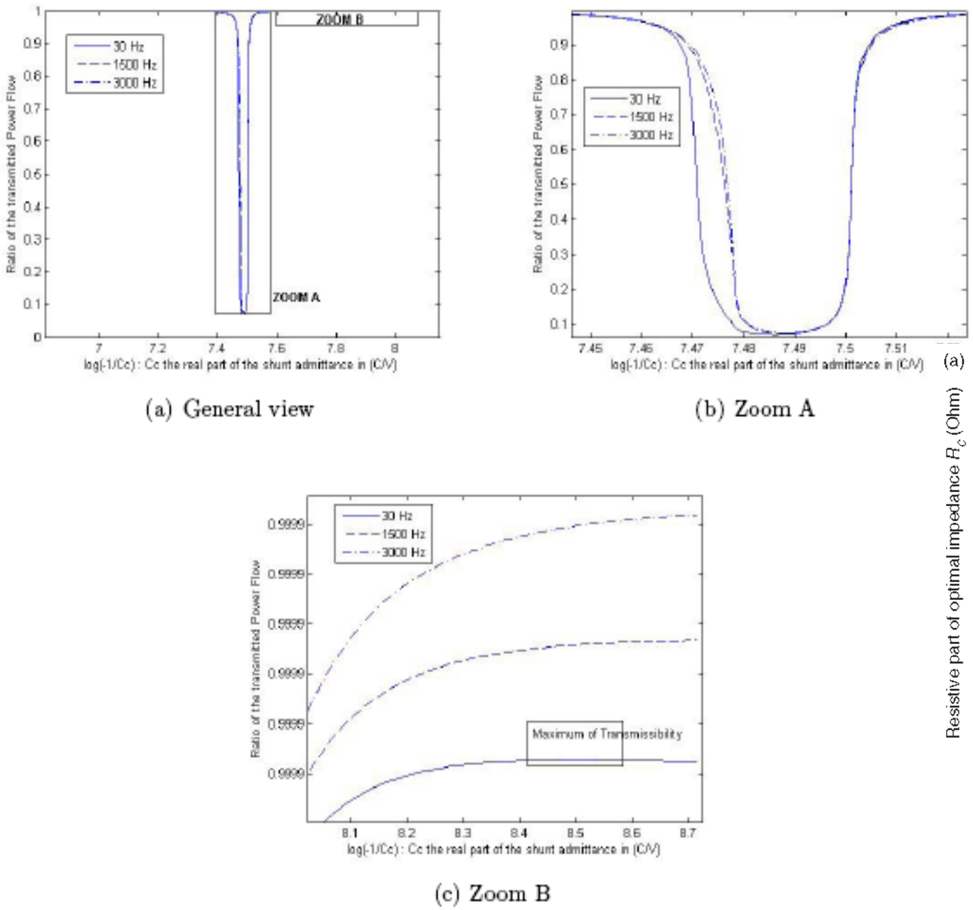
1D Wave Guide : Diffusion operator



$$\begin{bmatrix} 0 & \Phi_A^{+T} J_n \Phi_A^- & -\Phi_A^{+T} J_n \Phi_B^+ & -\Phi_A^{+T} J_n \Phi_B^- & N_A^{+T} \\ \Phi_A^{-T} J_n \Phi_A^+ & 0 & -\Phi_A^{-T} J_n \Phi_B^+ & -\Phi_A^{-T} J_n \Phi_B^- & 0 \\ \Phi_B^{+T} J_n \Phi_A^+ & \Phi_B^{+T} J_n \Phi_A^- & 0 & -\Phi_B^{+T} J_n \Phi_B^- & 0 \\ \Phi_B^{-T} J_n \Phi_A^+ & \Phi_B^{-T} J_n \Phi_A^- & -\Phi_B^{-T} J_n \Phi_B^+ & 0 & N_B^{-T} \\ N_A^{+T} & 0 & 0 & N_B^{-T} & 0 \end{bmatrix} \begin{bmatrix} \alpha_A^+ \\ \alpha_A^- \\ \alpha_B^+ \\ \alpha_B^- \\ \xi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ U_o \end{bmatrix}$$

Modeling aspects : Wave Finite Elements WFE

1D Wave Guide : Diffusion operator

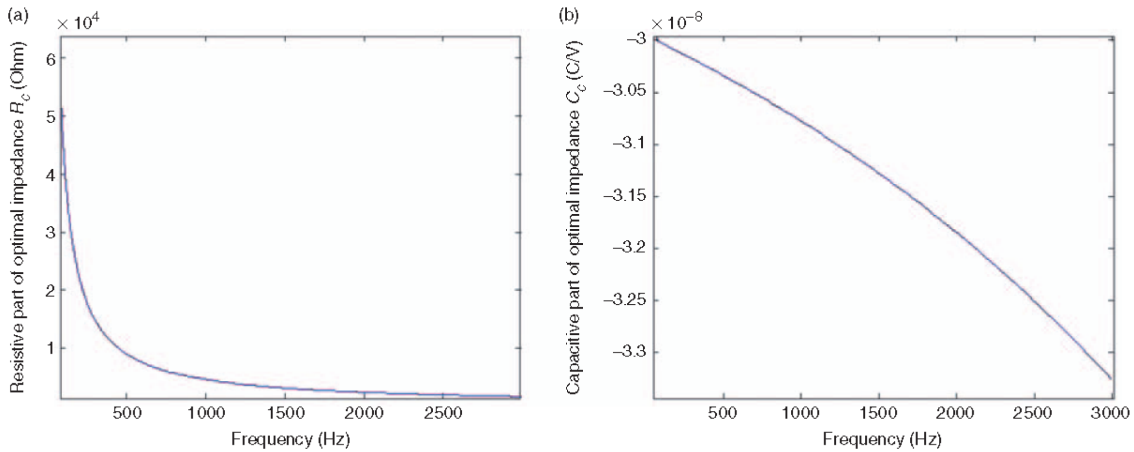


(a) General view

(b) Zoom A

(c) Zoom B

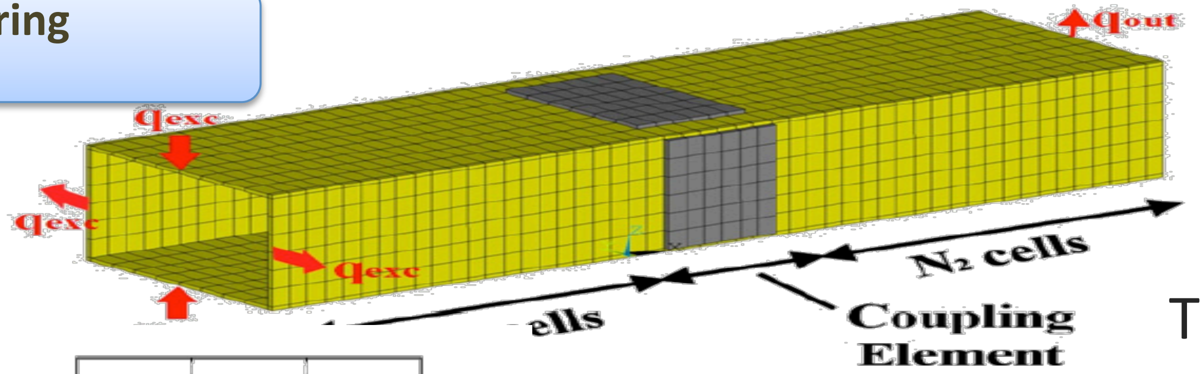
Figure III.11: Ratio of the transmitted flexural power flux as a function of negative capacitance shunt at 30 Hz, 1500 Hz and 3000 Hz



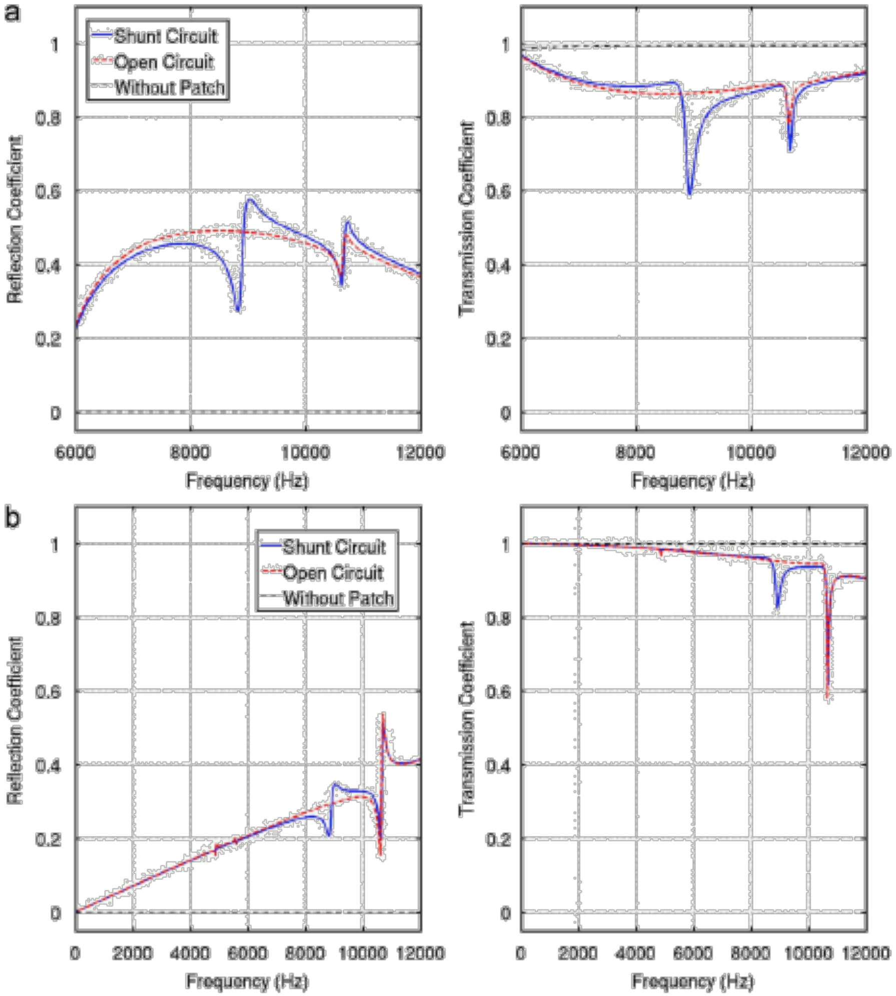
quasi C negative + resistance

Modeling aspects : Wave Finite Elements WFE

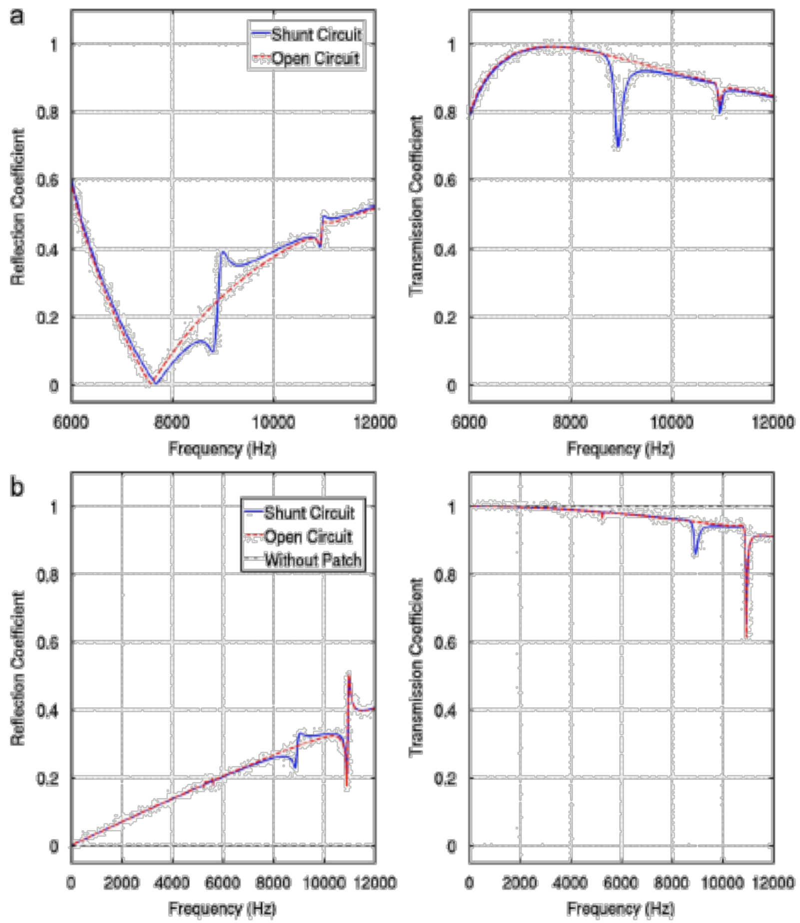
1D Wave Guide : Scattering computation



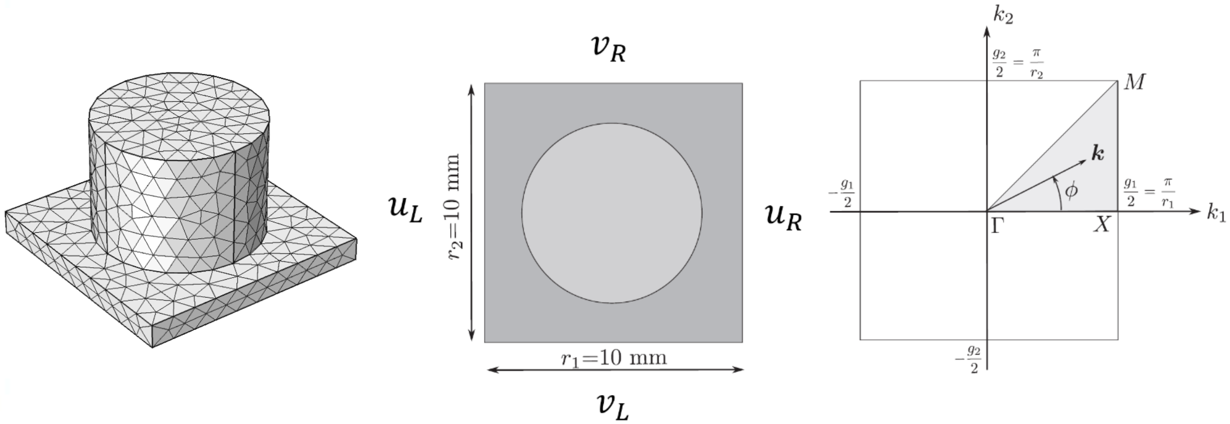
Reflection



Transmission



Modeling aspects : Shift cell operator



The physics

$$\rho(\boldsymbol{x})\omega^2\boldsymbol{w}(\boldsymbol{x}) + \nabla\boldsymbol{C}(\boldsymbol{x})\nabla_{sym}(\boldsymbol{w}(\boldsymbol{x})) = 0 \;\forall \boldsymbol{x} \in \mathbb{R}^3$$

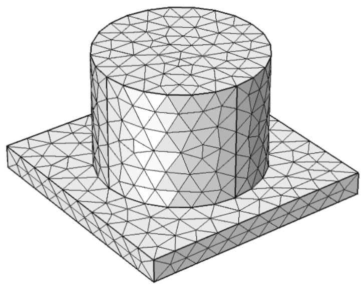
Floquet-Bloch modes

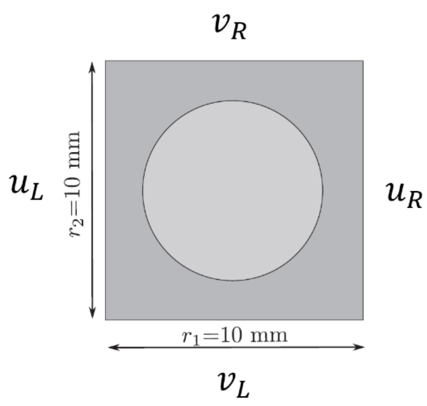
$$\boldsymbol{w}(\boldsymbol{x}) = \boldsymbol{w}_{n,k}(\boldsymbol{x},\boldsymbol{k})e^{i\boldsymbol{k}.\boldsymbol{x}}$$ + « Floquet-Bloch » periodic conditions

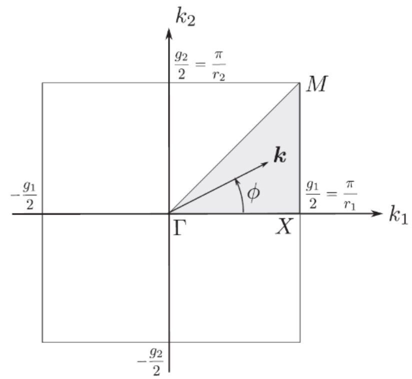
The ”shifted-cell” formulation

$$\frac{\partial}{\partial \boldsymbol{x}} \rightarrow \frac{\partial}{\partial \boldsymbol{x}} + i\boldsymbol{k}$$ in the cell + symmetric boundary conditions

A. Bensoussan, J.L. Lions, and G. Pananicolaou. Asymptotic Analysis for Periodic Structures. North Holland, 1978.
C. Wilcox. Theory of bloch waves. Journal d’Analyse Mathématique, 33:146–167, 1978. ISSN 0021-7670



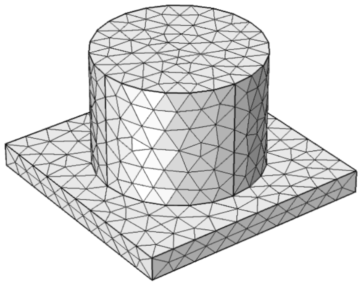


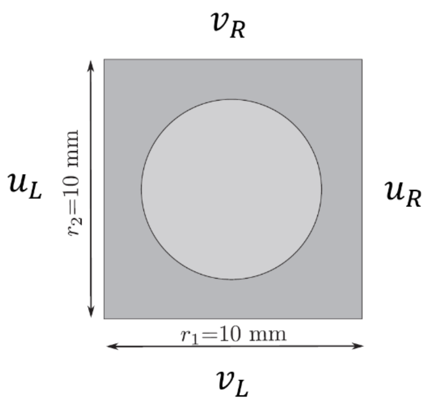


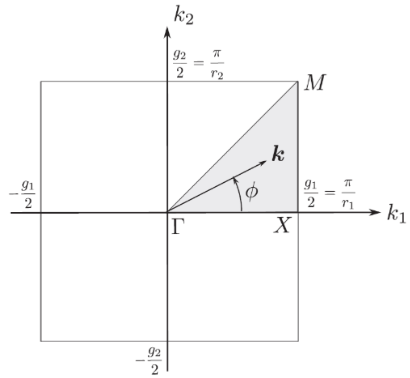
The (shifted) physics

$$
\begin{aligned}
\rho(\boldsymbol{x})\omega_n(\boldsymbol{k})^2\boldsymbol{w}_{n,k}(\boldsymbol{x}) + \nabla\boldsymbol{C}(\boldsymbol{x})\nabla_{sym}(\boldsymbol{w}_{n,k}(\boldsymbol{x})) & \\
-i\boldsymbol{C}(\boldsymbol{x})\nabla_{sym}(\boldsymbol{w}_{n,k}(\boldsymbol{x})).\boldsymbol{k} - i\nabla\boldsymbol{C}(\boldsymbol{x})\frac{1}{2}(\boldsymbol{w}_{n,k}(\boldsymbol{x}).\boldsymbol{k}^T + \boldsymbol{k}.\boldsymbol{w}_{n,k}^T(\boldsymbol{x})) & \\
+\boldsymbol{C}(\boldsymbol{x})\frac{1}{2}(\boldsymbol{w}_{n,k}(\boldsymbol{x}).\boldsymbol{k}^T + \boldsymbol{k}.\boldsymbol{w}_{n,k}^T(\boldsymbol{x})).\boldsymbol{k} & = 0 \quad \forall\boldsymbol{x} \in \Omega_R, \\
\boldsymbol{w}_{n,k}(\boldsymbol{x} - \boldsymbol{R}.\boldsymbol{n}) - \boldsymbol{w}_{n,k}(\boldsymbol{x}) & = 0 \quad \forall\boldsymbol{x} \in \Gamma_R.
\end{aligned}
$$

