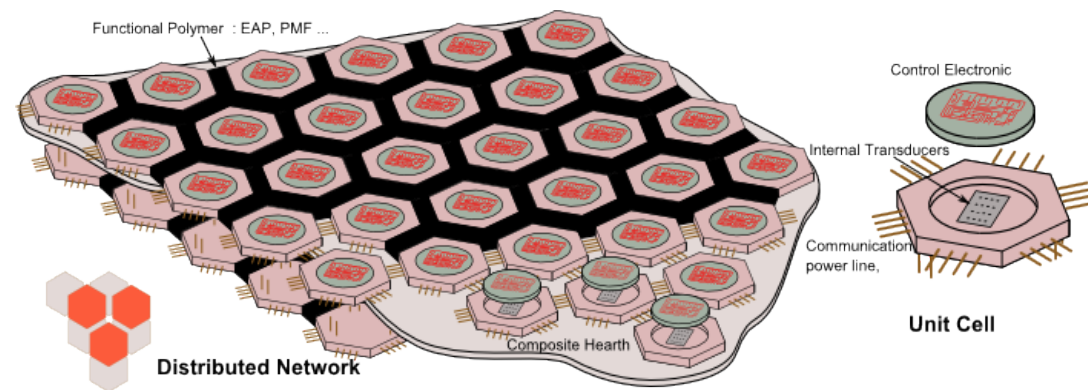


Les Métamatériaux Mécaniques : Architecturer pour fonctionnaliser...



Manuel COLLET (Senior Researcher CNRS, LTDS, ECL)

Morvan OUISSE (Professor ENSMM, FEMTO-ST Applied Mechanics)

Sami Karkar (LTDS, ECL)

Kaijun Yi (LTDS, ECL)

Kevin Billon (FEMTO-ST Applied Mechanics)

Laboratoire de
Tribologie et
Dynamique des
Systèmes

LTDS UMR 5513

<http://ltds.ec-lyon.fr>

With contributions from F. Tateo, G. Matten (FEMTO-ST), Fan Yu (LTDS), M. Ruzzene, K. Cunefare (GT)

« Green » technologies– structural weight reduction

(decrease CO₂ emission (5-15%), noise control....)

- Intensified dynamical environment
- Fatigue and damage : security
- Stability problem
- Adapted design methodologies

FR & EC research strategies, Clean Sky , DREAM EU Project s, AIAA's Emerging Technologies Committee (ETC) ...



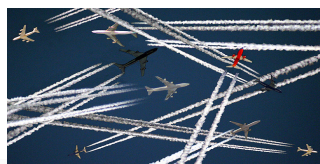
Transports



Nuclear



Civil Engineering



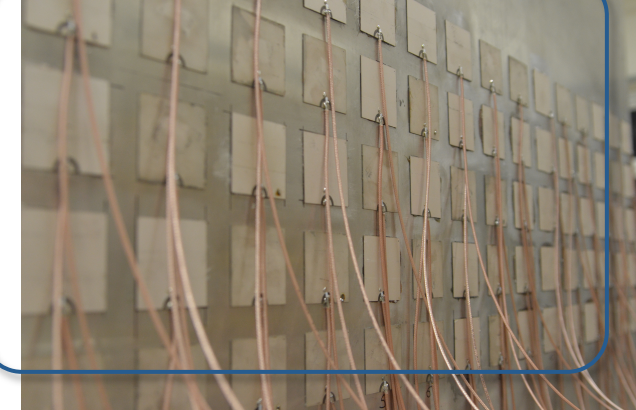
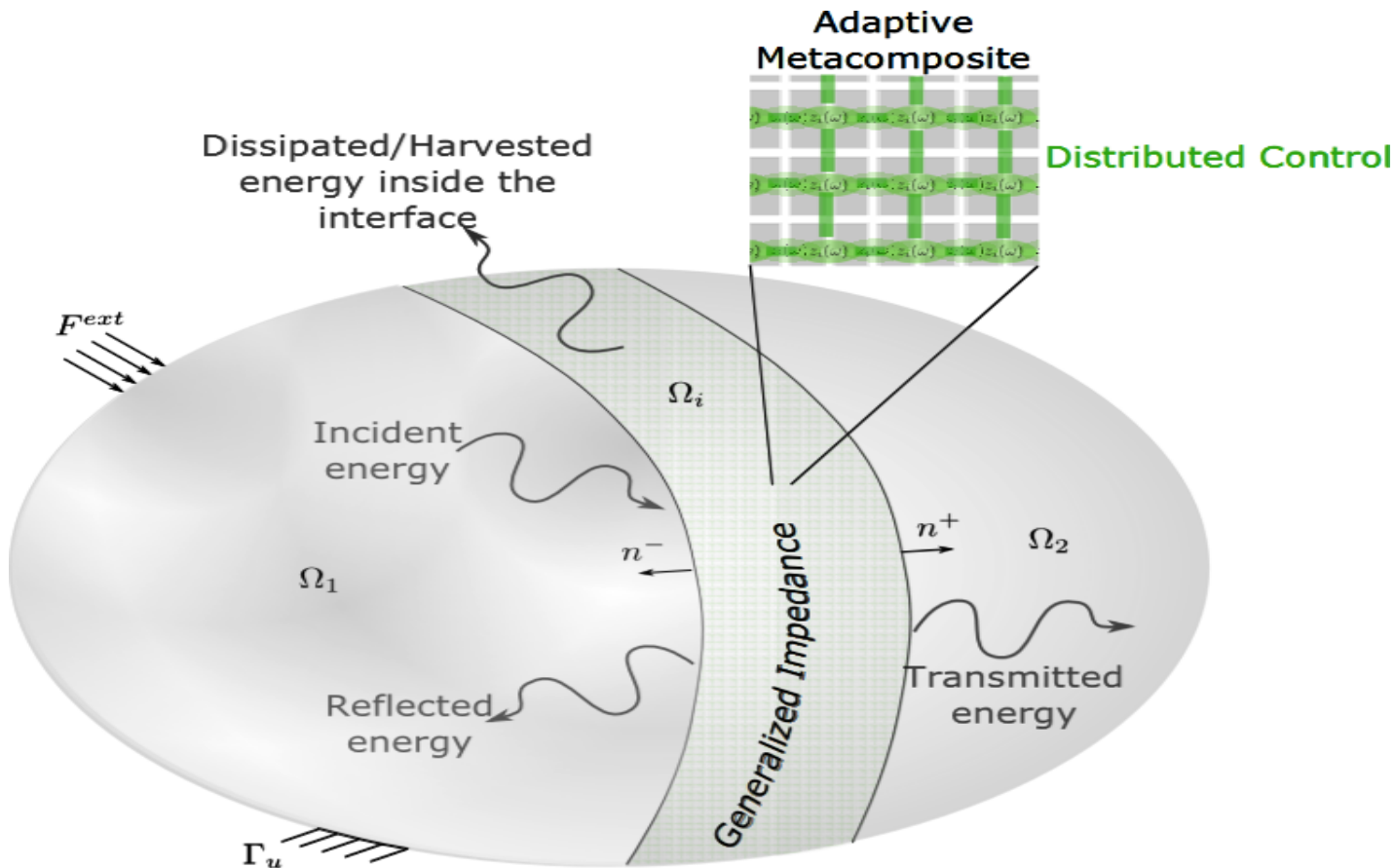
Aerospace

New Integrated functionalities

- Vibration Control
- Noise Control
- Structural Health Monitoring -SHM-
- NDE, PHM
- Waves trapping
- And more....