Modeling aspects : Shift cell operator







Weak form => FE formulation

$$(\boldsymbol{K}+\lambda\boldsymbol{L}(\boldsymbol{\Phi})-\lambda^2\boldsymbol{H}(\boldsymbol{\Phi})-\omega_n^2(\lambda,\boldsymbol{\Phi})\boldsymbol{M})\boldsymbol{w}_{n,k}(\boldsymbol{\Phi})=0$$

$$\lambda = ik$$

$$\boldsymbol{k} = k\begin{bmatrix}\sin(\theta)\cos(\phi)\\ \sin(\theta)\sin(\phi)\\ \cos(\theta)\end{bmatrix}$$

$$\boldsymbol{\Phi} = \begin{bmatrix}\sin(\theta)\cos(\phi)\\ \sin(\theta)\sin(\phi)\\ \cos(\theta)\end{bmatrix}$$

$$\boldsymbol{M} \rightarrow \int_{\Omega_R} \rho(\boldsymbol{x})\omega_n^2(\boldsymbol{k})\tilde{\boldsymbol{w}}_{n,k}(\boldsymbol{x})\boldsymbol{w}_{n,k}(\boldsymbol{x})d\Omega,$$

$$\boldsymbol{K} \rightarrow \int_{\Omega_R} \tilde{\boldsymbol{\varepsilon}}_{n,k}(\boldsymbol{x})\boldsymbol{C}(\boldsymbol{x})\boldsymbol{\varepsilon}_{n,k}(\boldsymbol{x})d\Omega,$$

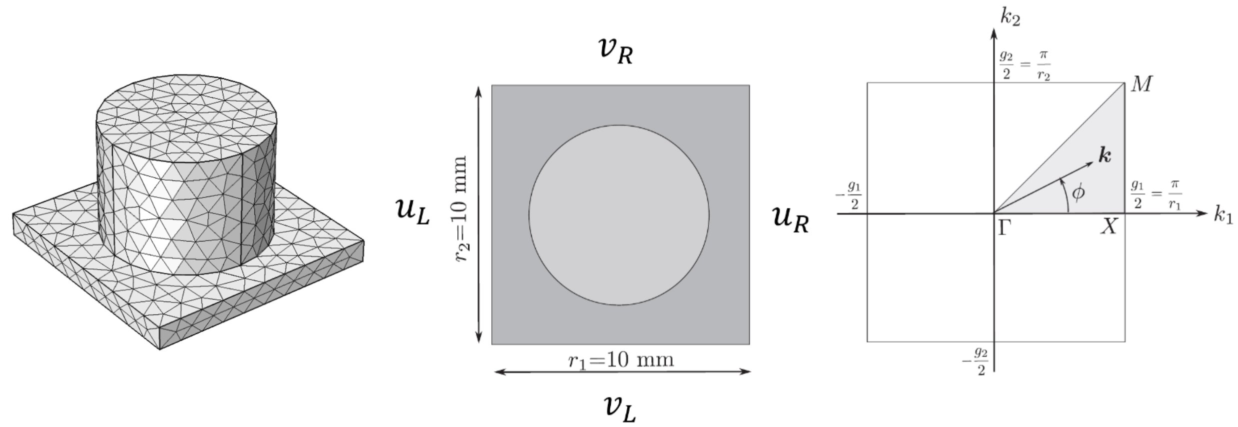
$$\boldsymbol{L} \rightarrow \int_{\Omega_R} -\tilde{\boldsymbol{\kappa}}_{n,k}(\boldsymbol{x})\boldsymbol{C}(\boldsymbol{x})\boldsymbol{\varepsilon}_{n,k}(\boldsymbol{x})+\tilde{\boldsymbol{\varepsilon}}_{n,k}(\boldsymbol{x})\boldsymbol{C}(\boldsymbol{x})\boldsymbol{\kappa}_{n,k}(\boldsymbol{x})d\Omega,$$

skew-symmetric

$$\boldsymbol{H} \rightarrow \int_{\Omega_R} \tilde{\boldsymbol{\kappa}}_{n,k}(\boldsymbol{x})\boldsymbol{C}(\boldsymbol{x})\boldsymbol{\kappa}_{n,k}(\boldsymbol{x})d\Omega.$$

symmetric semi-definite positive

Modeling aspects : Shift cell operator



Weak form => FE formulation

$$(\boldsymbol{K} + \lambda \boldsymbol{L}(\boldsymbol{\Phi}) - \lambda^2 \boldsymbol{H}(\boldsymbol{\Phi}) - \omega_n^2(\lambda, \boldsymbol{\Phi})\boldsymbol{M})\boldsymbol{w}_{n,k}(\boldsymbol{\Phi}) = 0$$

fix k_x and k_y, find ω and w

Rearrangement

$$(\boldsymbol{K} - \omega^2 \boldsymbol{M}) + \lambda_n(\omega, \boldsymbol{\Phi})\boldsymbol{L}(\boldsymbol{\Phi}) - \lambda_n^2(\omega, \boldsymbol{\Phi})\boldsymbol{H}(\boldsymbol{\Phi}))\boldsymbol{w}_{n,k}(\boldsymbol{\Phi}) = 0$$

fix ϕ and ω, find k and w

$\Rightarrow$ Solve a **QEP** to characterize **wave propagation** in any **frequency-dependent** (visco, poro, piezo…) **periodic structure**

Modeling aspects : Shift cell operator

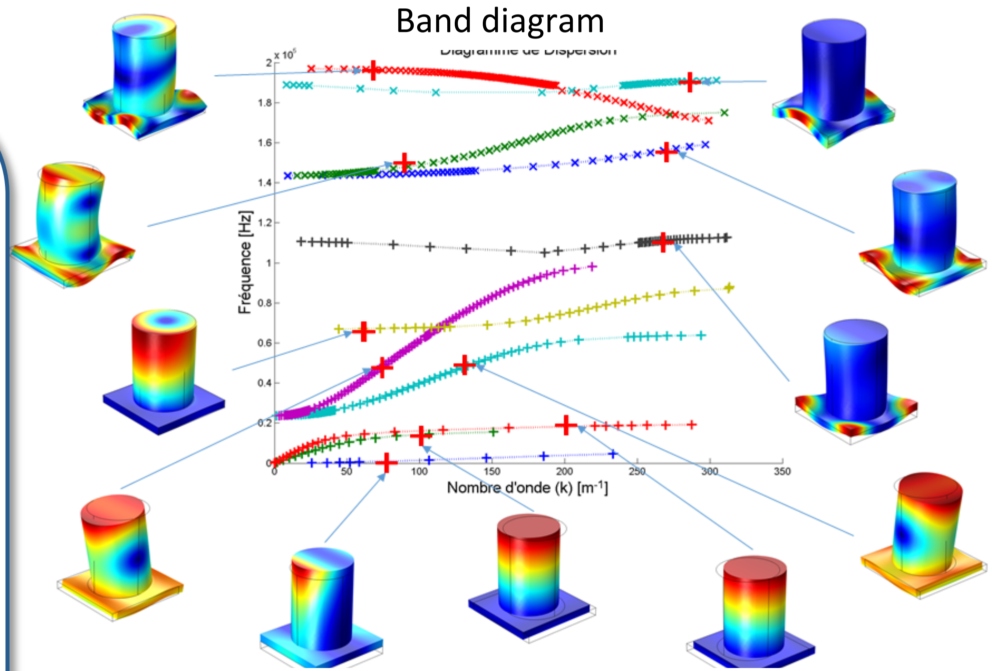
Some practical issues

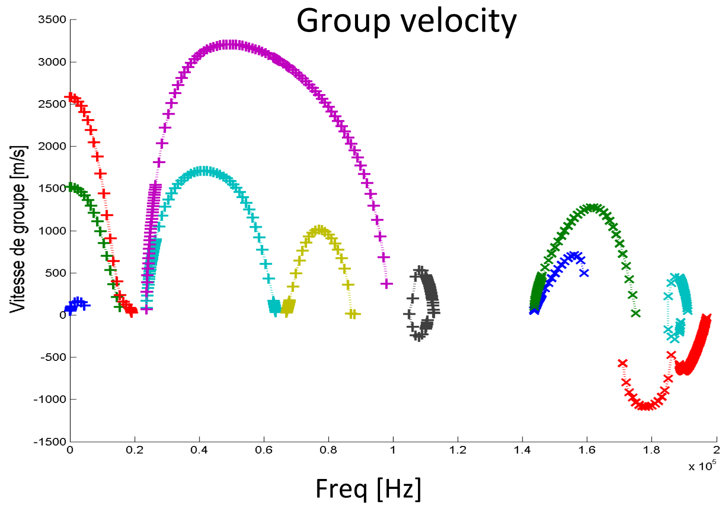
- All wave numbers are **complex**
  => need for suitable criteria to distinguish "**propagative**" and "**evanescent**" waves
- How to **track** a given wave when parameters change?
  => need for **correlation criteria**
- Computation of the **group velocity** for periodic **damped** structures?

$$C_g = \left(\frac{\partial \omega}{\partial k}\right) ???$$

- For an **homogeneous material** with $E(\omega) = f(\omega) E_0$ one has

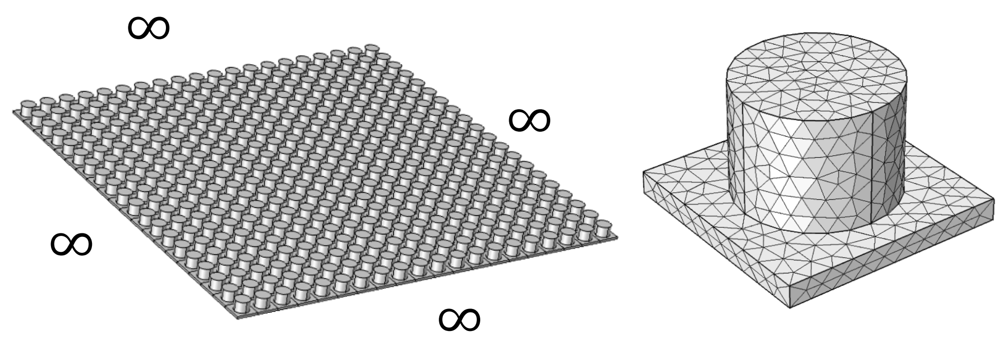
$$C_g = real\left(\frac{j \phi_i^{l\,T} \left[f(\omega)\left(-L_0 + L_0^T + 2\lambda_i H_0\right)\right] \phi_i^r}{\phi_i^{l\,T}\left[\omega^2 \left(\frac{\partial f / \partial \omega}{f(\omega)}\right) - 2\omega\right] M \phi_i^r} \right)$$



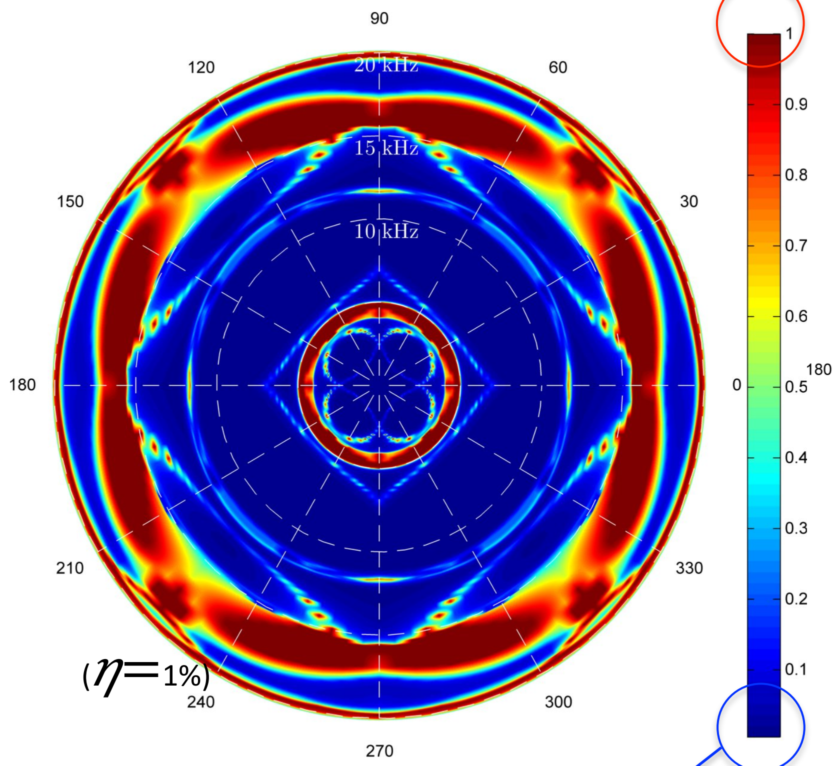


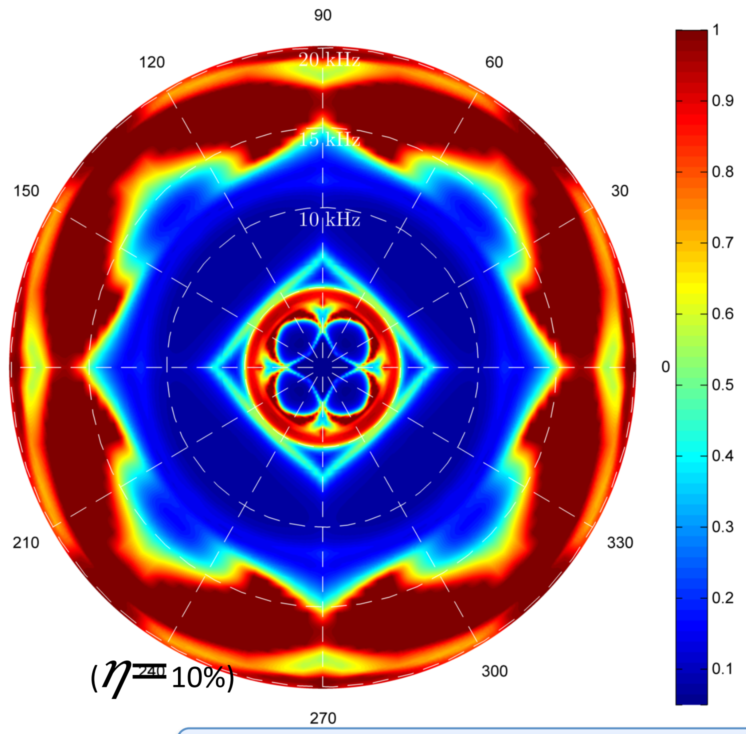
M. Collet, M. Ouisse, M. Ruzzene et M. Ichchou : Floquet-bloch decomposition for the computation of dispersion of two-dimensional periodic, damped mechanical systems. *International Journal of Solids and Structures*, 48(20):2837–2848, 2011. 10.1016/j.ijsolstr.2011.06.002.