Meyer et al.: Advanced Microsystems for Automotive Applications 2009 - Smart Systems for Safety, Sustainability and Comfort, Springer 2009

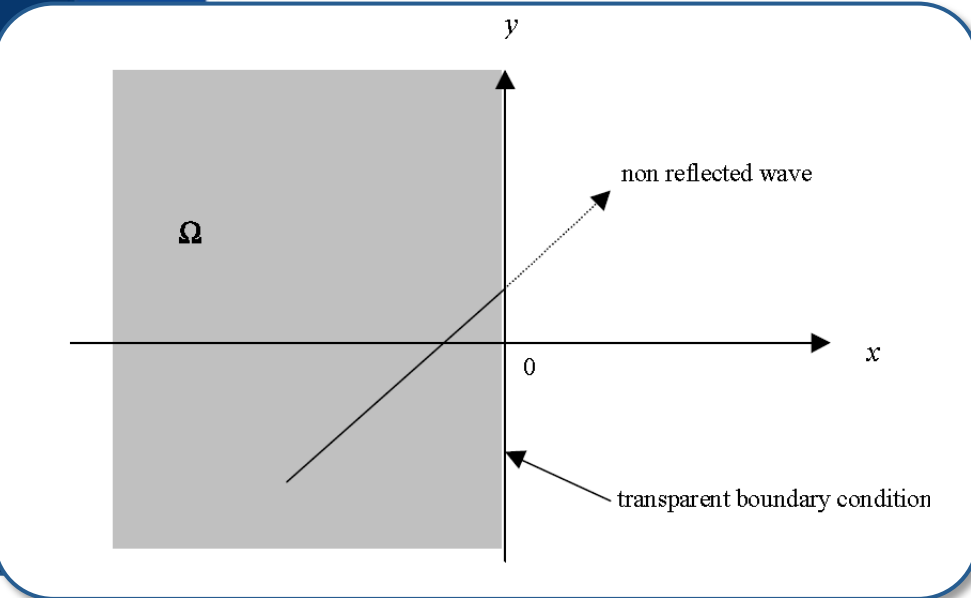
Synthesis of **generalized Impedance operator** using **distributed** (low cost, low energy) **individual** (communicating) cells



Metacomposites:
Synthesis of functional constitutive laws inside hybrid composite material by using distributed sets of smart cells

Scale of interest:
mm -> few cm





G. Monseny, LAAS report, Toulouse, 2002

$$\begin{cases} \partial_t^2 \theta - \Delta \theta = 0 & \text{on } \Omega_\theta \times \mathbb{R}_t^{+*} \\ \theta(0, y, t) := v(y, t) \\ w(y, t) := \partial_x \theta(0, y, t), \end{cases}$$

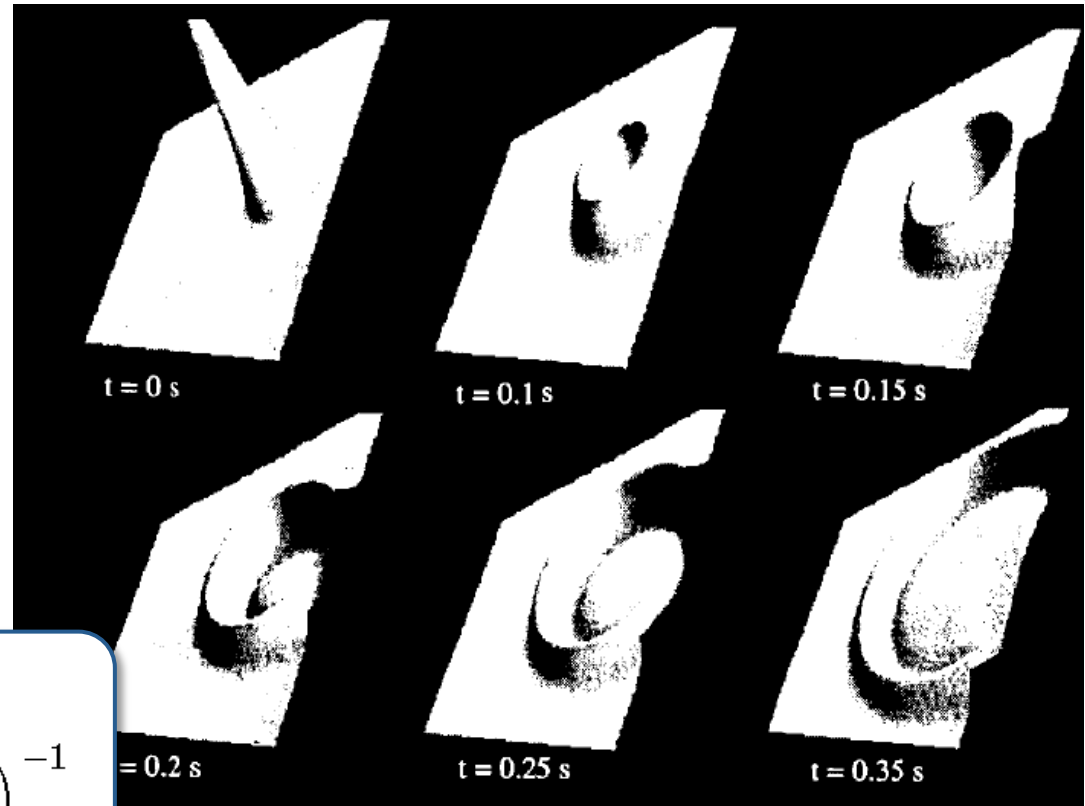
Acoustic System

$$v := \mathcal{K}w$$

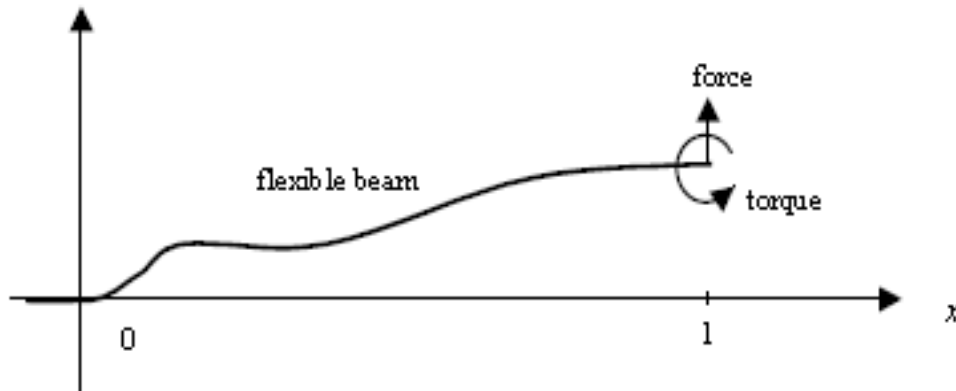
$$\mathcal{K} = - \left(\sqrt{\partial_t^2 - \partial_y^2} \right)^{-1}$$