Modeling aspects : Shift cell operator



All waves are highly evanescent



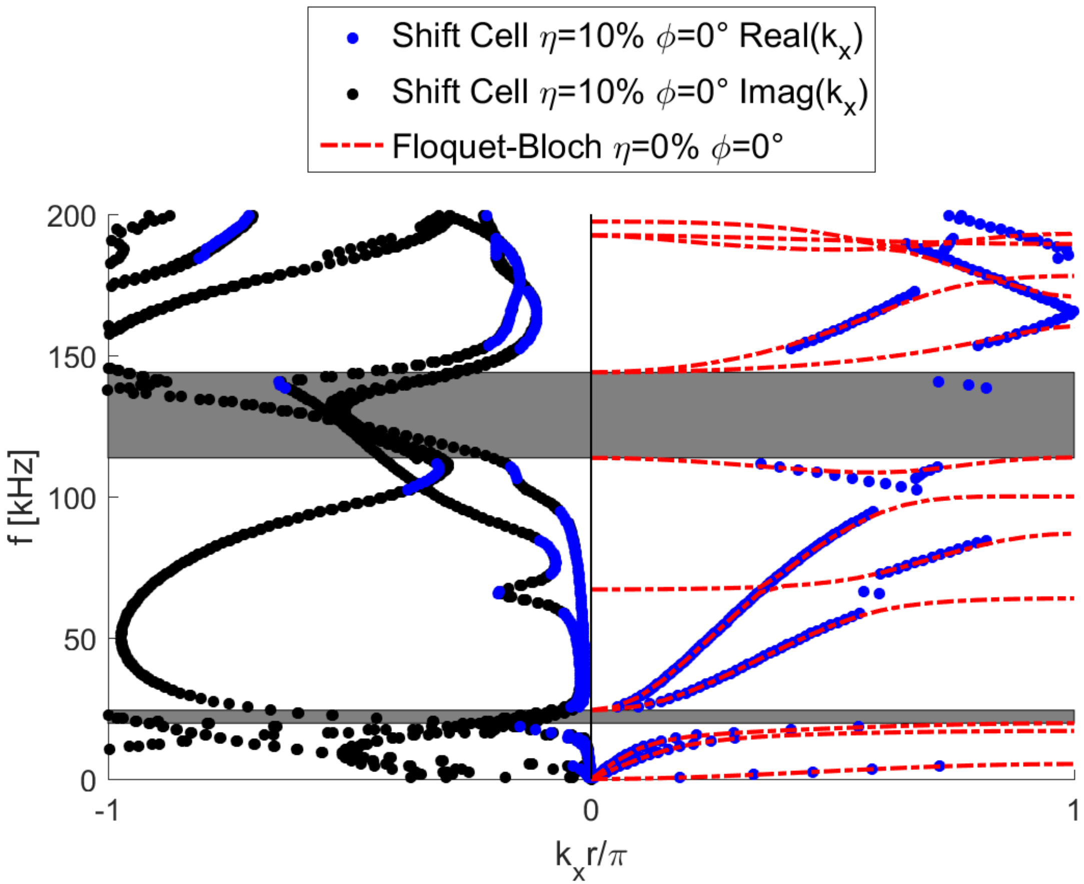


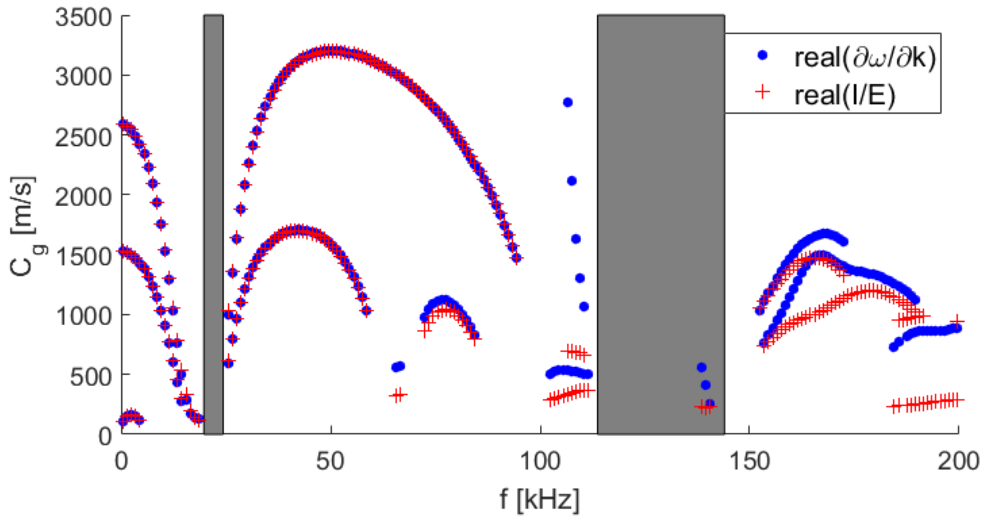
Directivity in terms of evanescence

Some waves are highly propagative

Dampen Medium

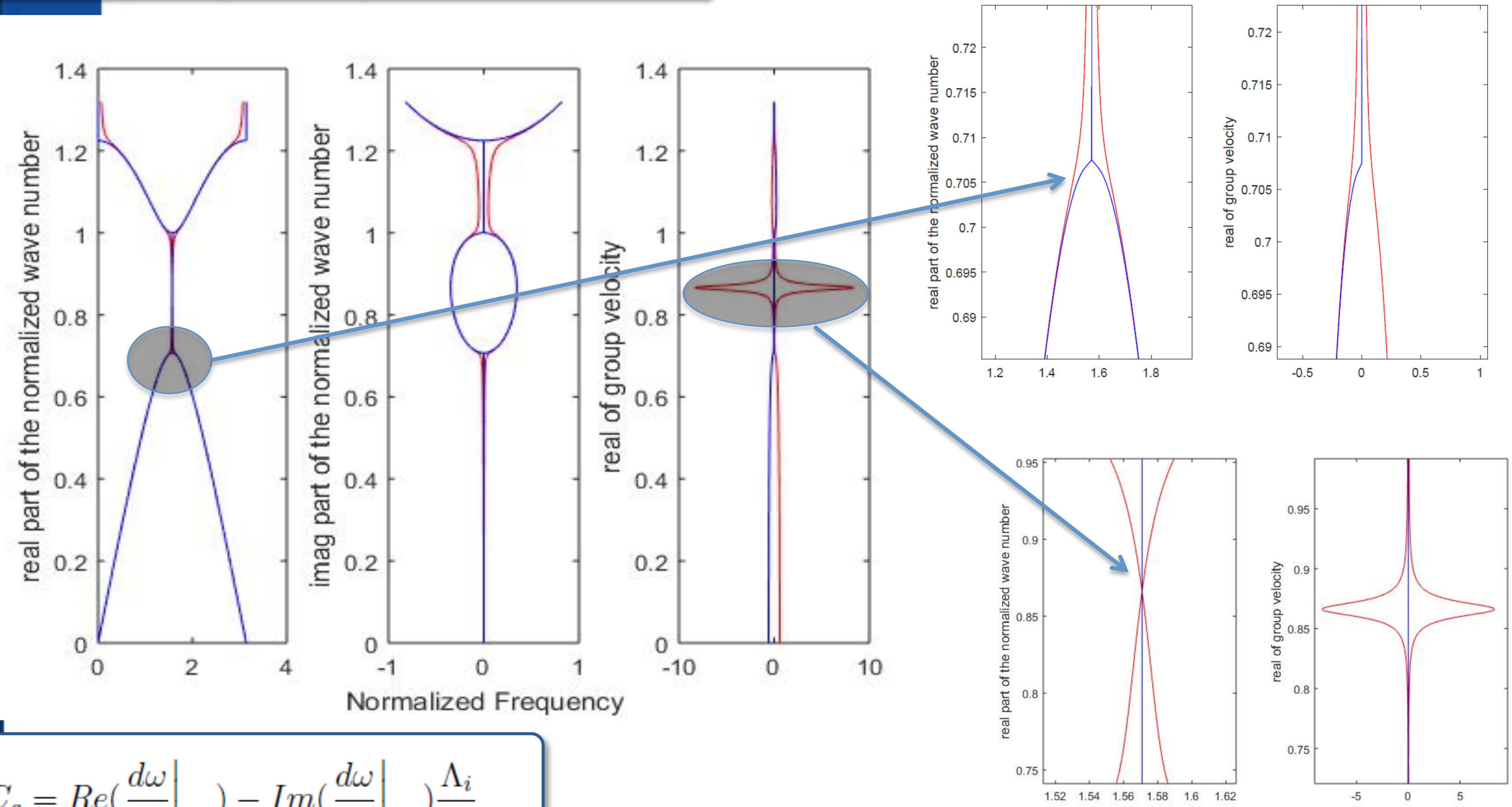
η=10%





Modeling aspects : Shift cell operator

Group velocity in dampen medium

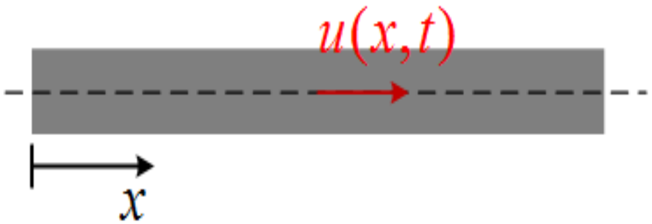


$$C_g = Re\left(\frac{d\omega}{dk}\Big|_{k_m}\right) - Im\left(\frac{d\omega}{dk}\Big|_{k_m}\right)\frac{\Lambda_i}{\Lambda_r}$$

Muschietti L and Dumb C T 1993 *Phys. Fluids B* **5** 1383–1397

Modeling aspects : Plane Wave Expansion PWE

Example : Time varying beam system



$$E(x,t) = E(x+\lambda_m, t+T_m) \qquad \rho(x,t) = \rho(x+\lambda_m, t+T_m)$$

$$\lambda_m = \frac{2\pi}{k_m}, T_m = \frac{2\pi}{\omega_m}$$

Time varying longitudinal motion

$$\frac{\partial}{\partial x}[E(x,t)\frac{\partial u(x,t)}{\partial x}] - \frac{\partial}{\partial t}[\rho(x,t)\frac{\partial u(x,t)}{\partial t}] = 0$$

Modeling aspects : Plane Wave Expansion PWE

Outline of Plane Wave Expansion method

- Fourier expansion of material parameters:

$$E(x,t) = \sum_{p=-\infty}^{p=+\infty} \hat{E}_p e^{ip(\omega_m t - k_m x)} \qquad \rho(x,t) = \sum_{p=-\infty}^{p=+\infty} \hat{\rho}_p e^{ip(\omega_m t - k_m x)}$$

$\hat{E}_p, \hat{\rho}_p$: Fourier coefficients

- Bloch expansion of the solution:

$$u(\mathrm{x},\mathrm{t}) = \mathrm{e}^{i(\omega \mathrm{t} - k\mathrm{x})} \sum_{q=-N}^{+N} U_q \, \mathrm{e}^{iq(\omega_m \mathrm{t} - k_m \mathrm{x})}$$

N : Truncation order

Outline of Plane Wave Expansion method

- Quadratic Eigenvalue Problem (QEP):

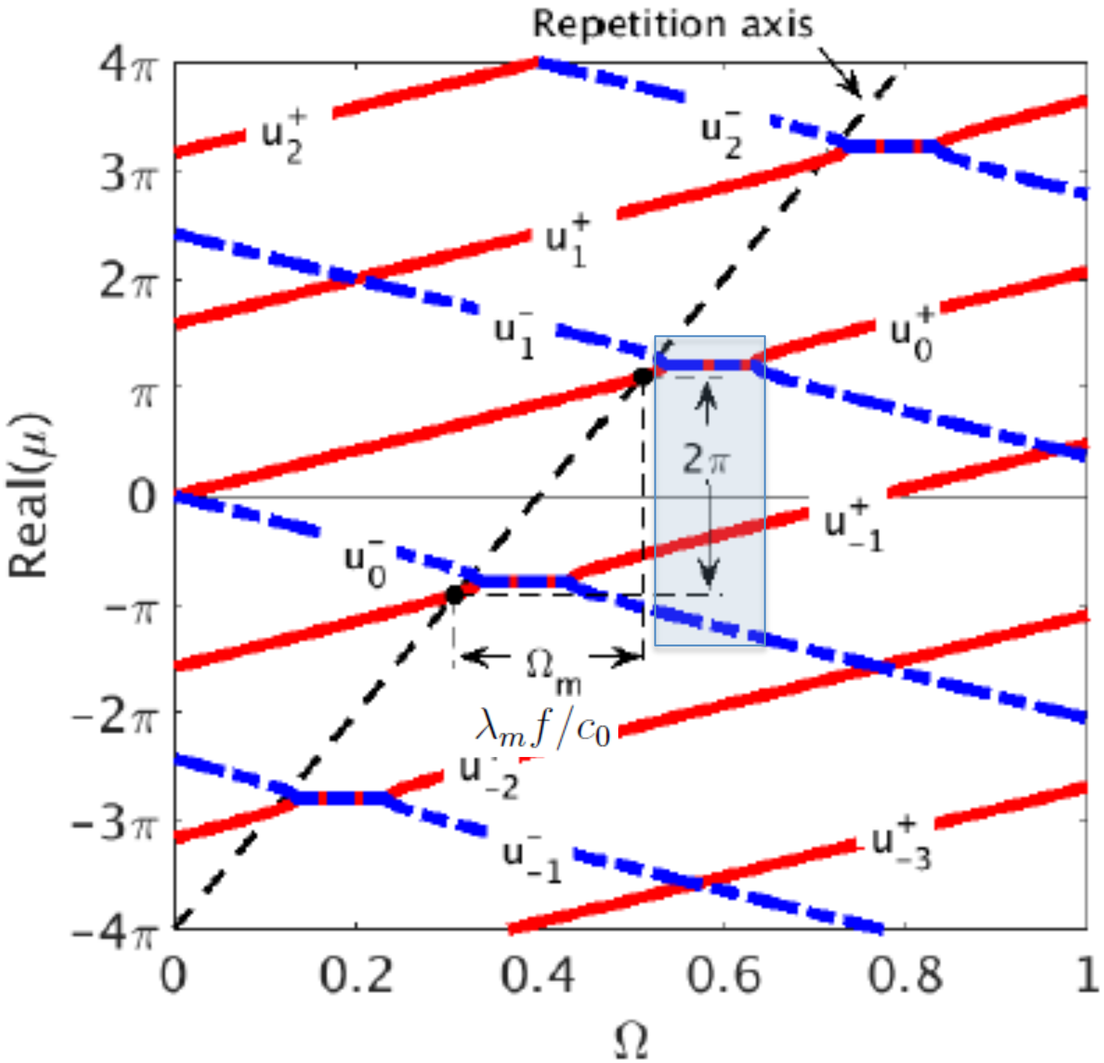
$$\sum_{q=-\infty}^{\infty} (k + qk_m)(k + nk_m)\hat{E}_{n-q}U_q = (\omega + n\omega_m)^2 \rho_0 U_n$$

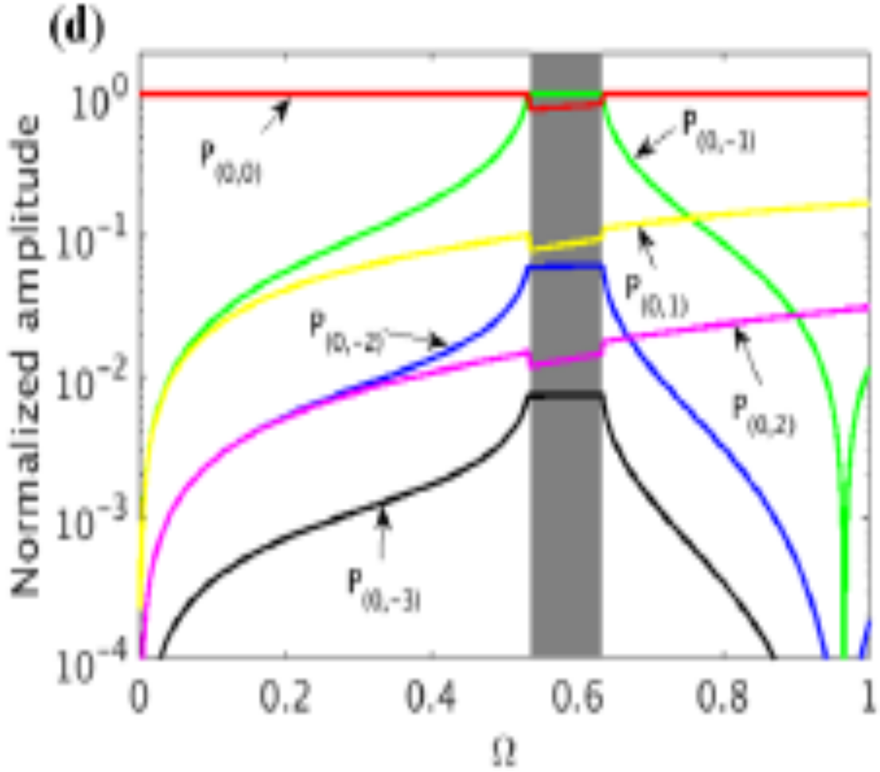
$$u_n^+(x,t,k_n^+,\omega) = \sum_{q=-N}^{+N} U_{(n,q)}^+ e^{i[(\omega+q\omega_m)t-(k_n^+ + qk_m)x]}$$

$$u_n^-(x,t,k_n^-,\omega) = \sum_{q=-N}^{+N} U_{(n,q)}^- e^{i[(\omega+q\omega_m)t-(k_n^- + qk_m)x]}$$

Modeling aspects : Plane Wave Expansion PWE

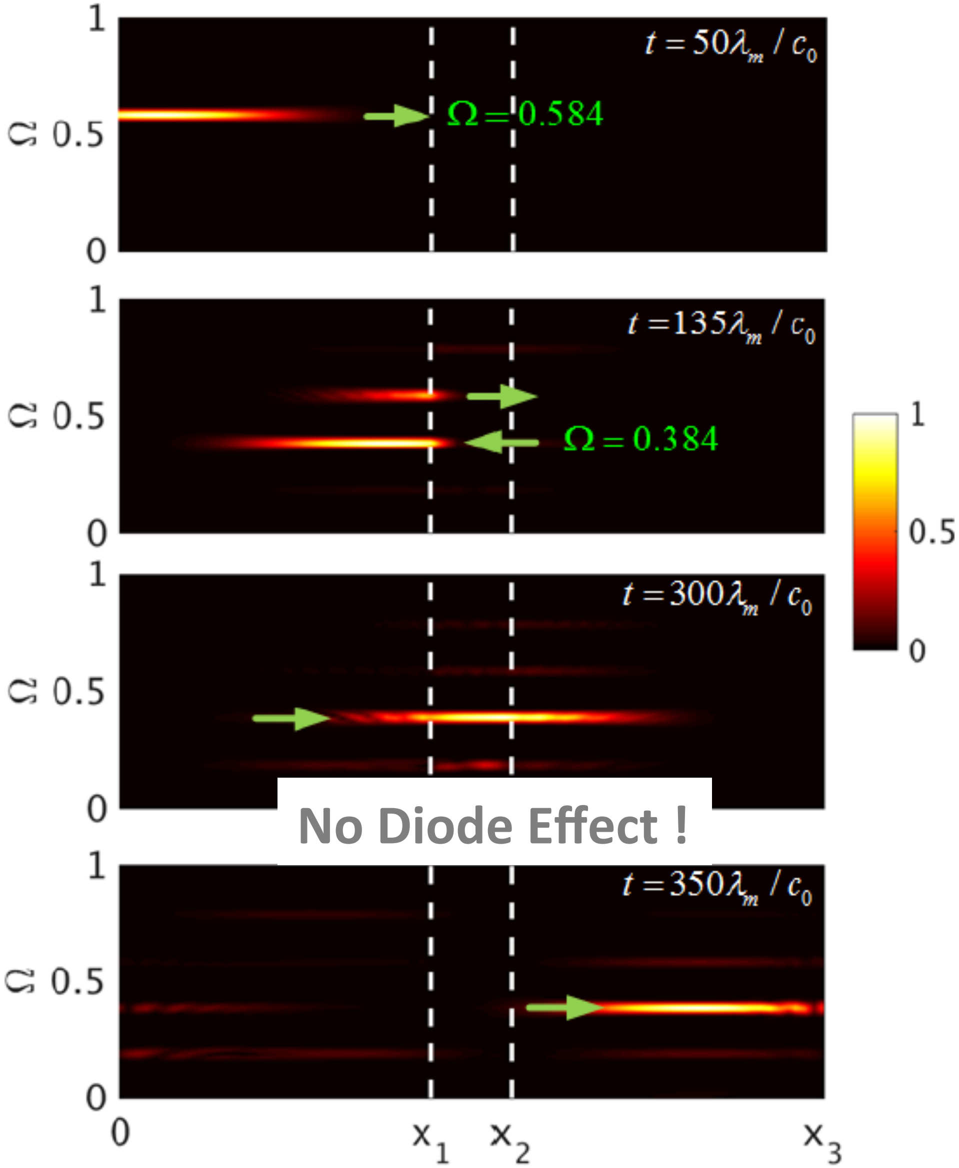
Example of obtained wave dispersion curves :

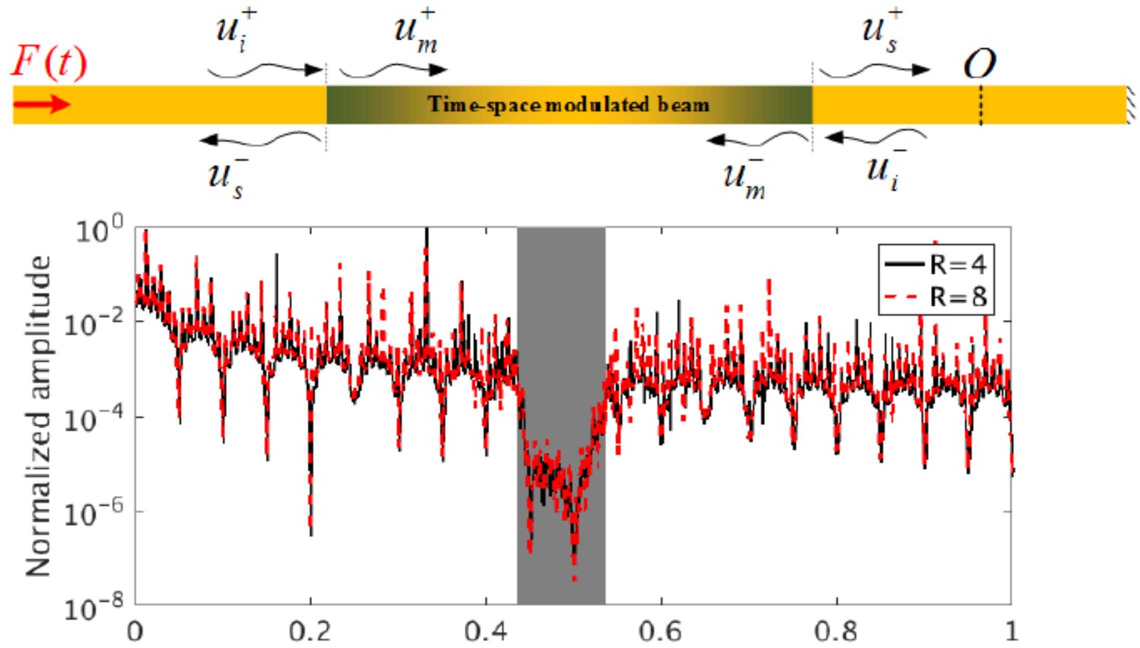




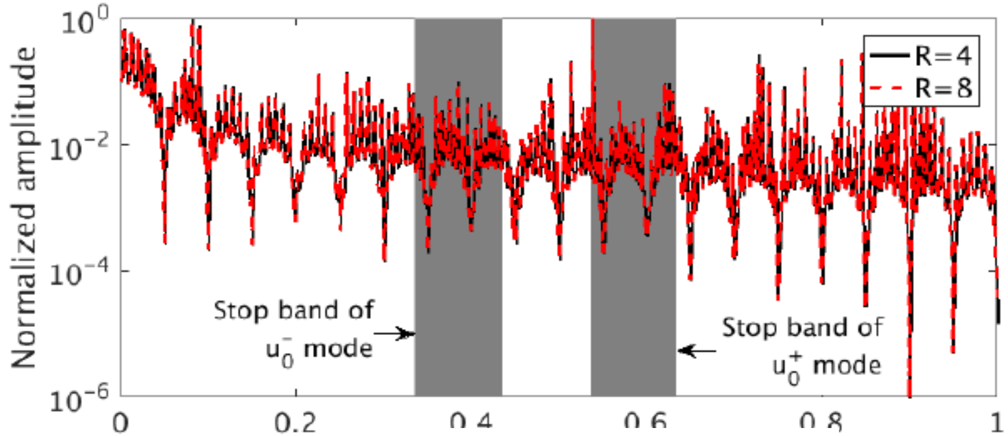
Modeling aspects : Plane Wave Expansion PWE

Example of obtained response :



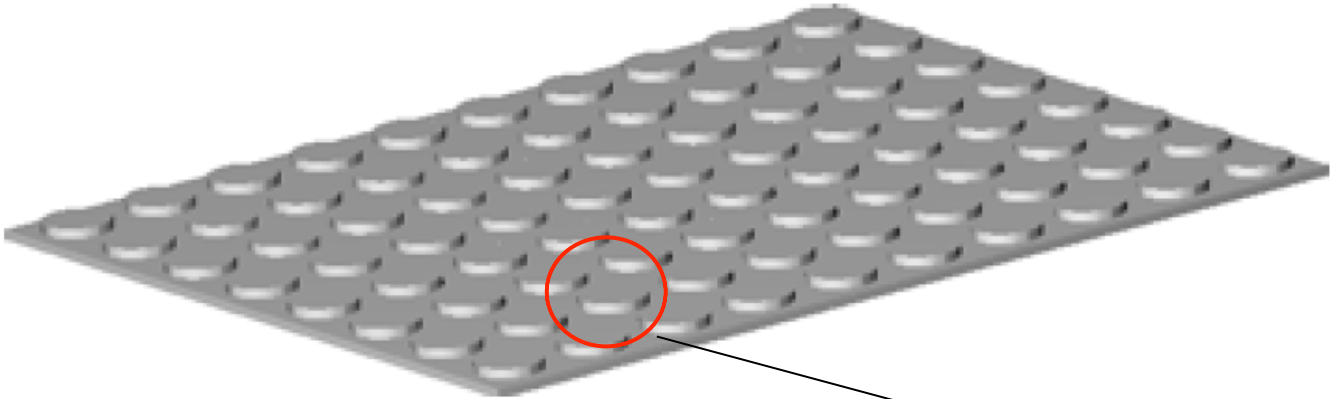


Only standard Bragg Effect



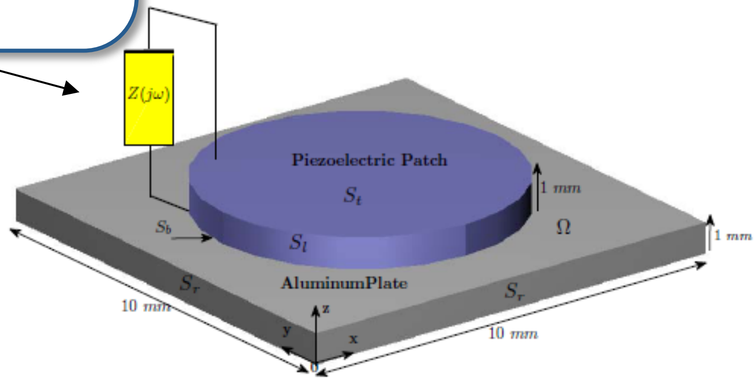
Modulating Beam

Design functional structures using band gap properties : Waves diffusion

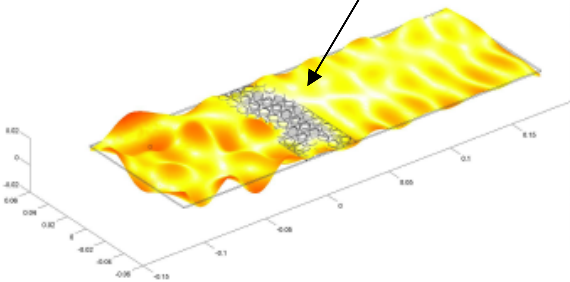


The metacomposite

PZT-Aluminum Composite

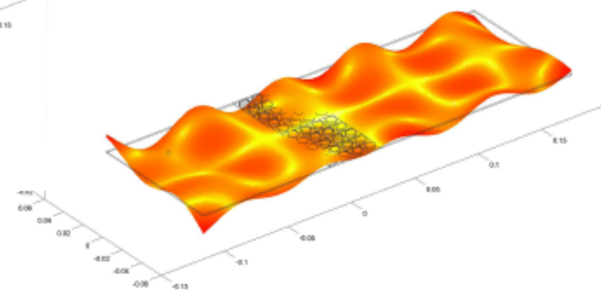


Interface with two configurations



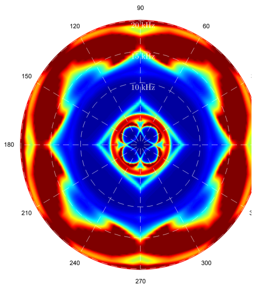
Vibration isolation

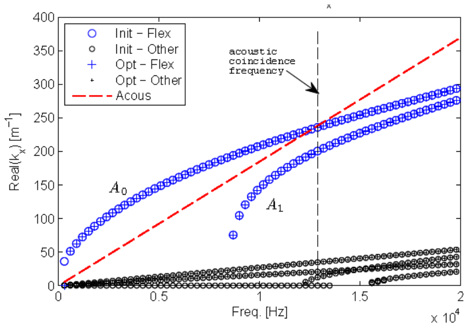
Vibration damping



…a distributed control device with no computation: combination of analog structural/electrical loops communicating through the structure

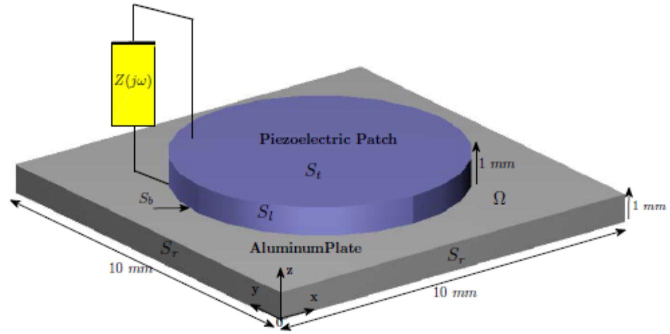
Design functional structures using band gap properties : Waves diffusion





- Efficient tool for computation of dispersion diagrams
- Multiphysics damped system

How to choose Z(w) for specific functionnalities?



Elementary cell

Minimize group velocity of flexural waves: vibration & acoustics limitation

Case REFL: Stop propagation of flexural waves

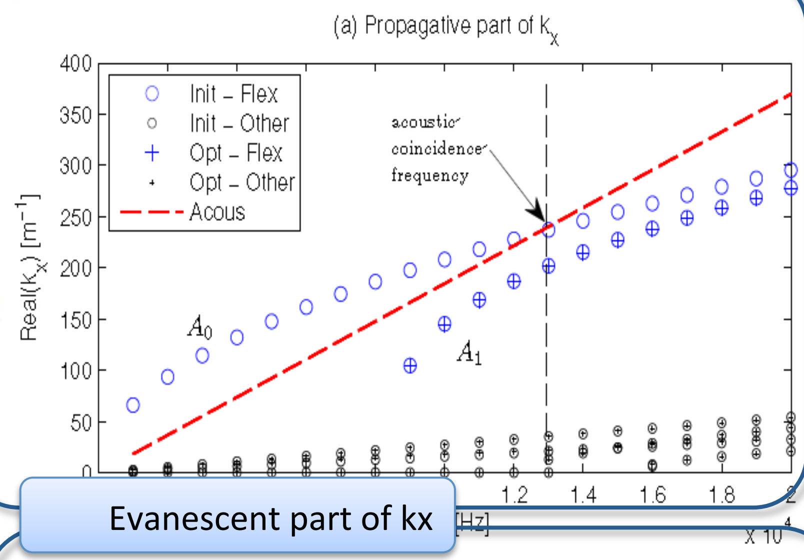
Maximize electric energy dissipation in shunt

Case ABS: Maximize dissipation

Optimization procedure: find optimal *Z(w)*

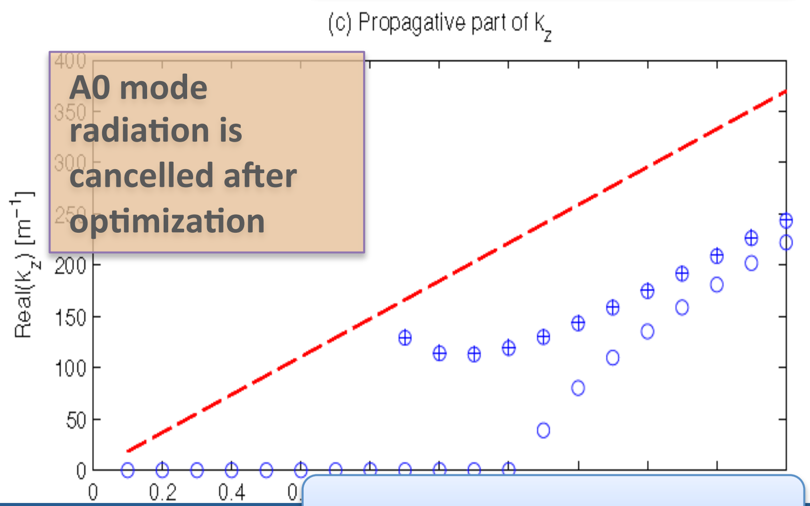
Design functional structures using band gap properties : Waves diffusion

Propagative part of kx

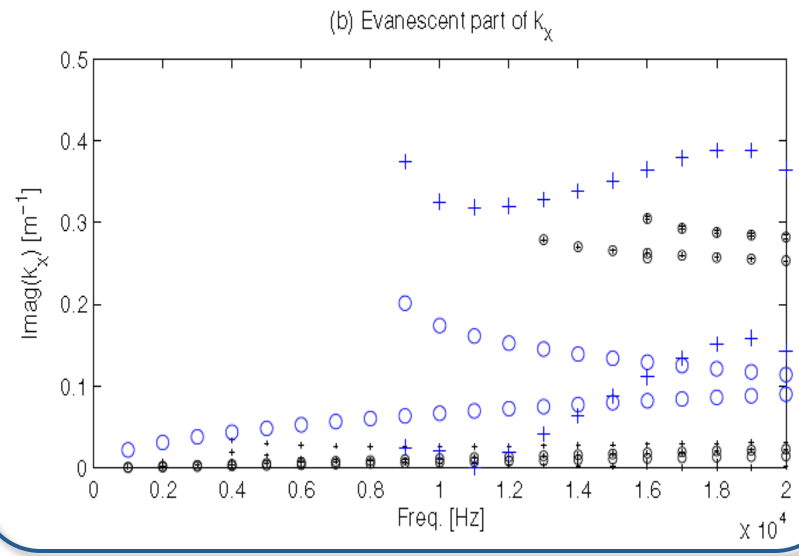


Propagative part of kz

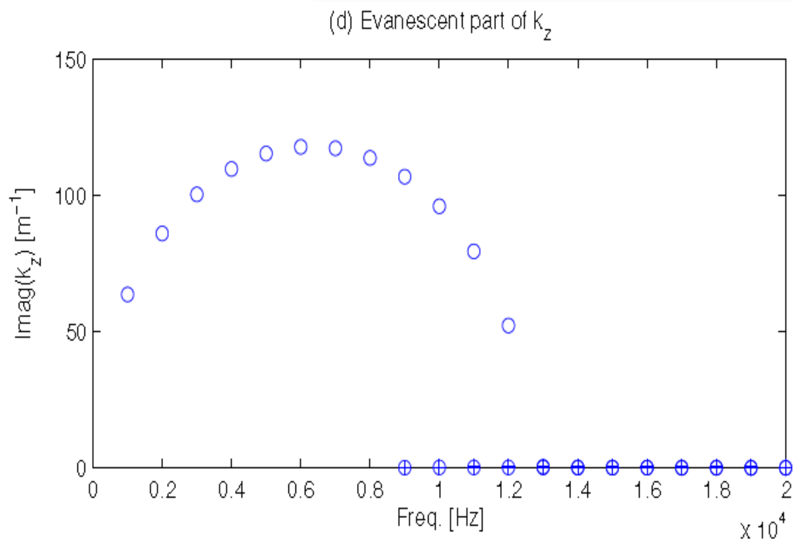
A0 mode radiation is cancelled after optimization



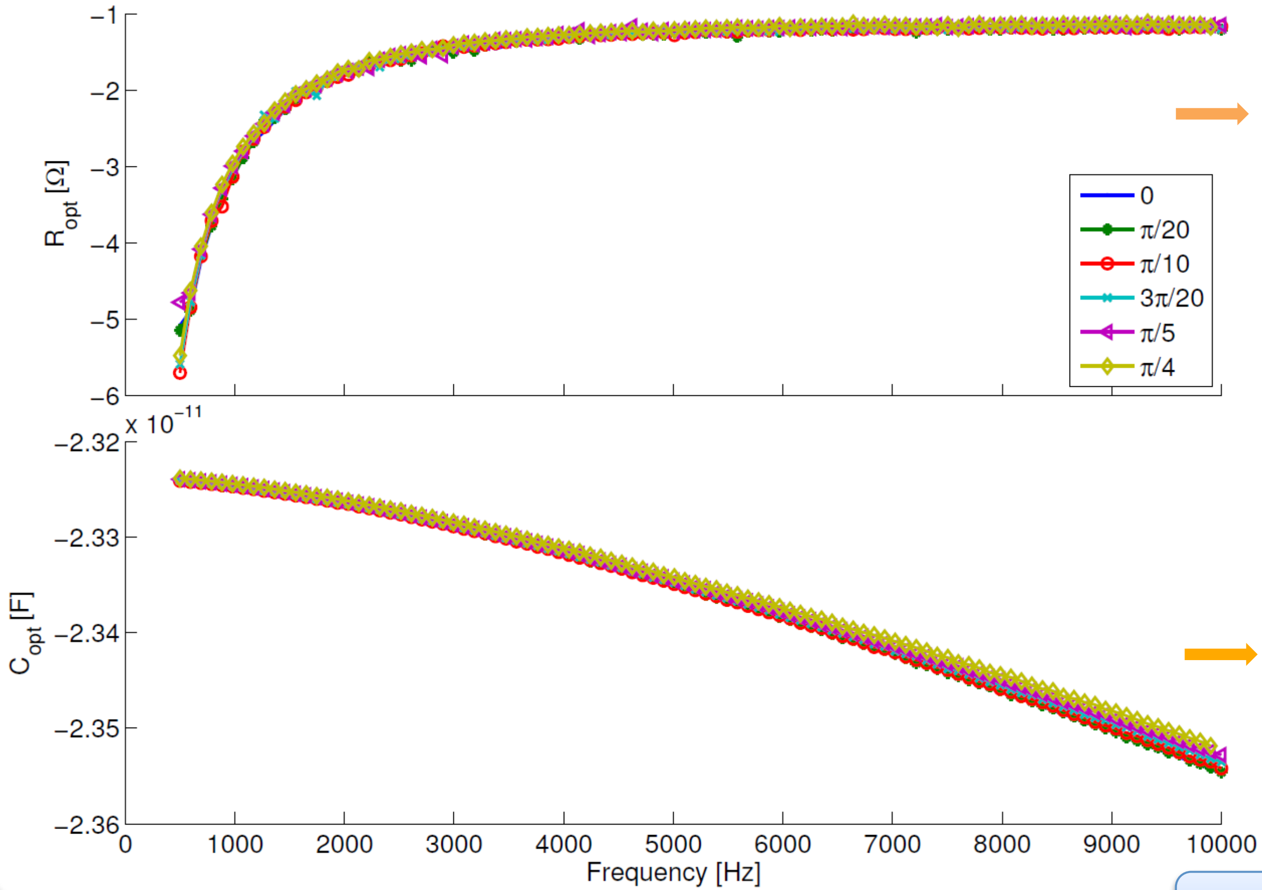
Evanescent part of kx



Evanescent part of kz



Design functional structures using band gap properties : Waves diffusion

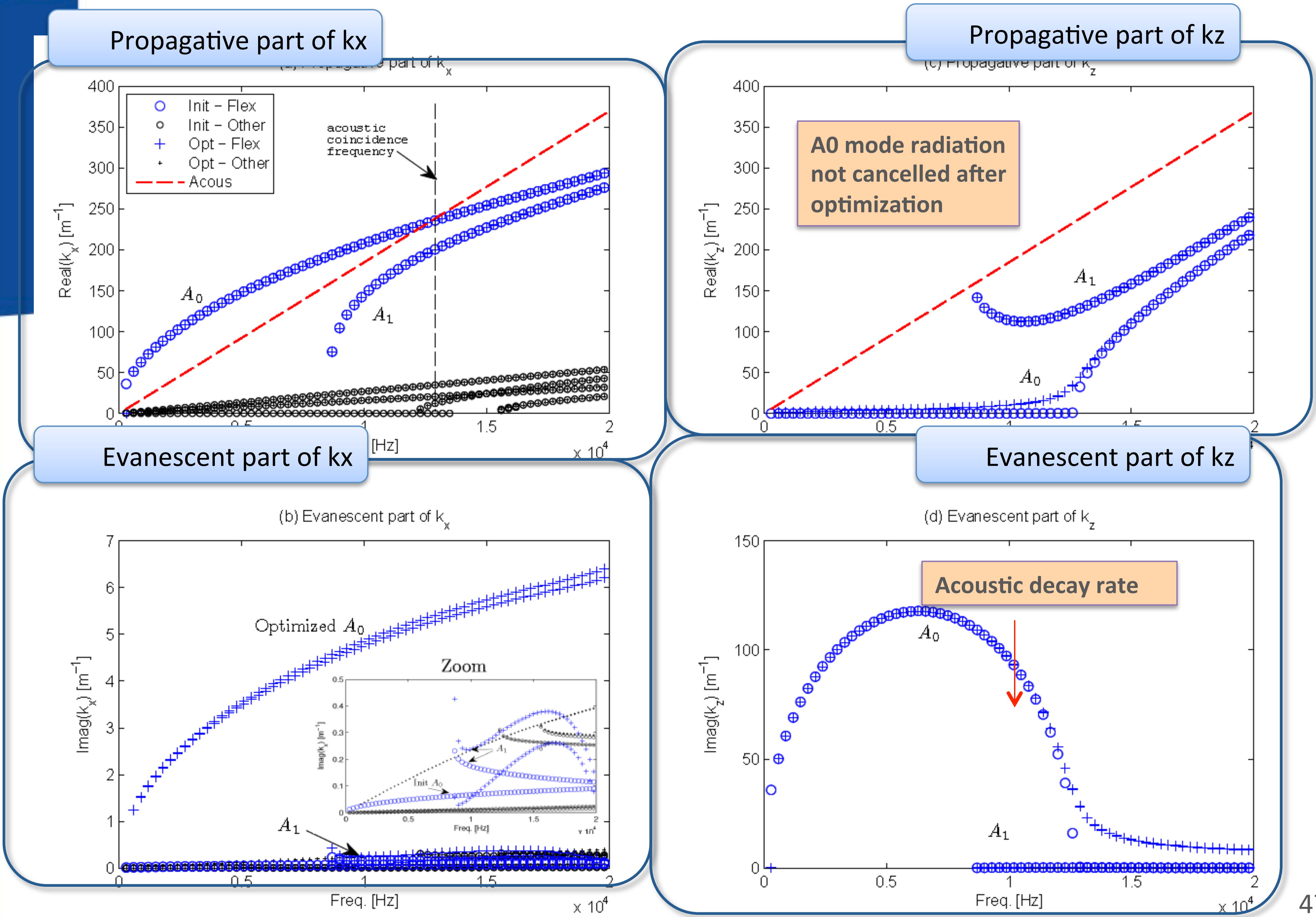


Reactive circuit (cancels structural damping)