Controlled System :

Pseudo derivative operator



Example : Generalized Impedance operator for beams



$$\begin{cases} \partial_t^2 \theta + \partial_x^4 \theta = 0, & x \in (0, 1) \\ \partial_x^2 \theta(1, t) = -\partial_t \theta(1, t) - \sqrt{2} \partial_t^{1/2} \partial_x \theta(1, t) \\ \partial_x^3 \theta(1, t) = \sqrt{2} \partial_t^{3/2} \theta(1, t) + \partial_t \partial_x \theta(1, t), \end{cases}$$

$$\begin{cases} \partial_t^2 \theta + \partial_x^4 \theta = 0 \\ \theta(x, 0) = \theta_0(x) \\ \partial_t \theta(x, 0) = \theta_1(x) \\ \theta(0, t) = 0 \\ \partial_x \theta(0, t) = 0 \\ \partial_x^2 \theta(1, t) = u(t) \\ \partial_x^3 \theta(1, t) = v(t), \end{cases}$$

Mechanical System

G. Monseny, LAAS report, Toulouse, 2002

Controlled Mechanical System :

Pseudo derivative operator

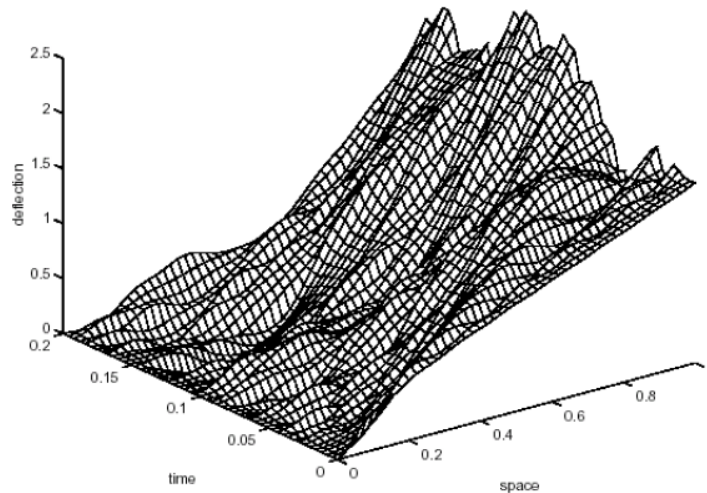


Figure 2: Autonomous beam

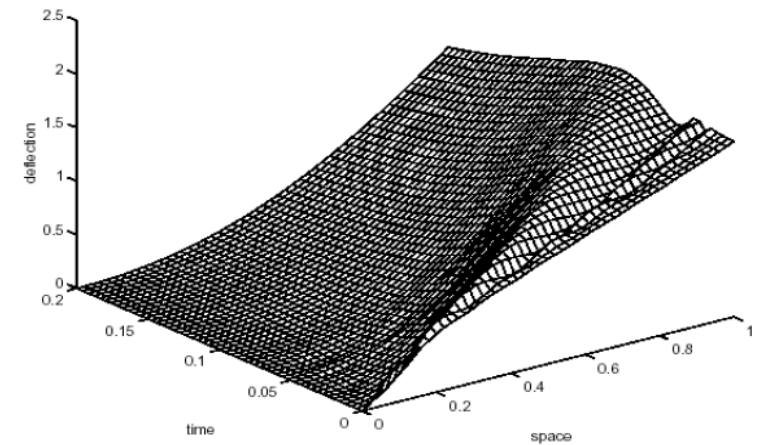
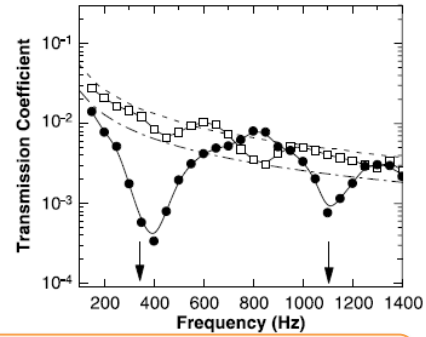
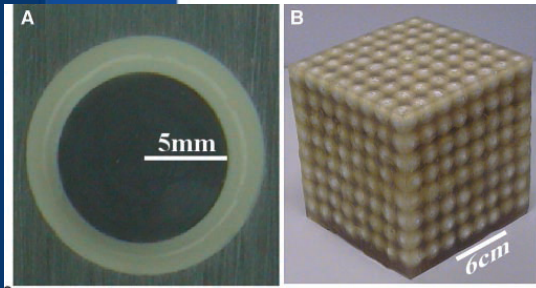
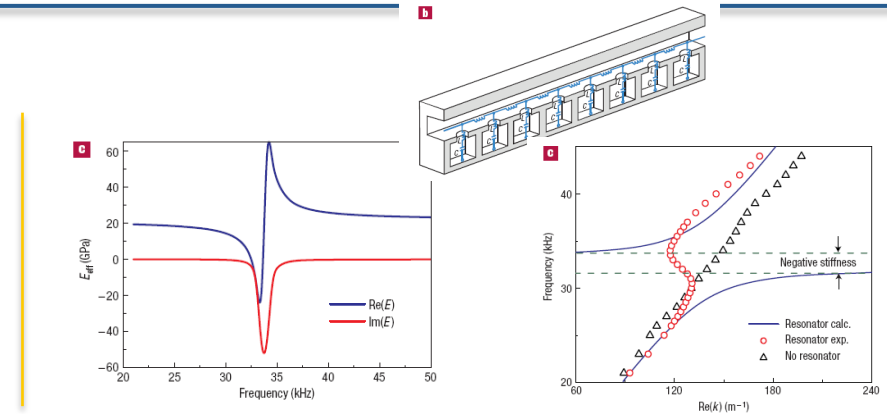