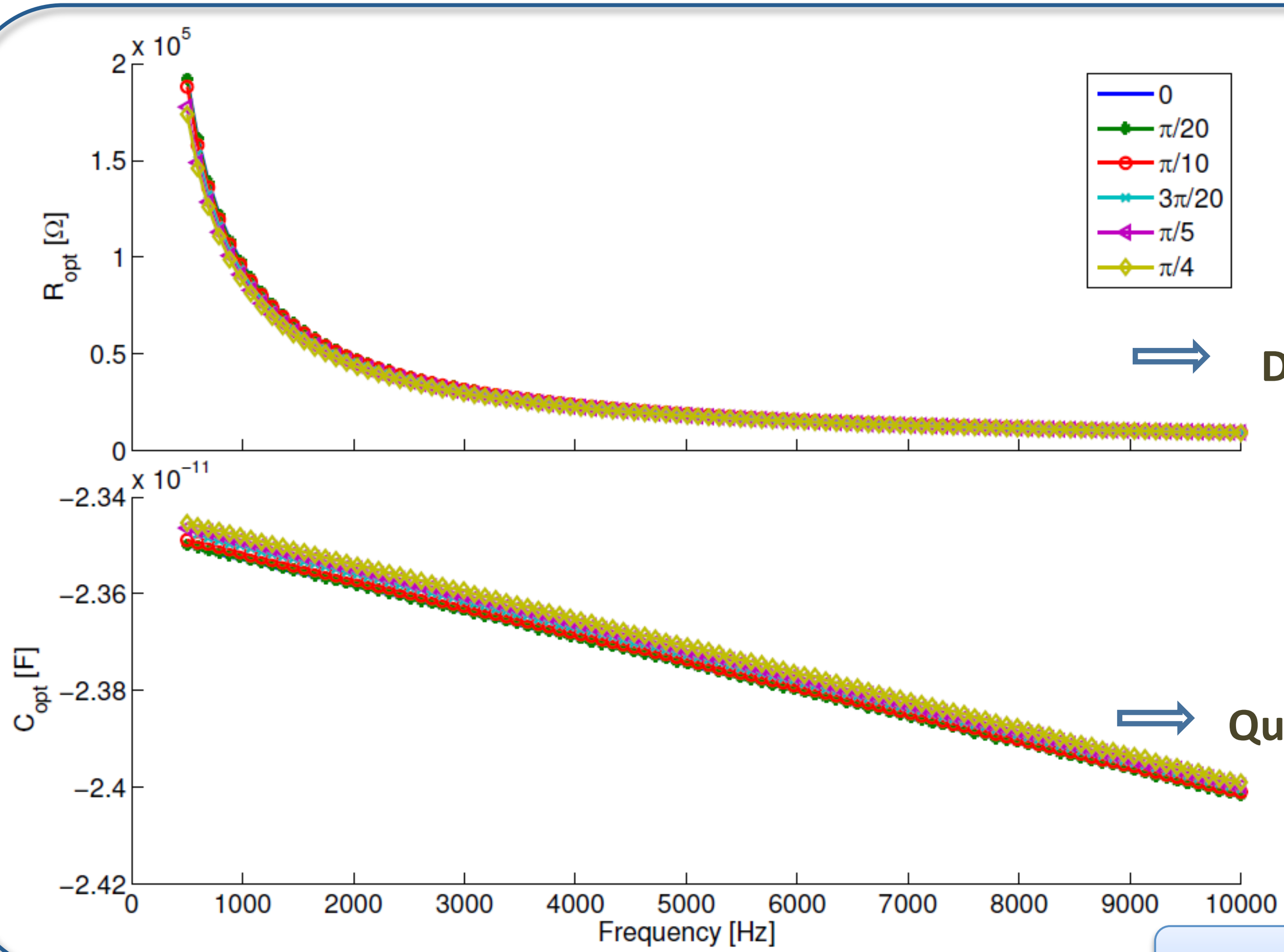
Quasi constant Cneg

Optimal electrical impedance

Design functional structures using band gap properties : Waves diffusion



Design functional structures using band gap properties : Waves diffusion

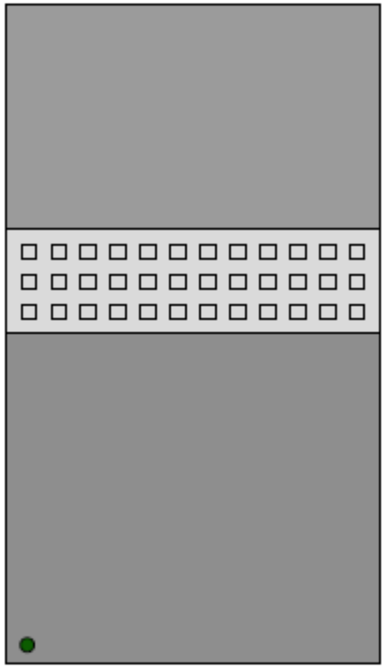


⟹ **Dissipative circuit**

⟹ **Quasi constant Cneg**

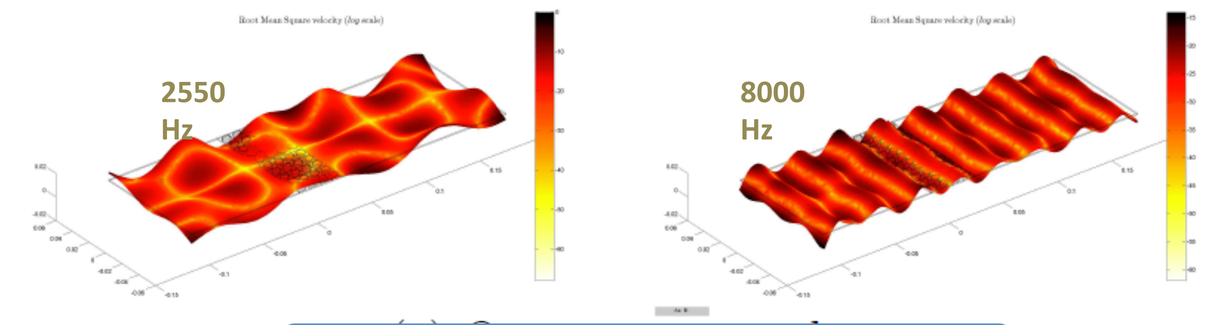
Optimal electrical impedance

Design functional structures using band gap properties : Waves diffusion

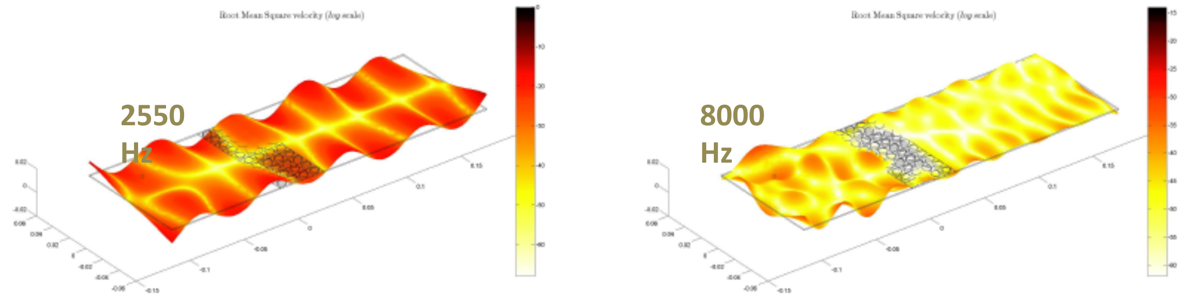


- Non-resonant shunt
- A0 wavelength @ 5 kHz = 30 cm
- Length of active interface : 35 cm

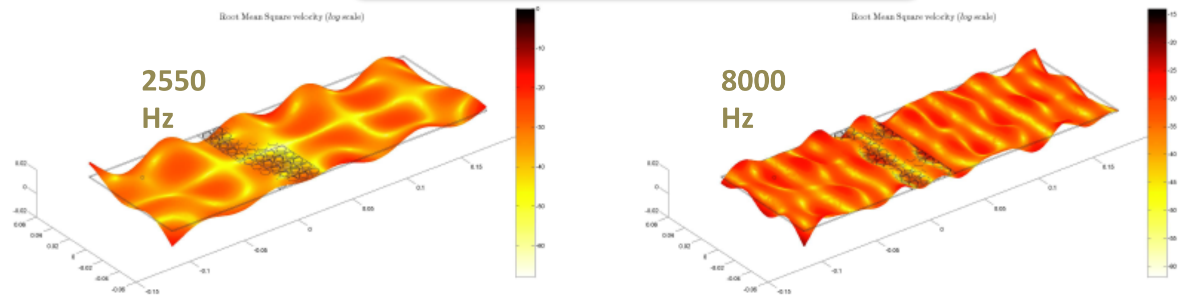
- Reconfiguration: only change the value of resistance in shunt circuit