Figure 3: Beam with feedback Z_0

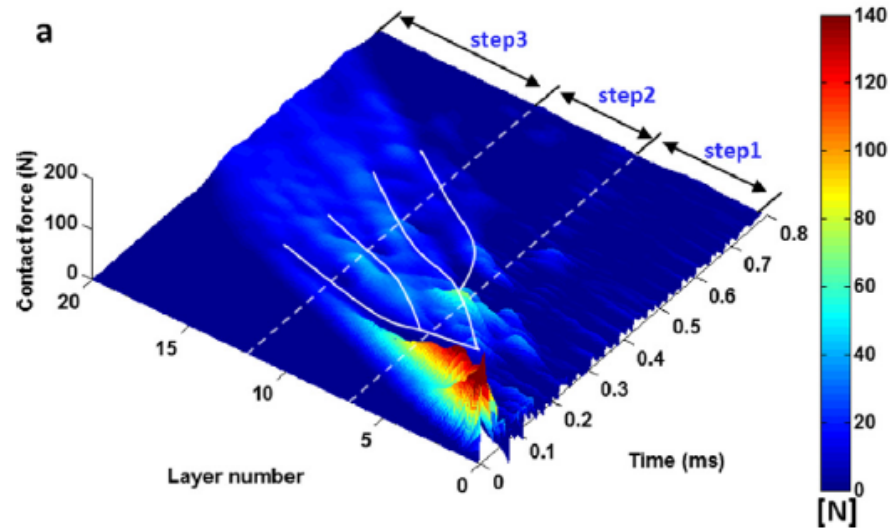
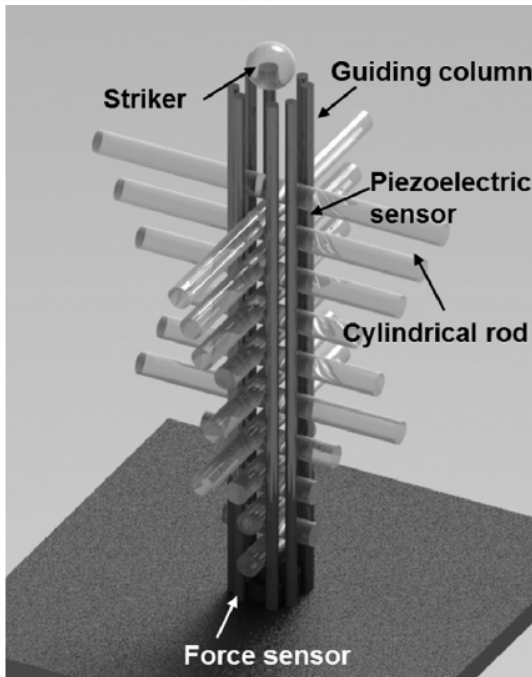


Liu Z. et al. "Locally Resonant Sonic Materials" *Science*, Vol. 289, September 2000, pp. 1734-1736.

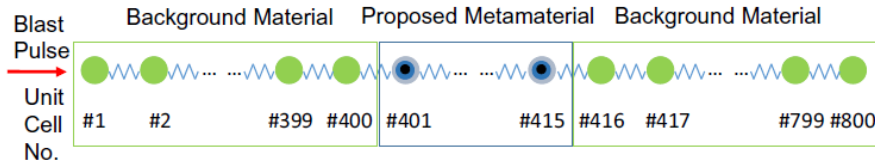
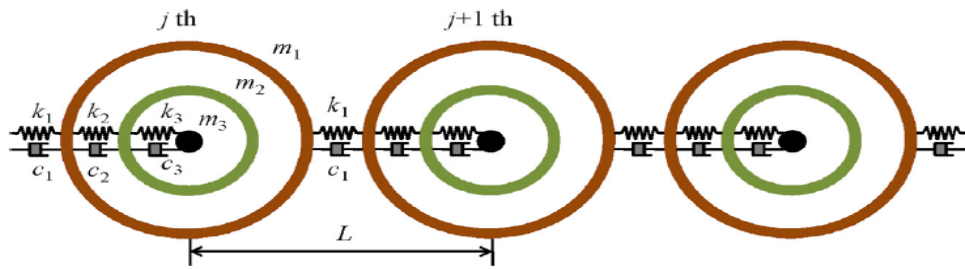


N. Fang et al., "Ultrasonic Metamaterials With Negative Modulus" *Nature Materials*, Vol. 5, June 2006, pp. 452-456

Classical Sonic Métamatériaux

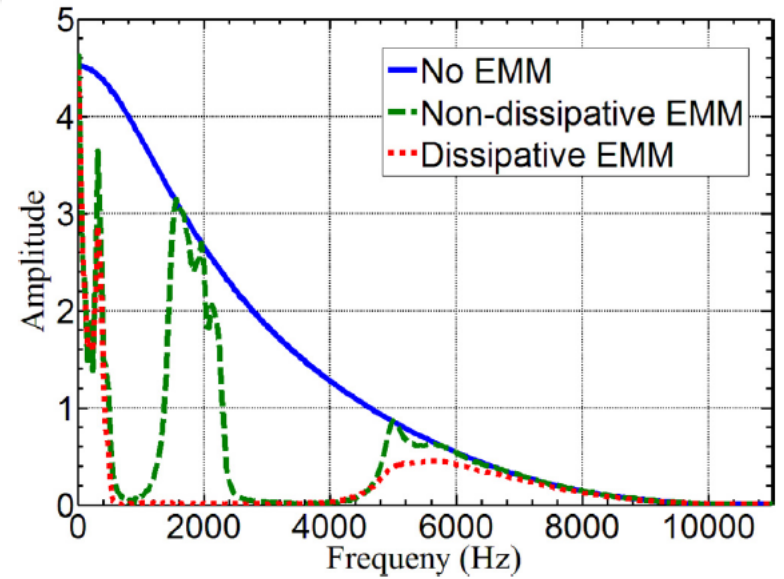


Impact and blast mitigation using locally resonant woodpile metamaterials
Eunho Kim, Jinkyu Yang, HuiYun Hwang, Chang Won Shul

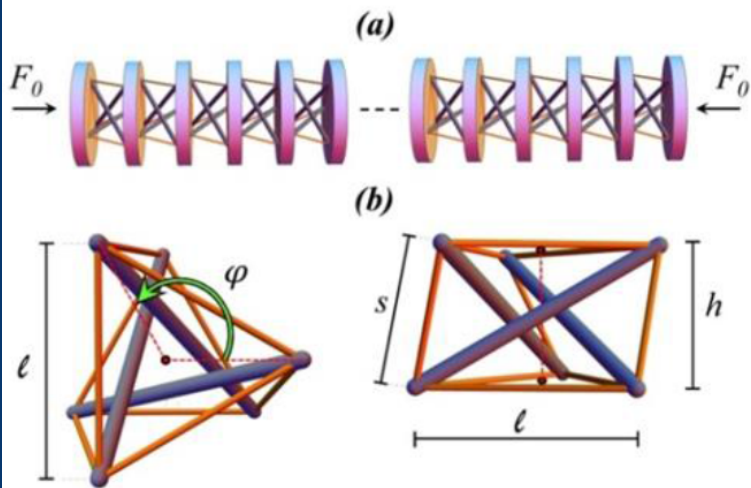


Dissipative elastic metamaterials for broadband wave mitigation at subwavelength scale

Y.Y. Chen^a, M.V. Barnhart^a, J.K. Chen^a, G.K. Hu^b, C.T. Sun^c, G.L. Huang^{a,*}

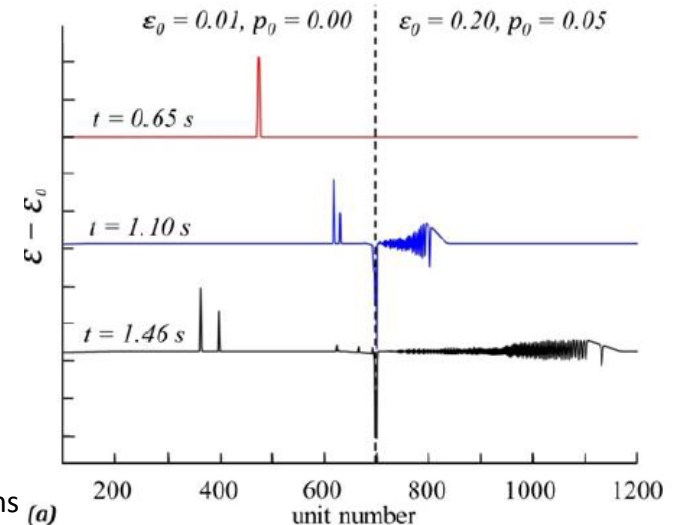


Classical Sonic Métamatériaux



Strongly nonlinear discrete systems with unprecedented tenability: chain of tensegrity prisms and lumped masses (a)

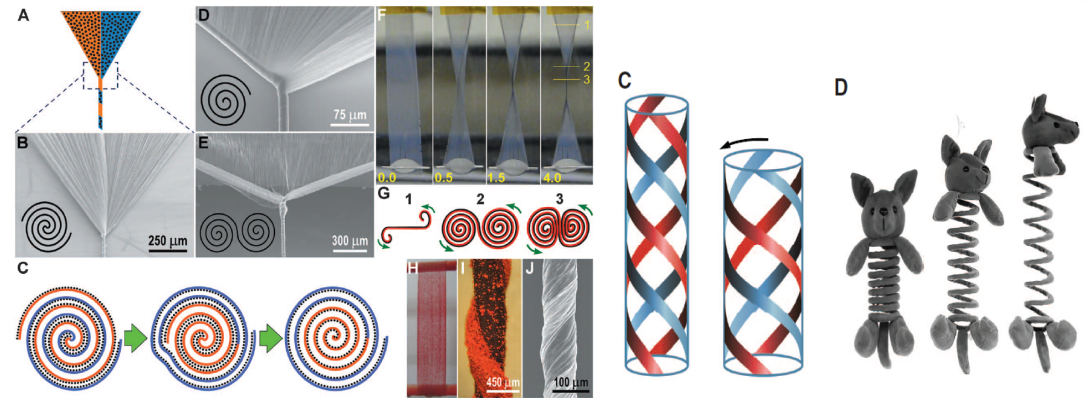
F. Fraternali, G. Carpentieri, A. Amendola, R. E. Skelton, and V. F. Nesterenko, Multiscale tunability of solitary wave dynamics in tensegrity metamaterials, arXiv 1409.7097



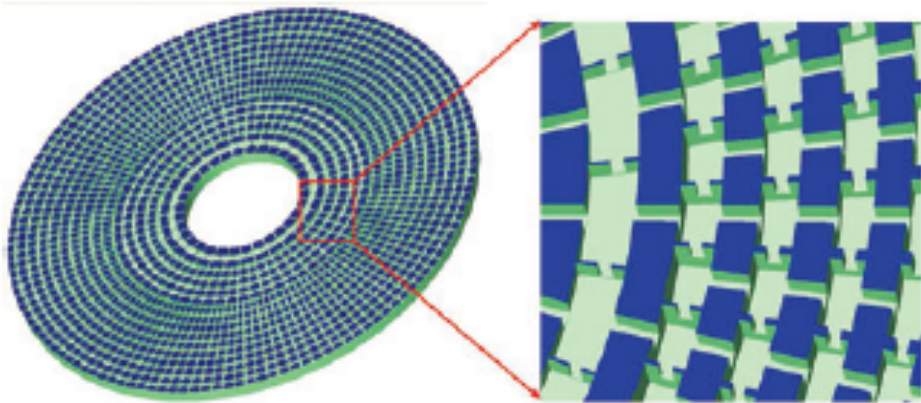
Material Architecture for Structural Optimization



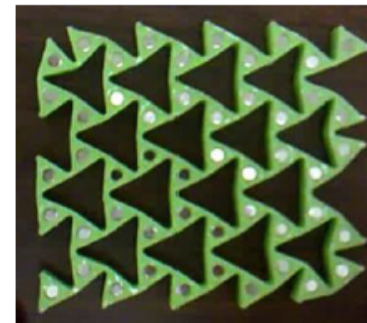
A sequence of images demonstrating the self-folding of a 4D printed multi-material single strand into the world of MIT



M.D Limaet al Bistralling Nanotube Sheets and Functional Guests into Yarns, Science 331, 51, 2011