Uncontrolled structure @ 2.5 & 8 kHz

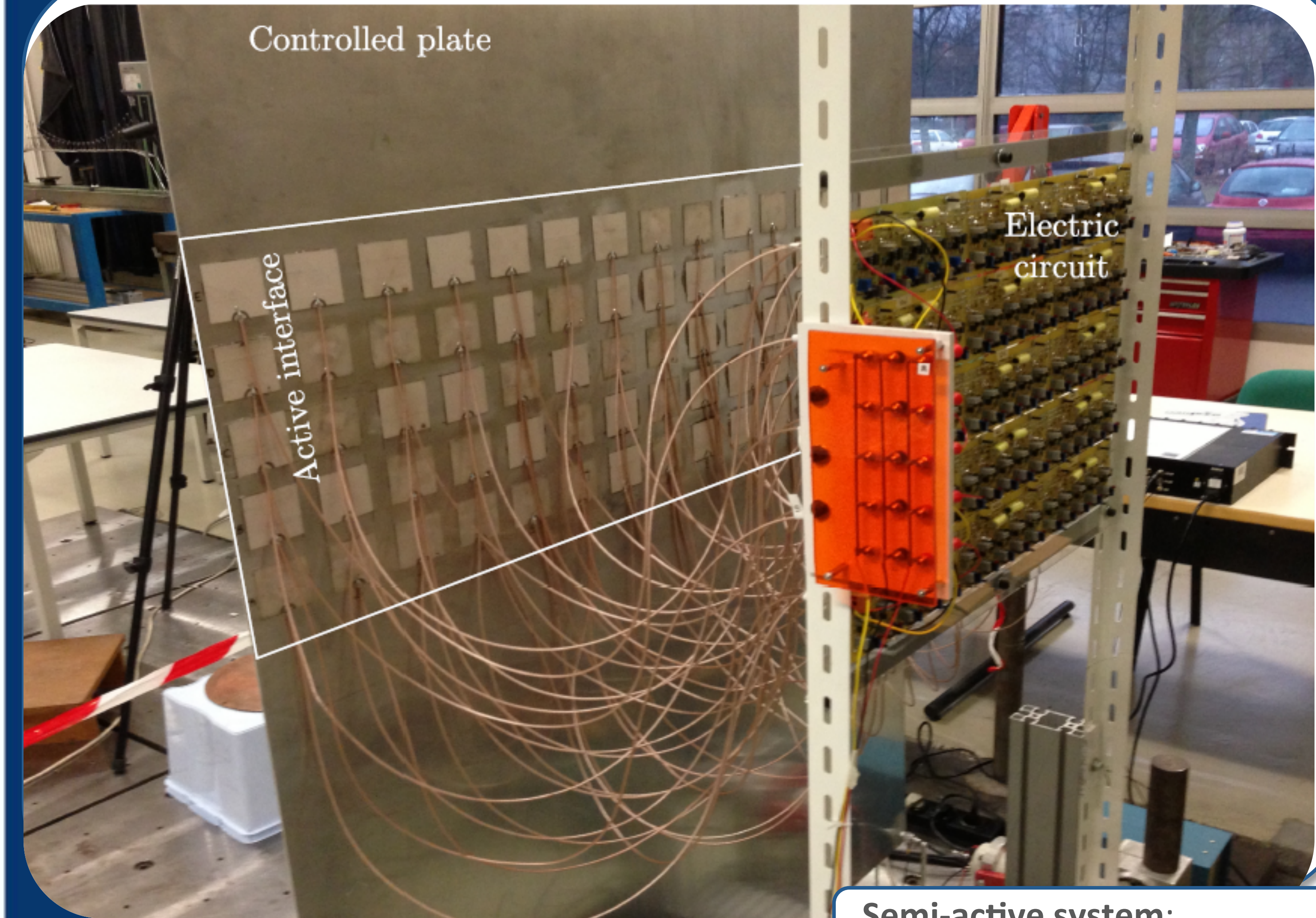


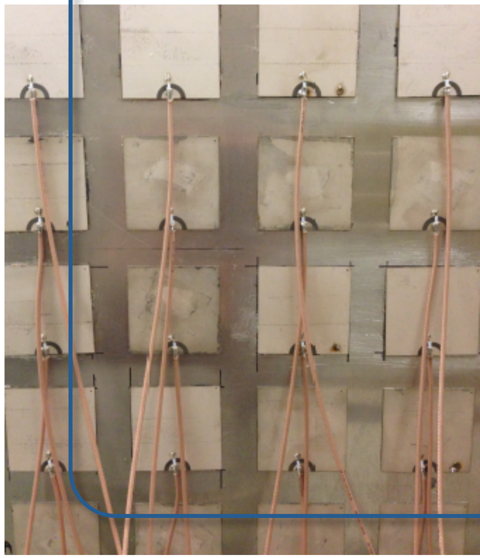
Case REFL @ 2.5 & 8 kHz

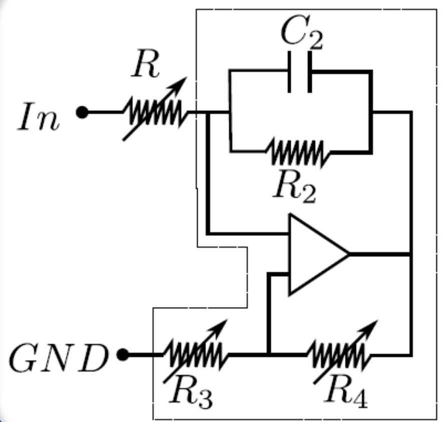


Case ABS @ 2.5 & 8 kHz

Design functional structures using band gap properties : Waves diffusion



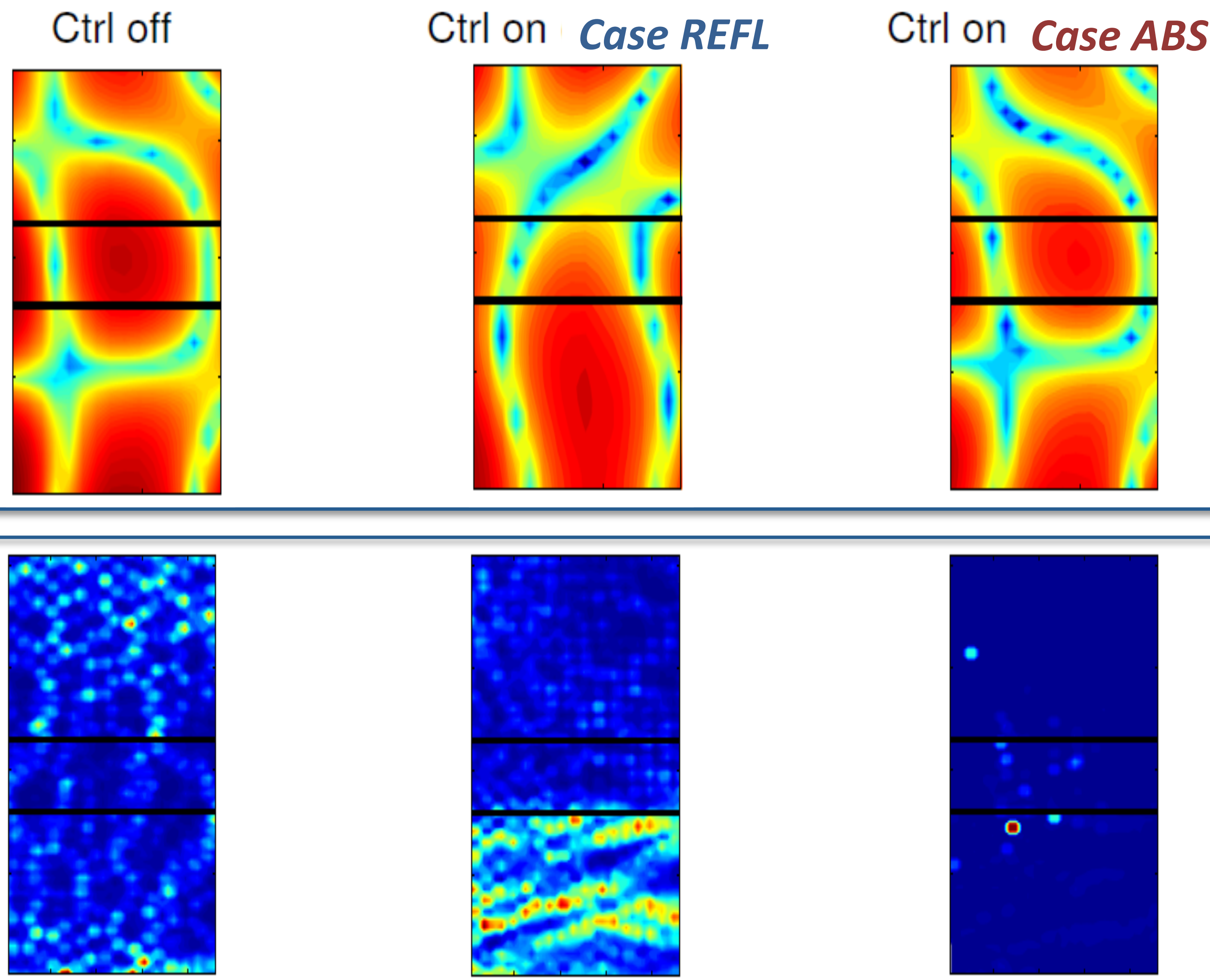




Semi-active system:
- No control loop => robustness
- Need only to power Op-Amps

Design functional structures using band gap properties : Waves diffusion

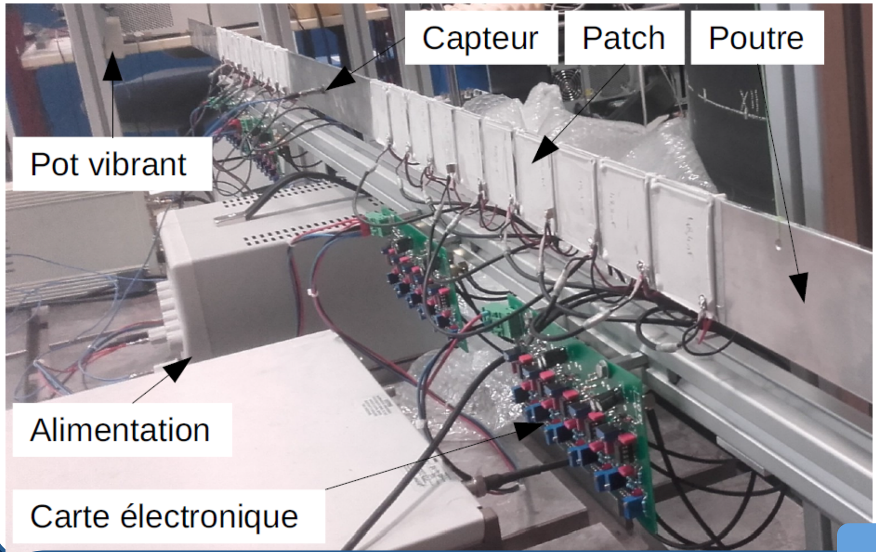
Measurement @ 25 Hz

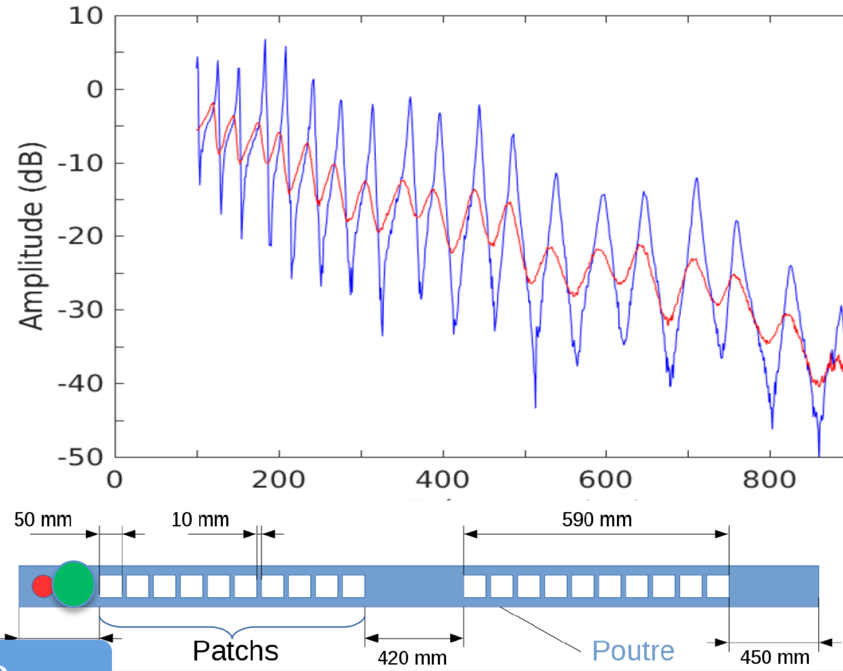


Measurement @ 3000 Hz

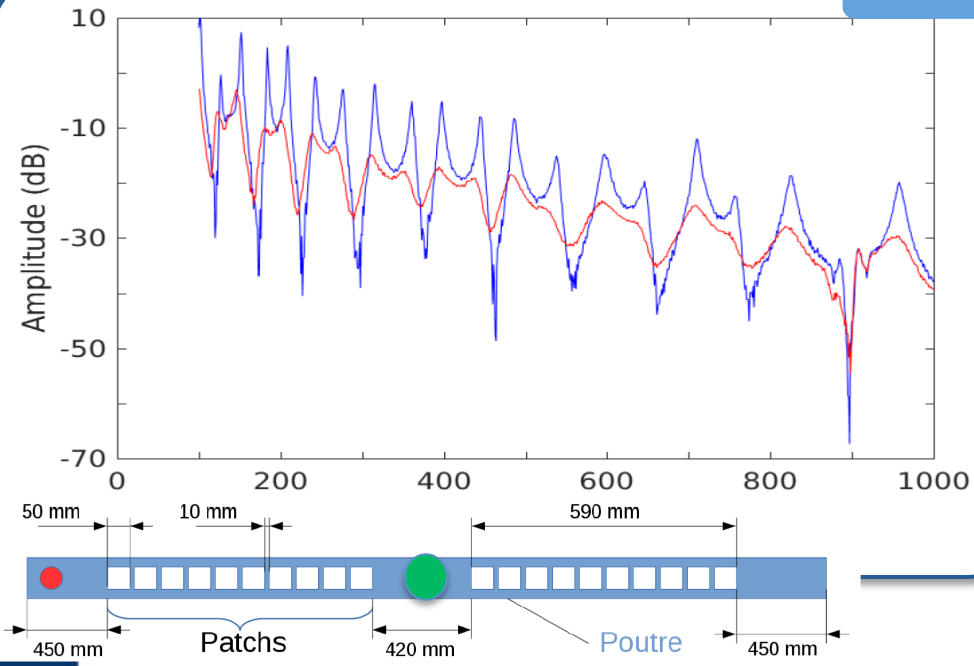
Design functional structures using band gap properties : Waves diffusion

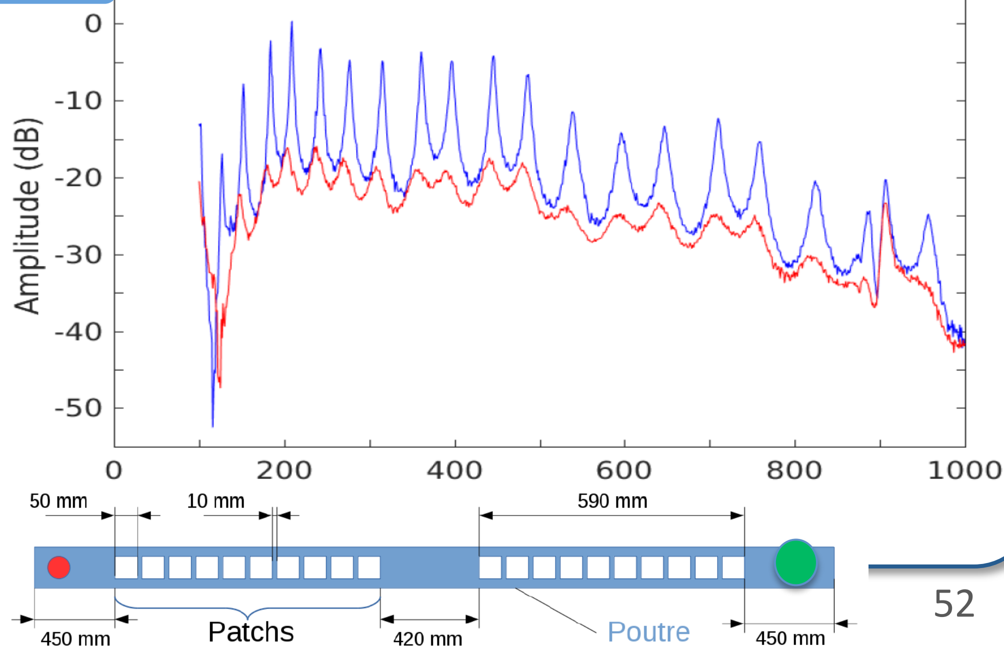
A beam with two interfaces: Low Frequency





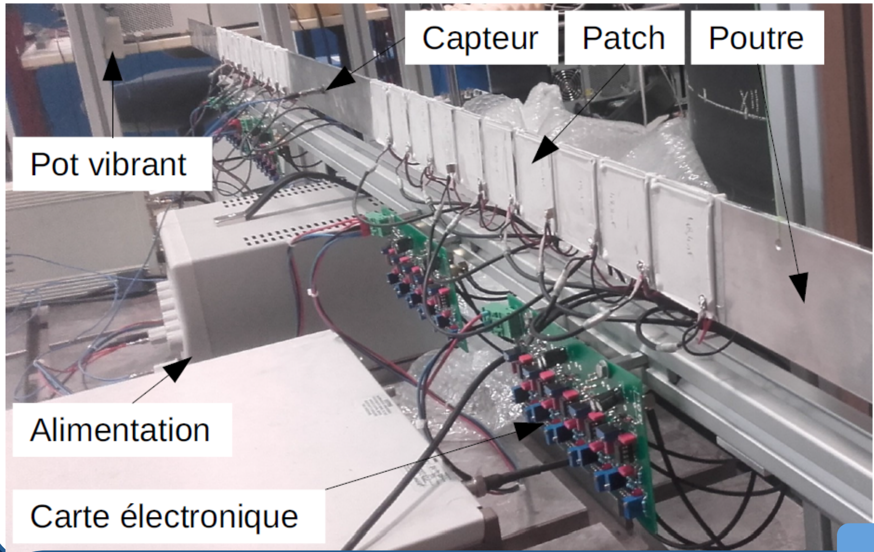
R=500 Ω

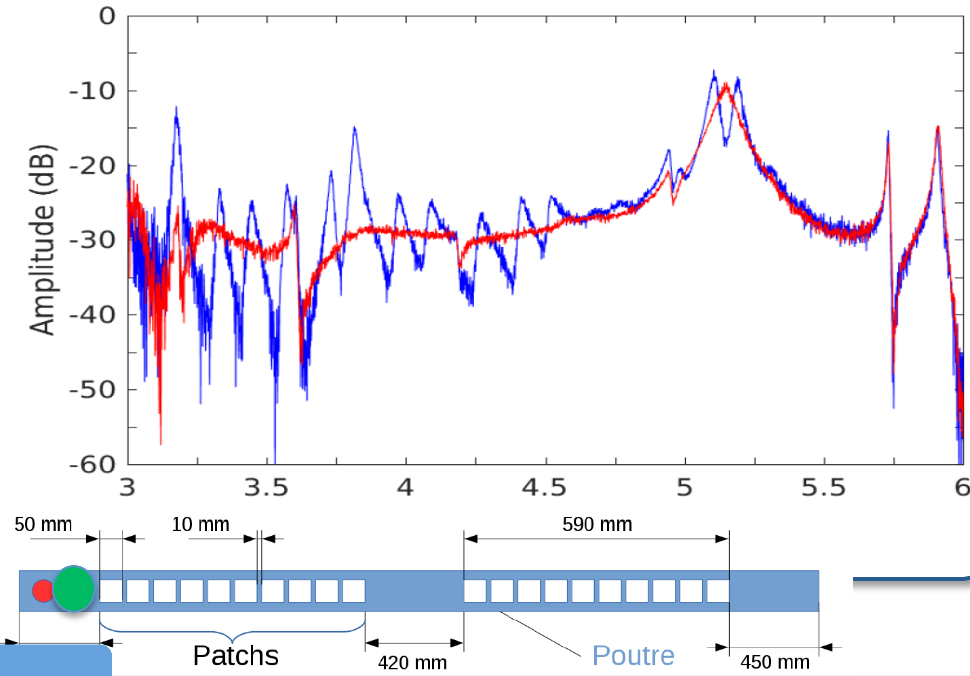




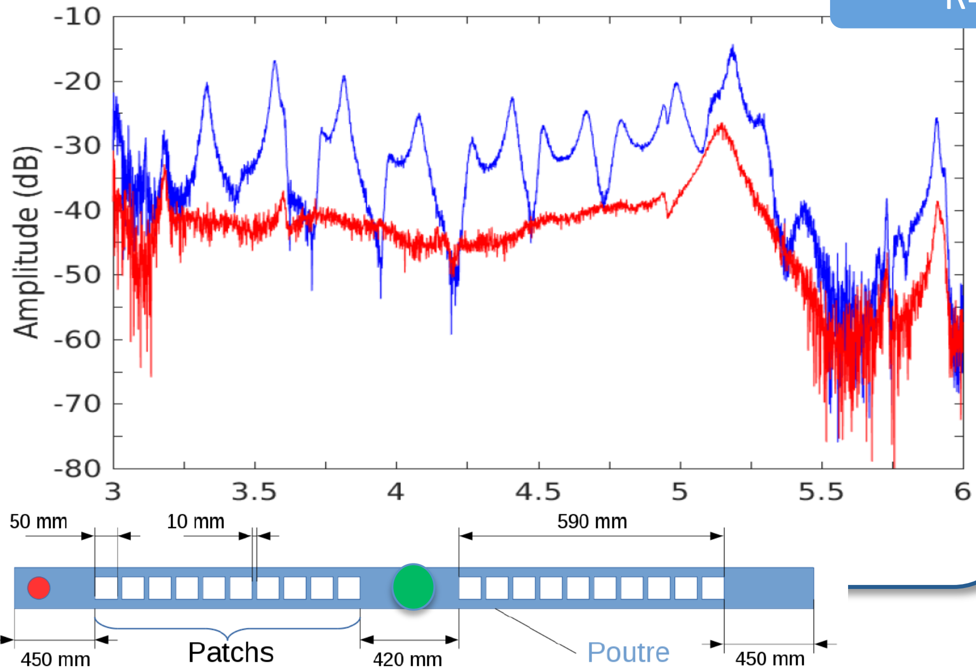
Design functional structures using band gap properties : Waves diffusion

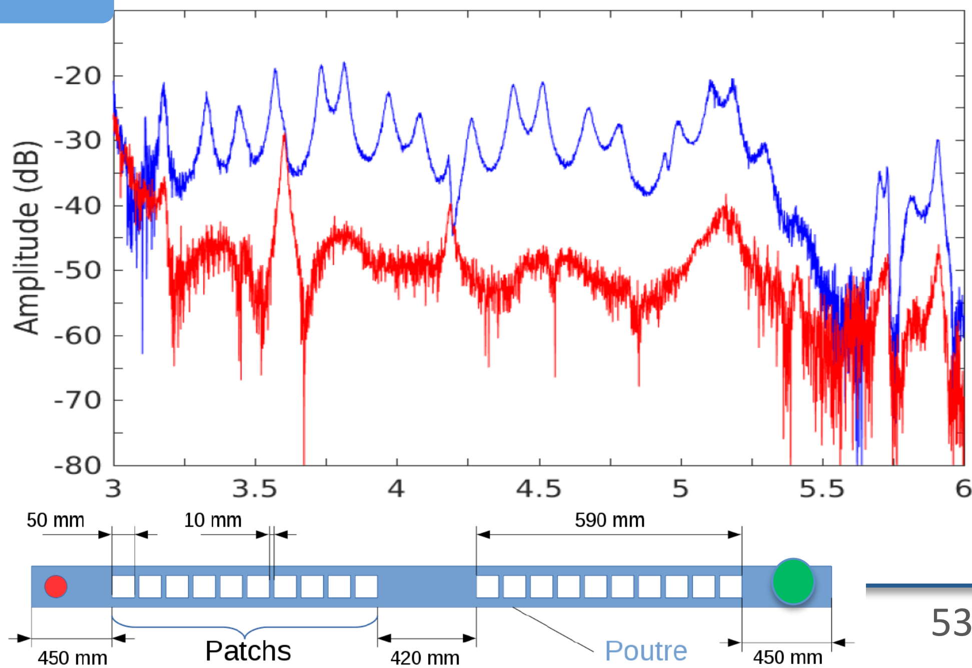
A beam with two interfaces: High Frequency





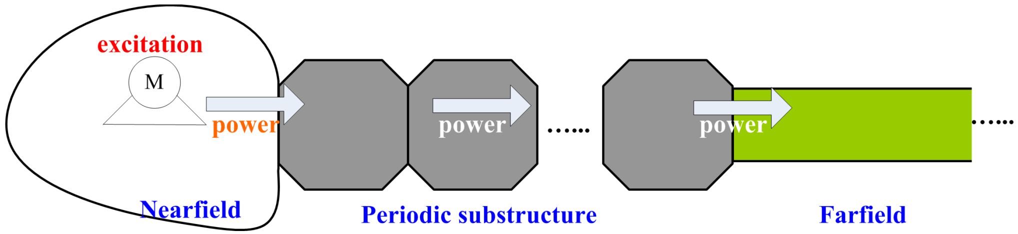
R=40 Ω



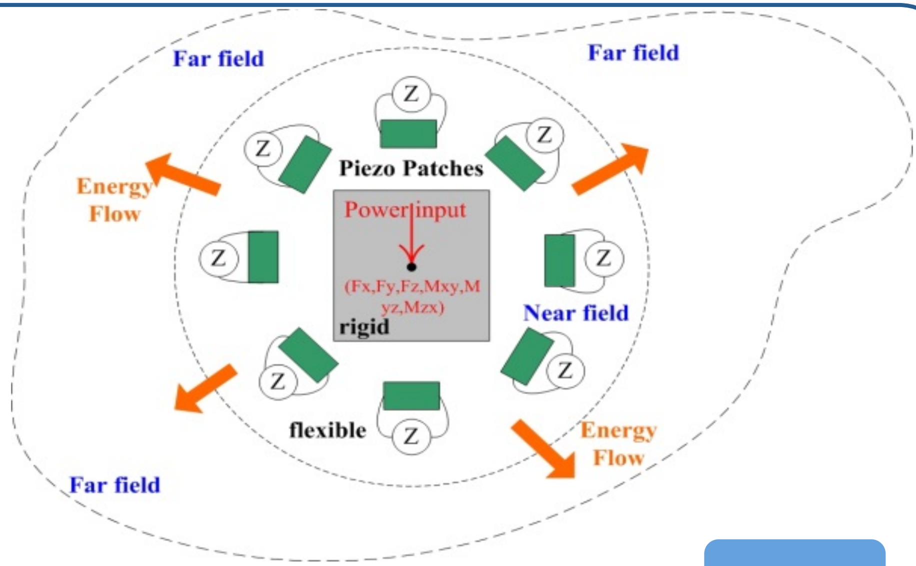


Design functional structures using band gap properties : The boundary

1D

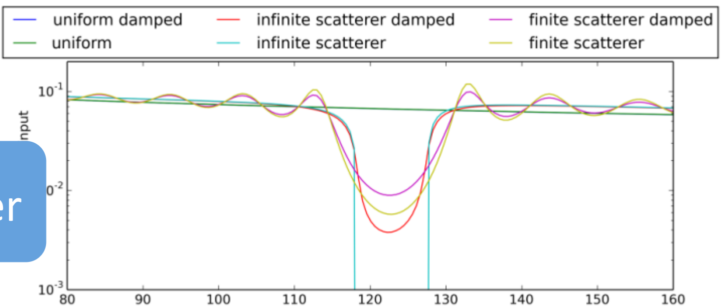


2D

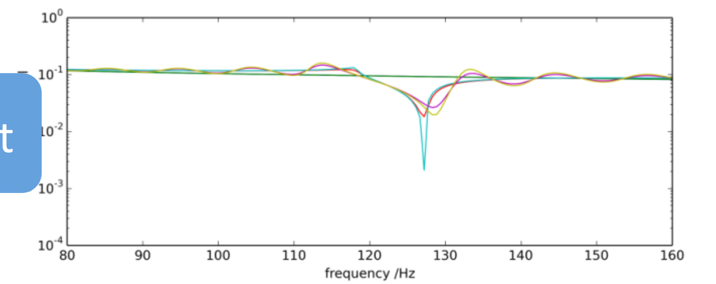


Local modes not excited

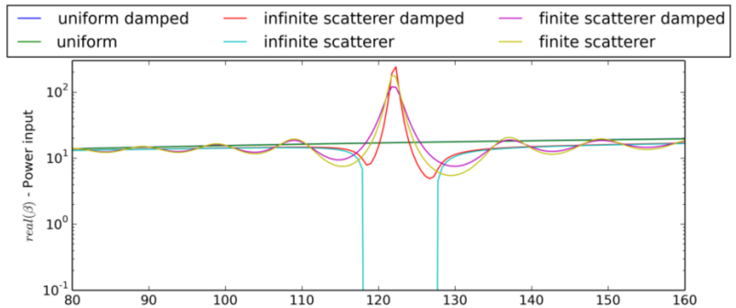
Output Power

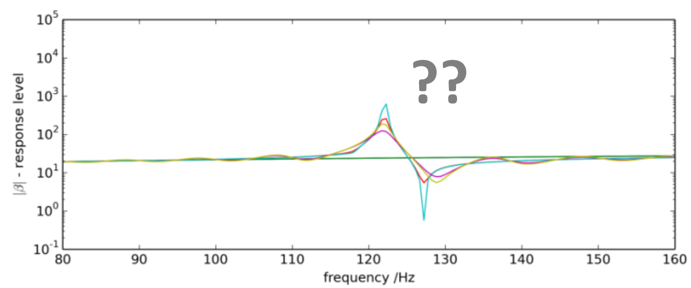


Displacement



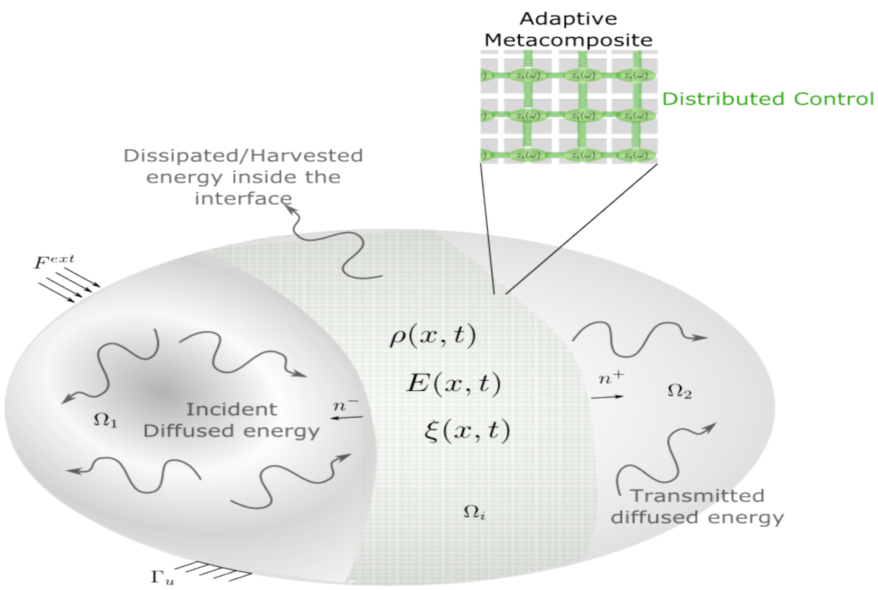
Local modes excited



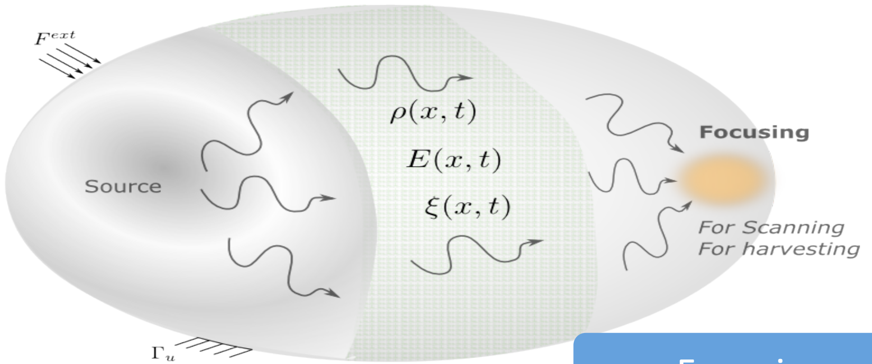


Beyond Band Gap

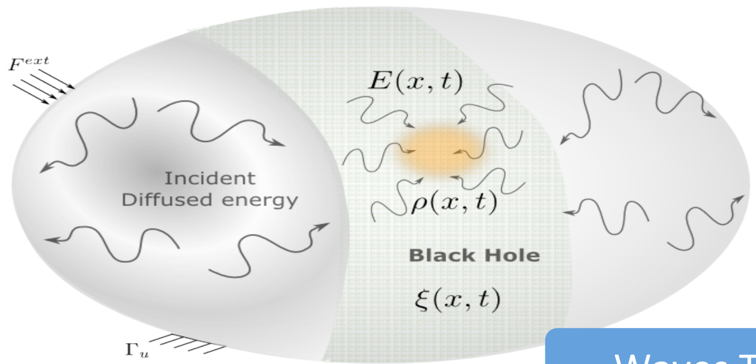
Design Graded Time Variant and Non-Linear architected Materials for Finite structure