S. Zhang, C. Xia and N. Fang,
"Broadband acoustic cloak for ultrasound waves", PRL, 106,2,2 4301, 2011

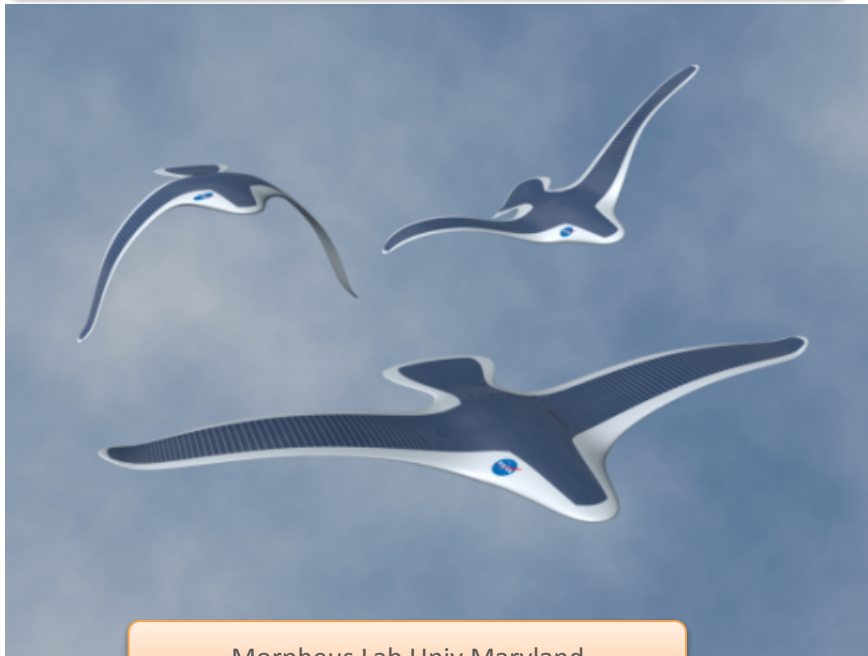


M. D. Schaeffer; M. Ruzzene, "Wave propagation in 2D magneto-elastic kagome lattices",
Proc. SPIE 9064, SMS 2014

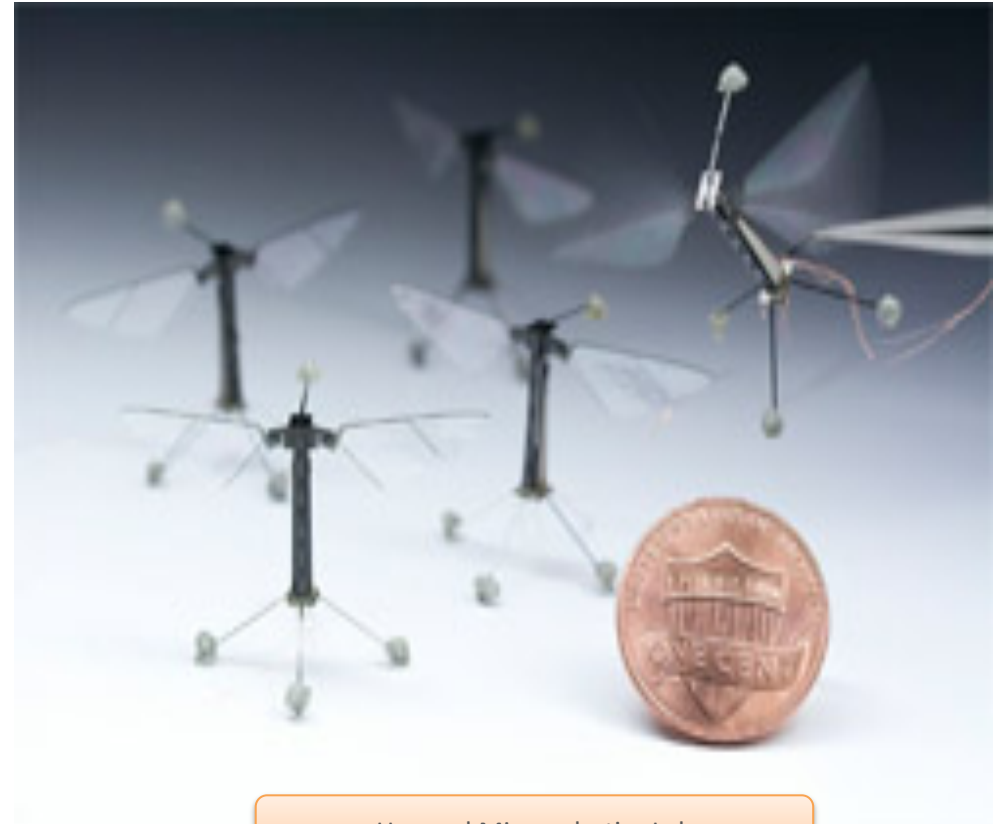
Material Architecture for Structural Optimization



Research in Programmable Matter Directed by Carnegie Mellon and Intel

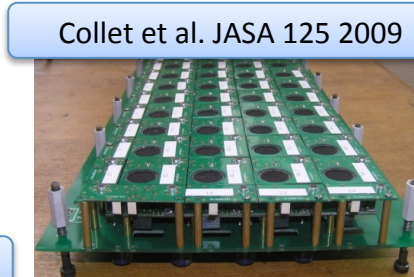
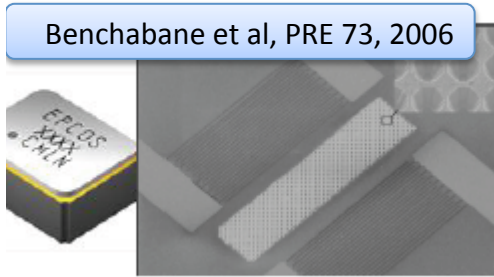
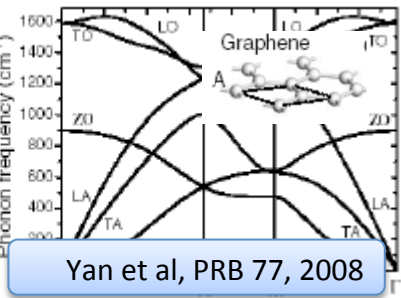


Morpheus Lab Univ Maryland



Harvard Microrobotics Lab

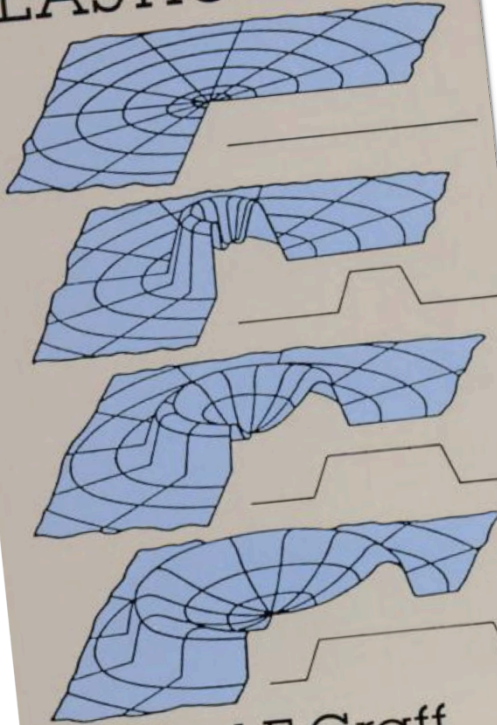
Structure	Physical properties	Waves support	Gap
Crystalline solids	Periodic arrangement of atoms $\sim 5 \text{ \AA}$	Electrons (Ψ) Schrödinger eq.	Absence of electron states
Photonic crystal	Periodic modulation of ϵ, μ (macro scale)	EM (E,B) Maxwell eqs.	Absence of states of the EM field
Phononic crystal	Periodic modulation of ρ, E, ν (macro scale)	Elastic (u) Elasticity eqs.	Absence of states of the elastic field



An arbitrary choice of 3 top-level references

Graff, 1975

WAVE MOTION IN ELASTIC SOLIDS



Karl F. Graff

Mead, JSV, 1996

WAVE PROPAGATION IN CONTINUOUS PERIODIC STRUCTURES: RESEARCH CONTRIBUTIONS FROM SOUTHAMPTON, 1964-1995

D. J. MEAD†

Institute of Sound and Vibration Research, University of Southampton, Southampton SO17 1BJ, England

(Received 1 November 1995)

After brief reference to some early studies by other investigators, this paper focuses mainly on methods developed at the University of Southampton since 1964 to analyze and predict the free and forced wave motion in continuous periodic engineering structures. Beginning with receptance methods which have been applied to periodic beams and rib-skin structures, it continues with a method of direct solution of the wave equation. This uses Floquet's principle and has been applied to beams and quasi-one-dimensional periodic plates and cylindrical shells. Sample curves of the propagation and attenuation constants pertaining to these structures are presented. A limited discussion of the method best suited to the prediction of sound radiated from a vibrating periodic structure, which has been some theorems and variational principles relating to periodic structures which have been developed at Southampton, and which form a basis for finding natural frequencies of finite structures or for computing free and forced wave motion by energy methods. This has led to the finite element method (in its standard and hierarchical forms) being used to study wave motion in genuine two-dimensional and three-dimensional structures. Examples of this work are shown. The method of phased array receptance functions is then introduced as possibly the easiest way of setting up exact equations for the propagation constants of uniform quasi-one-dimensional periodic structures. A summary is finally presented of the limited and early work performed at Southampton on simple disordered periodic structures.