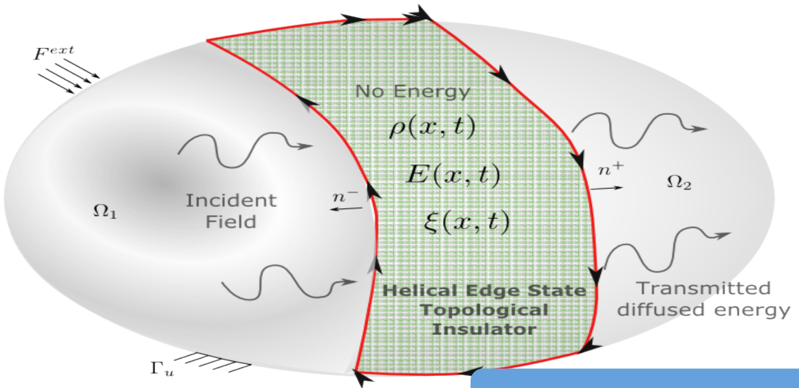
Energy diffusion



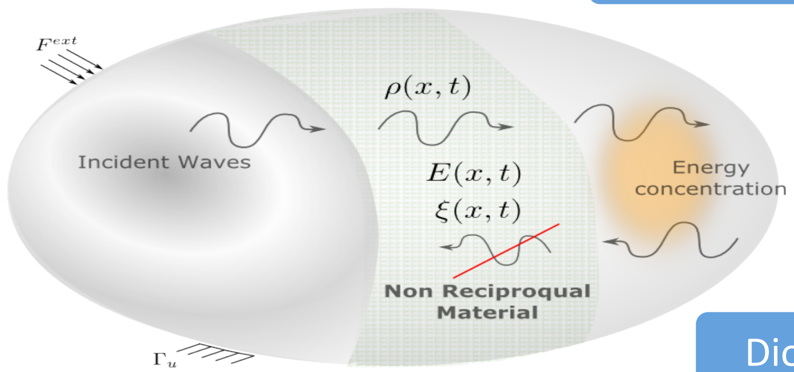
Focusing



Waves Traps

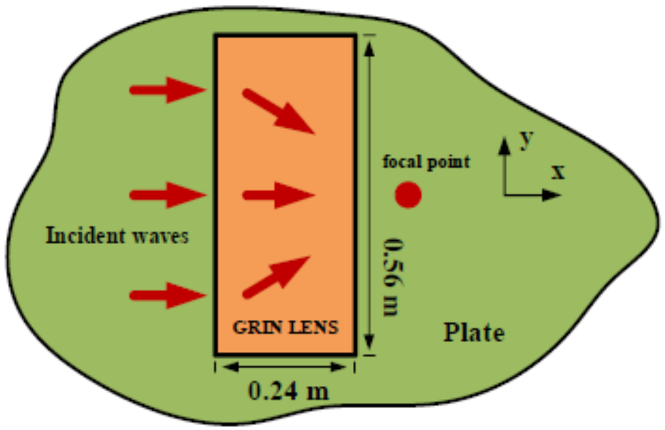


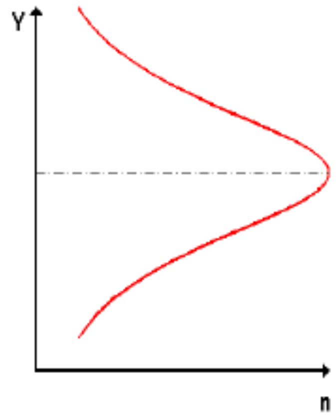
Insulator



Diodes

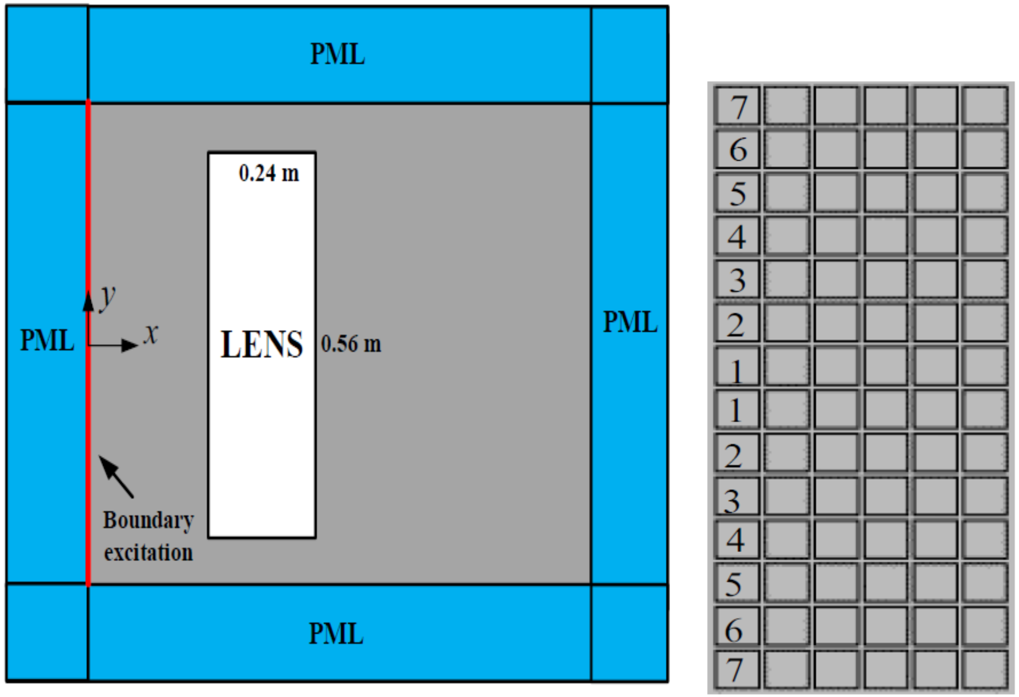
Beyond Band Gap : Adaptive Lens

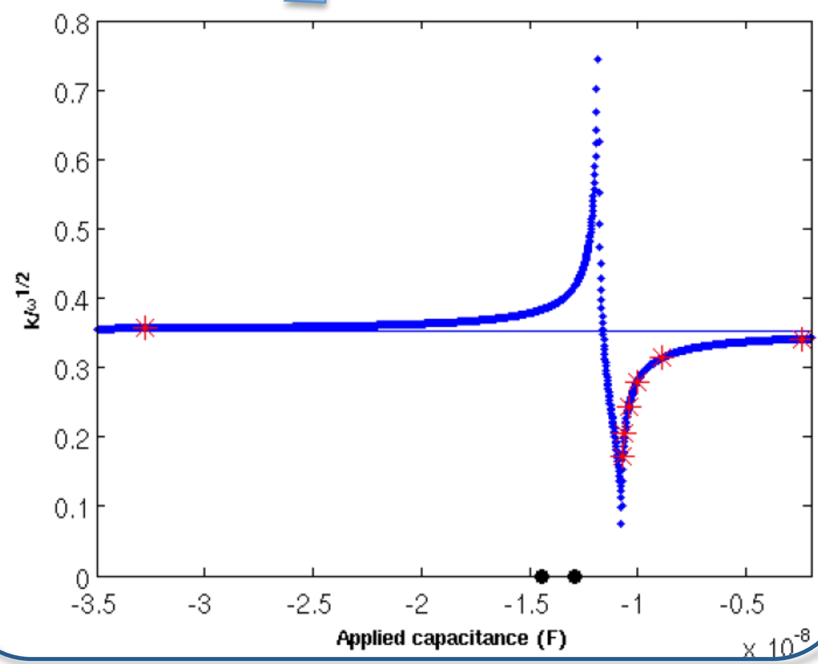




Refractive index

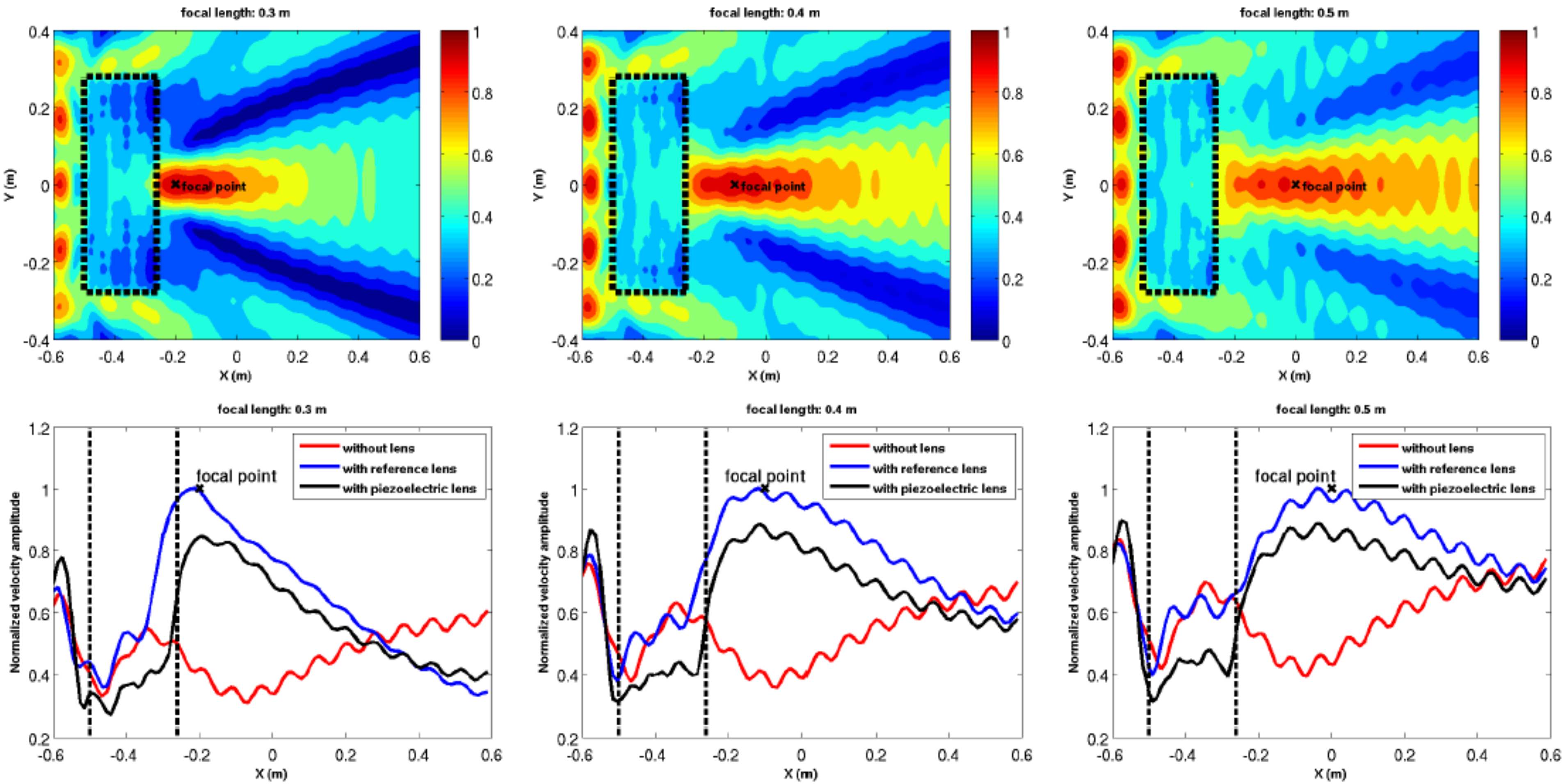
$$n(y) = \mathrm{sech}[\alpha(y - y_0)]$$





Beyond Band Gap : Adaptive Lens

Tunability of the focal point

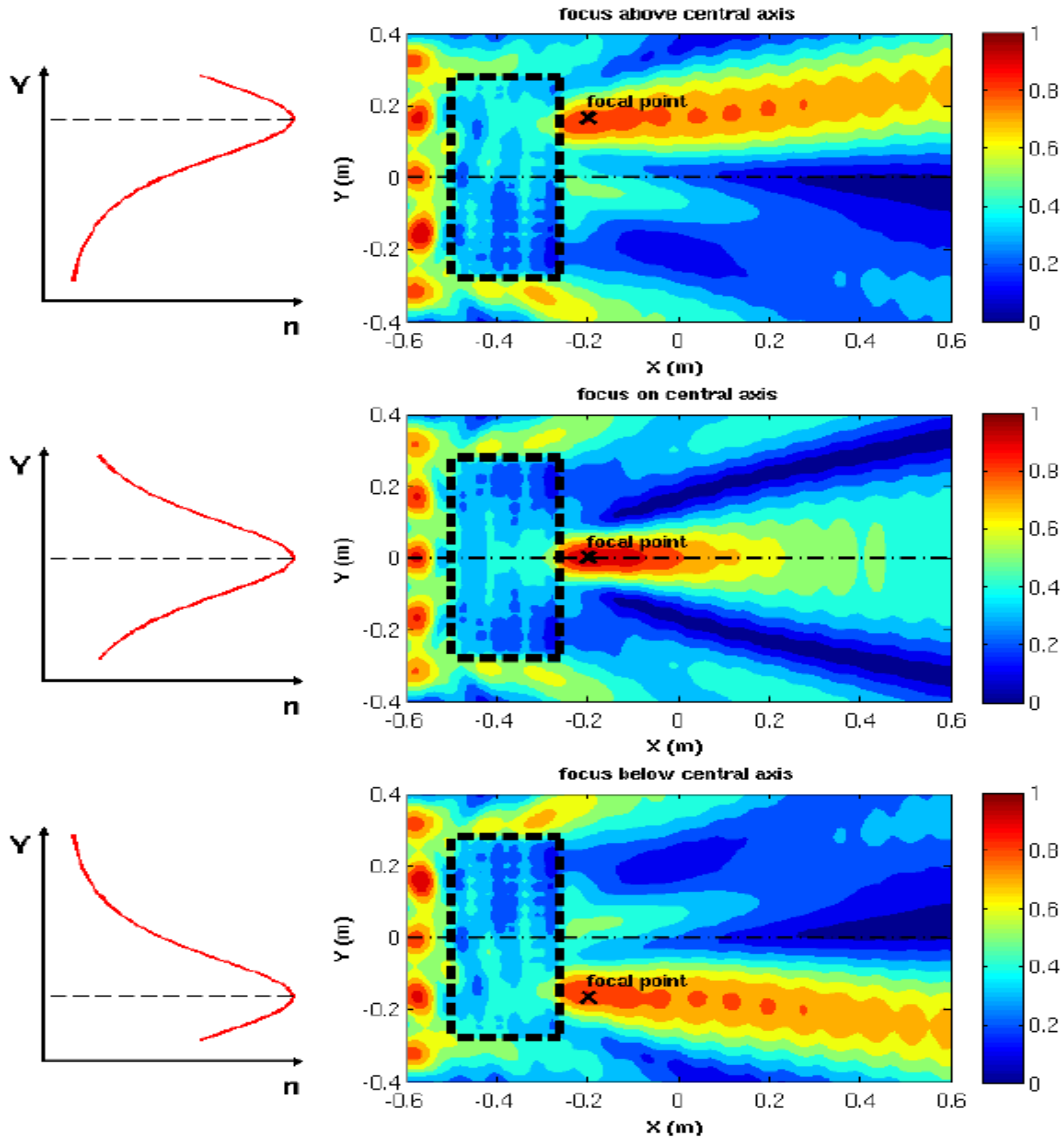


Focal length: 0.3 m | **Focal length: 0.4 m** | **Focal length: 0.5 m**

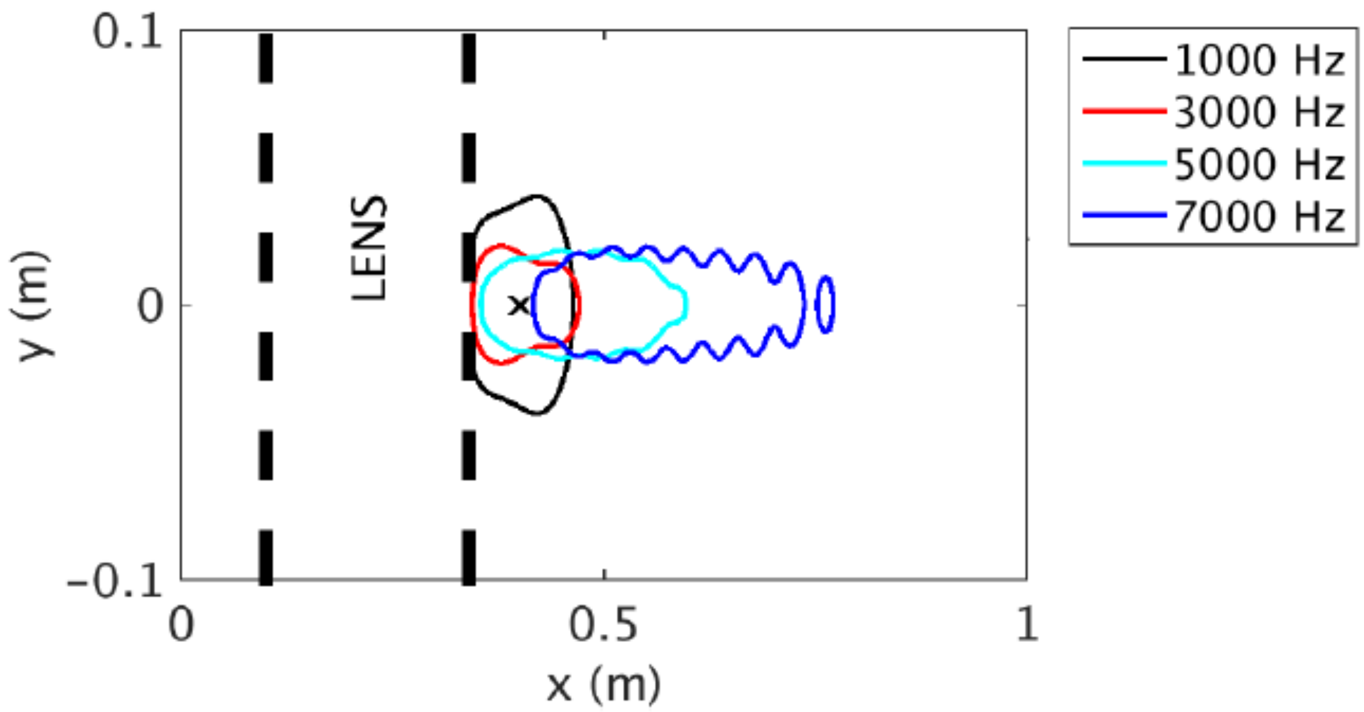
Beyond Band Gap : Adaptive Lens

Tunability of the focal point

At 2000 Hz



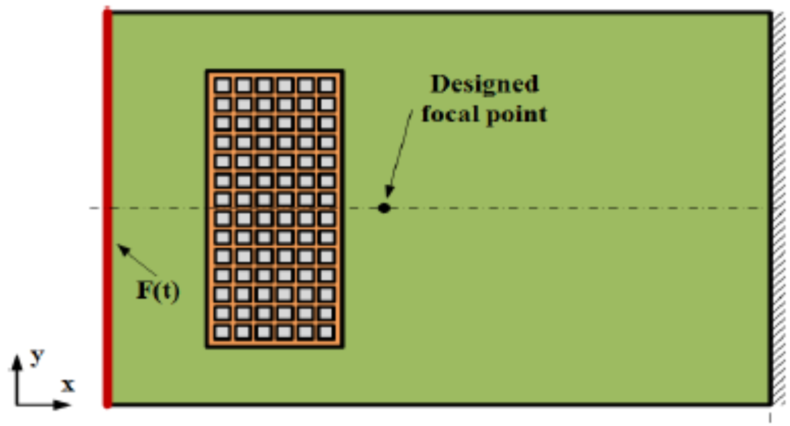
Energy Concentration zones at different frequencies



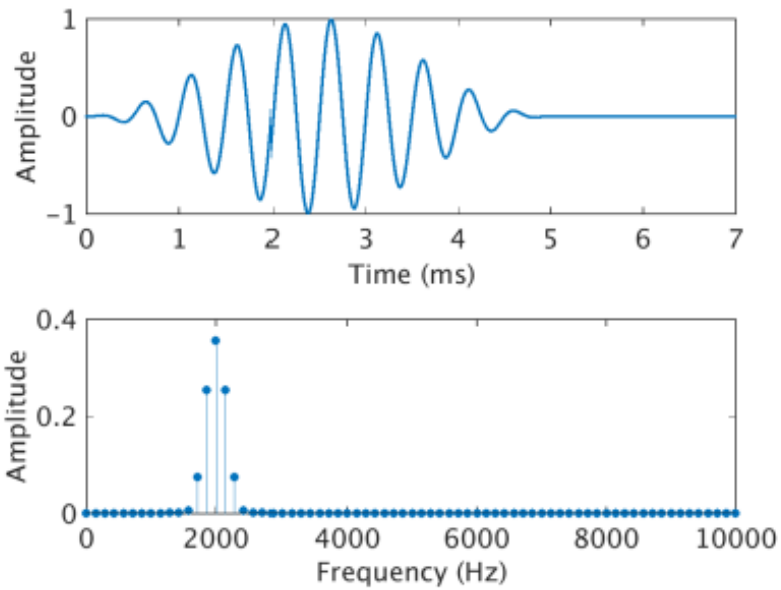
- At higher frequencies, the energy concentration zone shifts to the right.
- An overlapping zone after the designed focal point can be observed.

Beyond Band Gap : Adaptive Lens

Model: focal_length=0.3 m



Tone burst excitation:

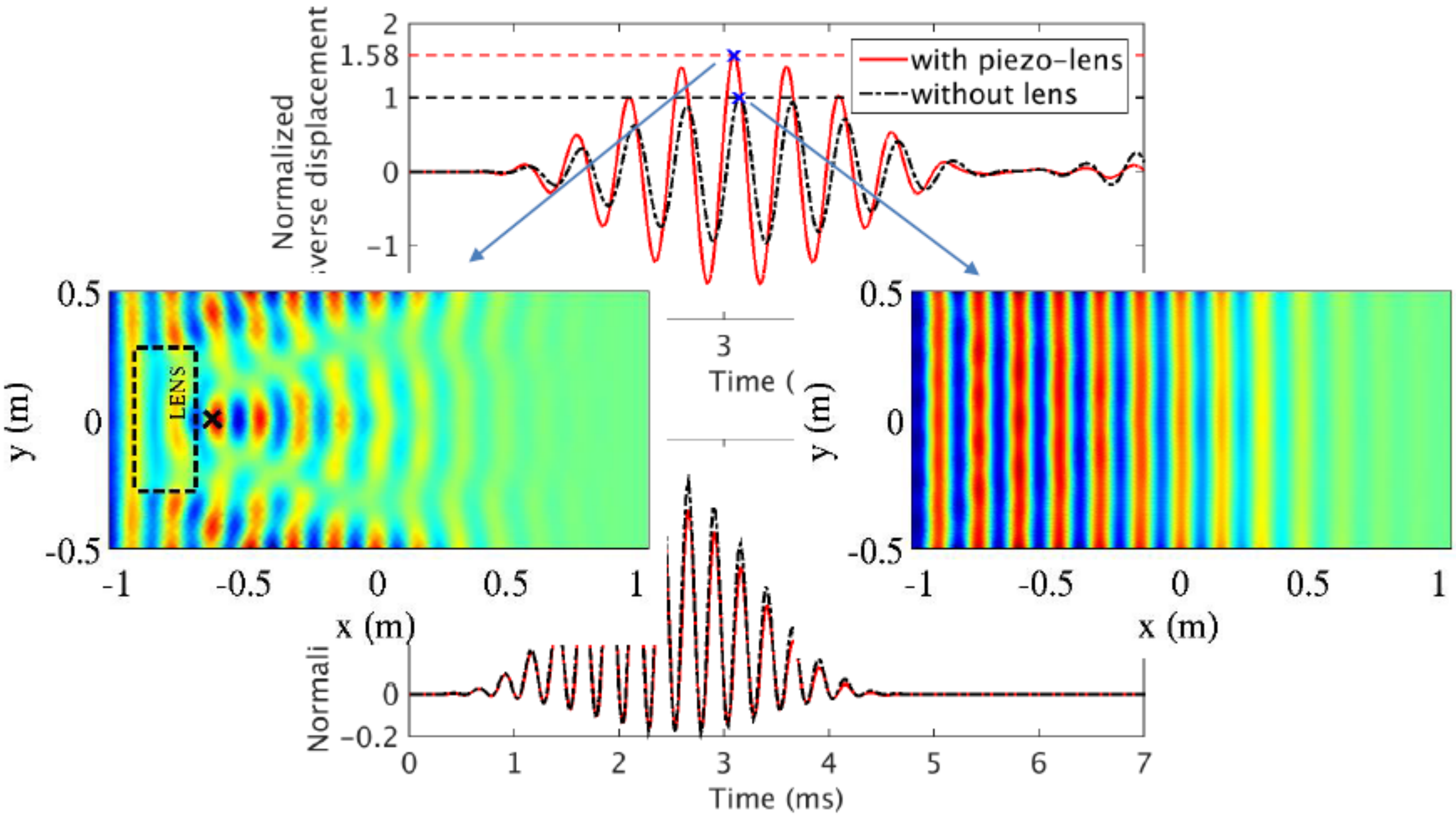


$$f_c = 2000Hz$$

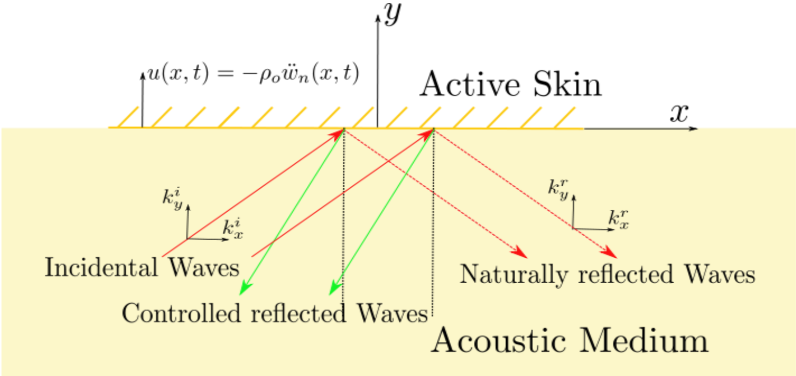
$$f_{max} = 2286Hz$$

Beyond Band Gap : Adaptive Lens

- Transverse response at focal point and input power:



Beyond Band-Gap : Reciprocity breaking and diode



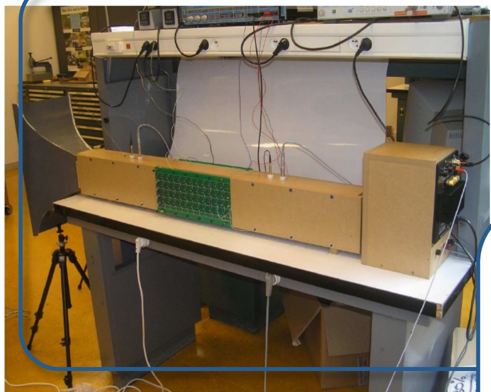
$$\begin{cases} \frac{1}{{c_0}^2}\frac{\partial^2 p}{\partial t^2} - \Delta p = 0 \quad \text{on } \Re_y^{-*} \times \Re_x \times \Re_t^{*+} \\ \frac{\partial p(x,0,t)}{\partial y} = u(x,t) \\ y(x,t) = p(x,0,t) \end{cases}$$

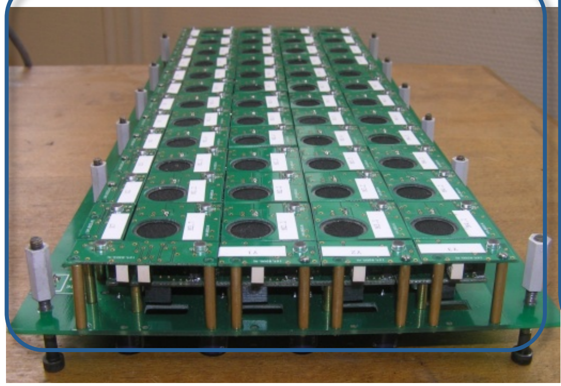
The physics

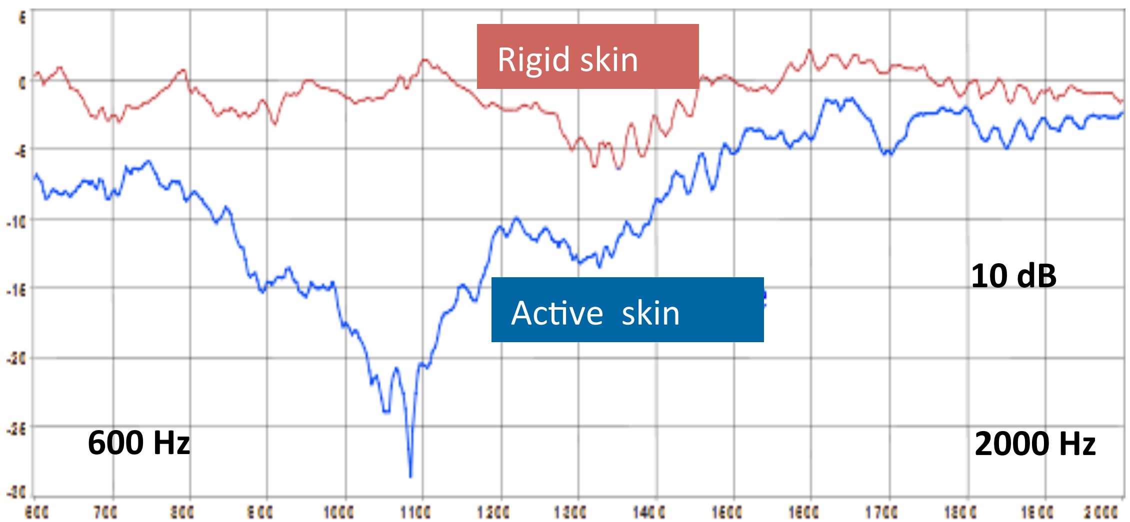
Control law that guarantees **kx<0**:

$$u(x,t) = -\left(\frac{1}{c_a}\frac{\partial p(x,0,t)}{\partial t} - \frac{\partial p(x,0,t)}{\partial x}\right)$$

Finite difference estimation of 1st-order derivatives

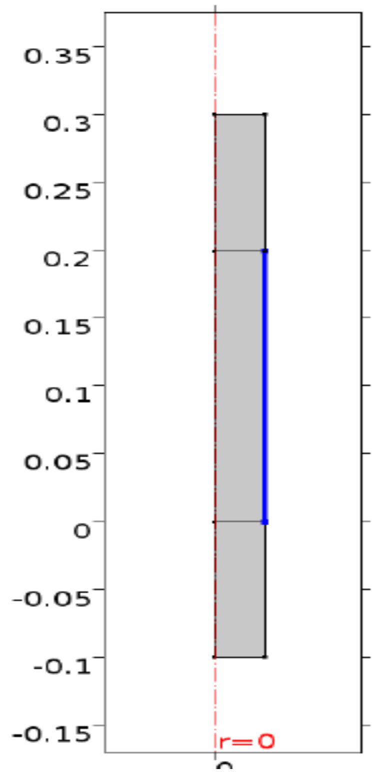




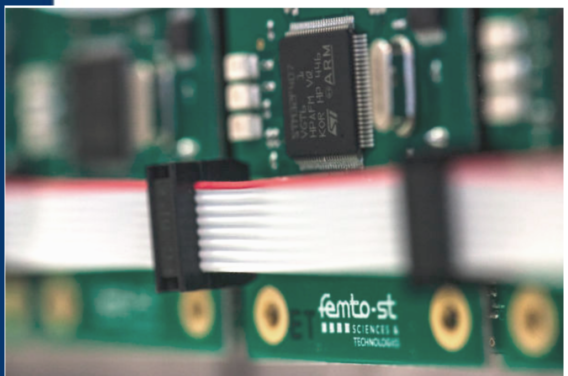


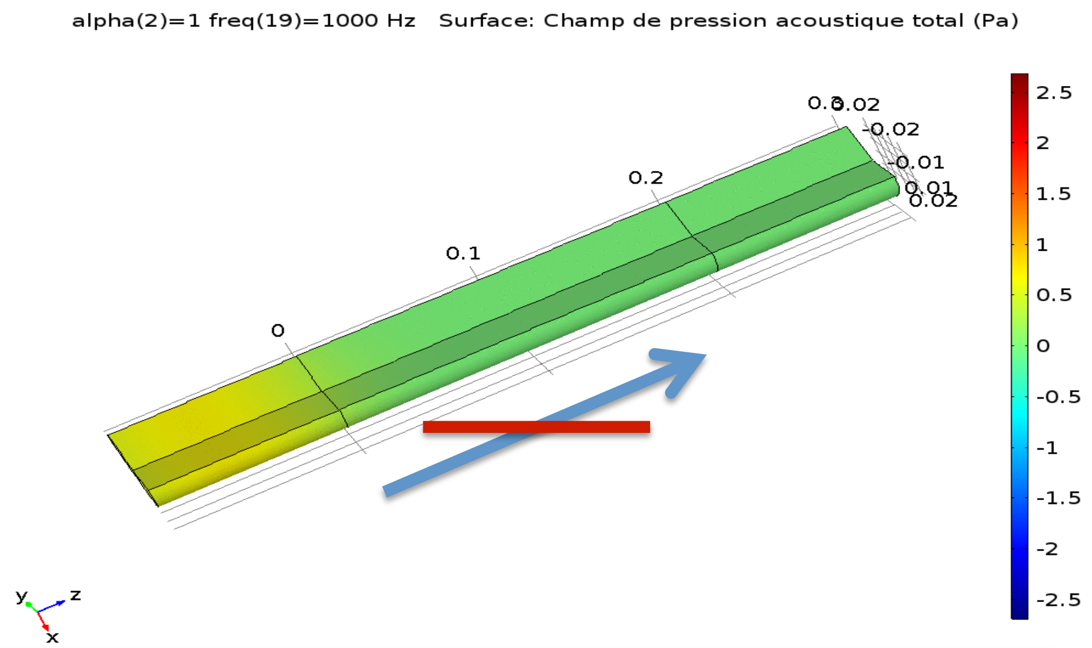
P. David, M Collet et al., SMS, 19(3), 2012

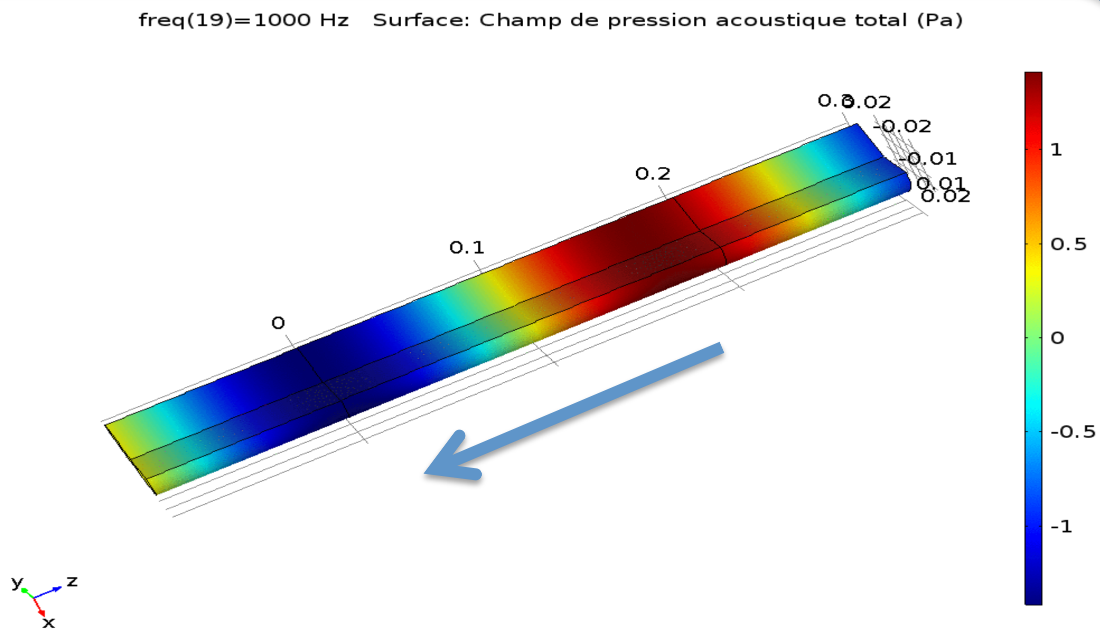
Beyond Band-Gap : Reciprocity breaking and diode



$$a_n = -\alpha(j\omega p/(\rho c) - \partial_{,z} p/\rho)$$







need more details?

M. Collet, M. Ouisse, F. Tateo
Adaptive Metacomposites for Vibroacoustic Control Applications
Cover of IEEE Sensors Journal 14(7), 2014
http://dx.doi.org/10.1109/JSEN.2014.2300052

F. Tateo, M. Collet, M. Ouisse, M. Ichchou, K.A. Cunefare, P. Abbe
Experimental characterization of a bi-dimensional array of negative capacitance piezo-patches for vibroacoustic control
Journal of Intelligent Material Systems and Structures, 2014
http://dx.doi.org/10.1177/1045389X14536006

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