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1. INTRODUCTION

Elfyn Richards' early vision and pioneering zeal in the study of aeroplane noise at Southampton had a "Coanda effect" upon those of us involved in structural teaching and research. We were inexorably drawn into the study of structural vibration caused by the noise of the early jet engines, which were shaking and shattering flimsy aeroplane structures. Something had to be done about it! While quieter engines were yet to be developed, less responsive and more fatigue-resistant structures had to be designed and built. "JR" gave much encouragement to three of us to work to this end—B. L. Clarkson, the late T. R. G. Williams and myself. His reputation and fund-raising ability drew the attention of the U.S. Air Force, which awarded us generous grants for vibration and

† Formerly of the Department of Aeronautics and Astronautics, 1952-1991.
495

Hussein, Leamy, Ruzzene, ASME Applied Mechanics Review, 2014

Dynamics of Phononic Materials and Structures: Historical Origins, Recent Progress, and Future Outlook

Mahmoud I. Hussein
Assistant Professor
Mem. ASME

Aerospace Engineering Sciences,
University of Colorado Boulder,
Boulder, CO 80309-0429
e-mail: mh@colorado.edu

Michael J. Leamy
Associate Professor

Mem. ASME
School of Mechanical Engineering,
Georgia Institute of Technology,
Atlanta, GA 30332-0405
e-mail: michael.leafmy@me.gatech.edu

Massimo Ruzzene¹
Professor

ASME Fellow
School of Aerospace Engineering,
School of Mechanical Engineering,
Georgia Institute of Technology,
Atlanta, GA 30332-0150
e-mail: ruzzene@me.gatech.edu

The study of phononic materials and structures is an emerging discipline that lies at the crossroads of a phononic medium is a material or structural system that usually exhibit some form of periodicity, which can be in the constituent dynamical characteristics geometry, or even the boundary conditions. As such, its overall dynamical characteristics are compactly described by a frequency band structure, in analogy to an electronic band structure. With roots extended to early studies of periodic systems by Newton and Rayleigh, the field has grown to encompass engineering configurations ranging from trusses and ribbed shells to phononic crystals and metamaterials. While applied research in this area has been abundant in recent years, treatment from a fundamental mechanical perspective, and particularly from the standpoint of dynamical systems, is needed in new directions. For example, techniques already developed for wave propagation advance the field in new directions. Similarly, numerical and experimental incorporation of damping and nonlinearities have recently been applied to wave propagation in phononic materials and structures. Similarly, numerical and experimental approaches originally developed for the characterization of conventional materials and structures are now being employed toward better understanding and exploitation of phononic systems. This article starts with an overview of historical developments and follows with an in-depth literature and technical review of recent progress in the field with special consideration given to aspects pertaining to the fundamentals of dynamics, vibrations, and acoustics. Finally, an outlook is projected onto the future on the basis of current trajectories of the field. (DOI: 10.1115/1.4026911)

Introduction

A periodic medium is a material or structural system that exhibits some form of spatial periodicity, which can be in the constituent material phases, or the internal geometry, or the boundary conditions. The study of vibrations and acoustics with origins in history in the field of Newton's first attempt to describe the propagation of sound in air [1] and Rayleigh's early study of continuous periodic structures [2]. The topology of periodic structures [2]. The topological physics due to the fundamental importance in condensed matter physics due to the study of atomic vibrations (and electronic structure) play in determining the properties of vibrations in a crystal lattice. Formally merged in the context of vibrational energy in an elastic medium as a quantum of vibrational energy in a discrete particle-like medium—which may be interpreted as the term has also been associated with the study of sound in a solid—the term has also been associated with the study of vibrations and acoustics, mainly in the context of periodic media. Subsequently, it has grown to become a term to refer to a periodic material, or structure, even if in the size scale of a large engineering system, as a "phononic material," or a "phononic structure." As such, there has been an emphasis on the relationship between the phonon physics and dynamics of periodic materials and structures, perhaps what stands out the most is the concept of a band structure diagram—a diagram that represents the relationship between frequency (or energy) and wavenumber, along multiple directions. In

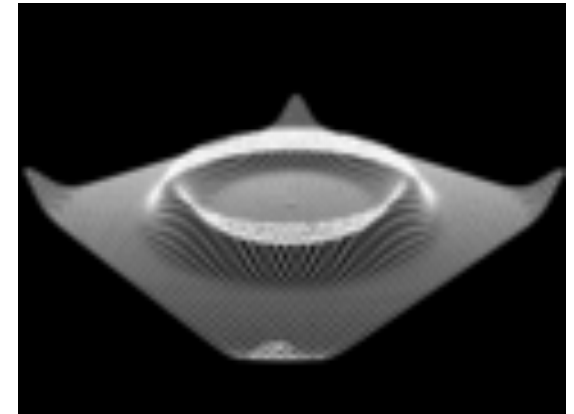
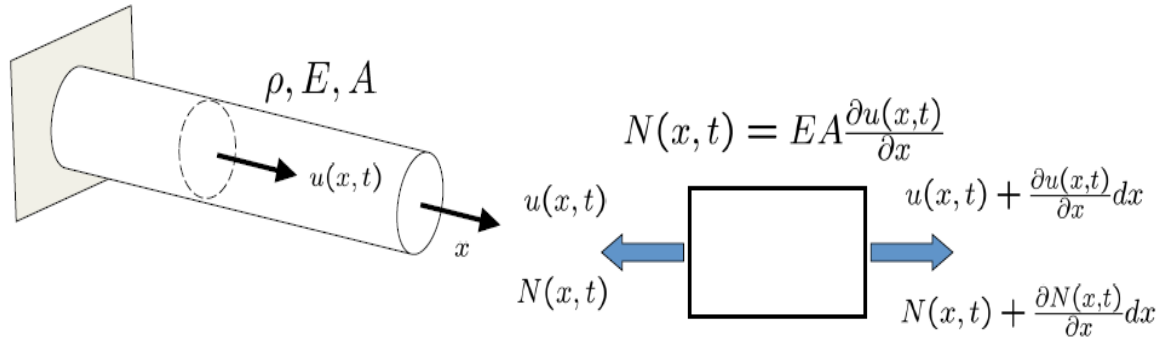
physics, this diagram represents the backbone of electronic structure theory, credited for forming a basis for the classification of crystals into metals, semiconductors, and insulators. In mechanics, a band diagram is precisely a representation of the dispersion relation describing the nature of free wave propagation in an elastic (or acoustic) medium.

Looking closely at the study of periodic systems in the past half-century, we find that researchers in vibrations and acoustics and more broadly in the mechanics community at large, have conducted a considerable amount of work on key theoretical foundations, concepts, and analysis techniques that are relevant to periodic systems in other nonmechanics disciplines. Arguably, two most motivating applications in mechanics, going back to the 1950s, and extending through the 1990s and beyond, have been composite materials (which conveniently have been modeled as periodic materials) [3-4] and aircraft structures (which naturally exhibit some degree of periodicity emanating from the presence of ribs introduced primarily for strengthening and stiffening purposes) [5,6]. Other areas related to mechanical and civil engineering include multiblade turbines [7-9], impact resistant foams [10-12], periodic foundations for building cellular materials [10-12], periodic foundations for building cellular materials [10-12], and multistory buildings and multispan bridges [13,14], and multistory buildings and multispan bridges [15]. In recent years, starting in the early 1990s, the field has experienced a resurgence with the introduction of phononic crystals [16-21], and, roughly a decade later, with the study of acoustic metamaterials [22], which may be also broadly considered as examples of phononic materials. A phononic crystal is a complex material consisting of one, two, or more materials arranged periodically in space. The material phases (solid and/or fluid) comprising the material is not much different from periodic composite materials studied earlier in the engineering literature, with the only difference that

¹Corresponding author.
Manuscript received April 17, 2013; final manuscript received December 30, 2013; published online May 2, 2014. Assoc. Editor: Chin An Tan.

1. Wave propagation – back to basics
2. Modeling aspects
 - a) Wave Finite Elements WFE
 - b) Shift cell operator for multiphysic coupled metamaterial
 - c) Plane Wave Expansion PWE
3. Design functional structures using band gap properties
 - a) Waves diffusion : Reflection and absorption
 - b) The boundary : a limit for the band gap efficiency
4. Beyond the Band –Gap
 - a) Lensing
 - b) Reciprocity breaking and diode
 - c) ...

1D Rod wave :



Wave Equation :

$$E \frac{\partial^2 u(x,t)}{\partial x^2} = \rho \frac{\partial^2 u(x,t)}{\partial t^2}$$

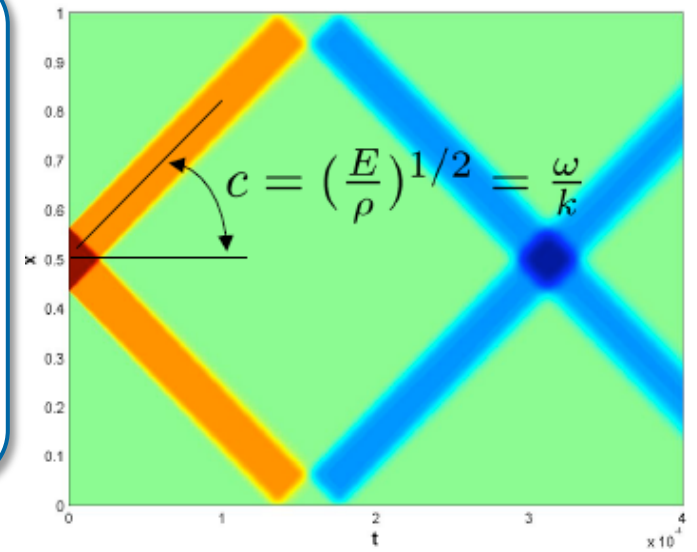
$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u(x,t)}{\partial t^2}$$

$$u(x, t) = \frac{1}{2} [U(x - ct) + U(x + ct)]$$

Wave velocity : Initial Value:

$$c = \left(\frac{E}{\rho}\right)^{1/2} \quad u(x, 0) = U(x)$$

$$\dot{u}(x, 0) = 0$$



From M Ruzzene Courses

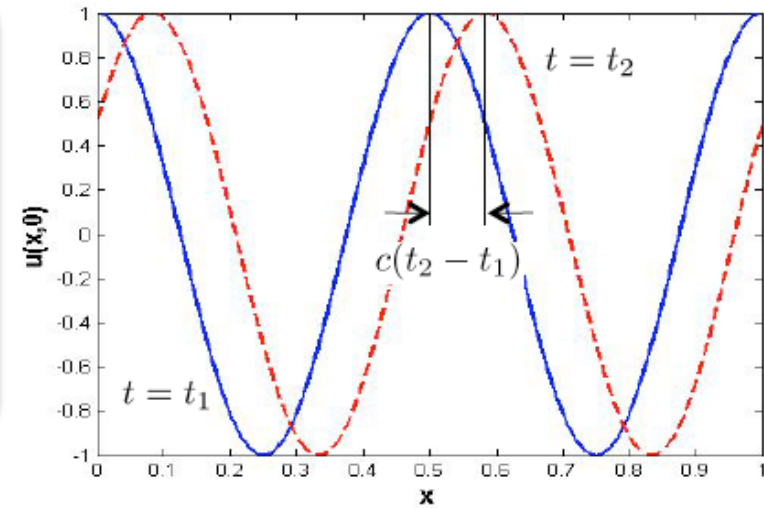
If initial condition : $u(x, 0) = U_0 \cos(kx)$

k : wavenumber - $\lambda = \frac{2\pi}{k}$: wavelength

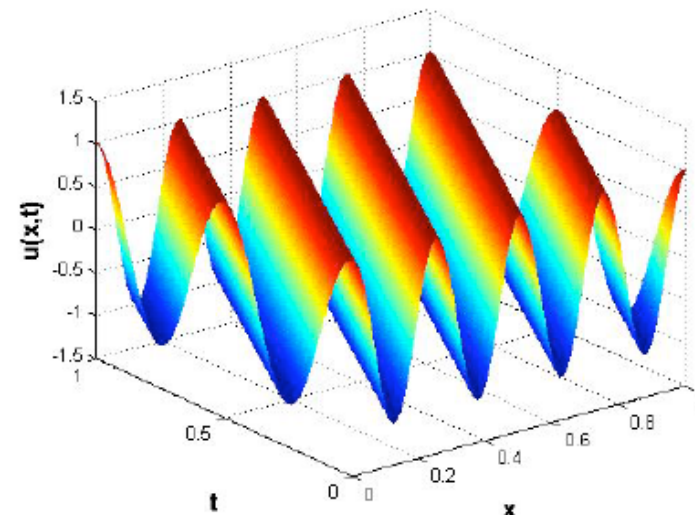
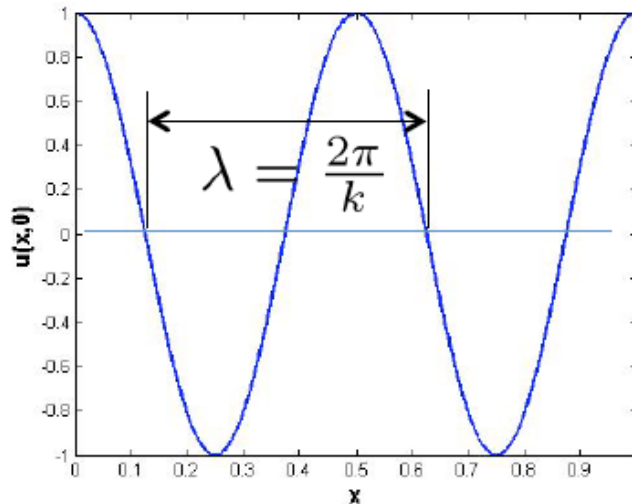
Solution is :

$$u(x, t) = \frac{U_0}{2} [\cos(kx - \omega t) + \cos(kx + \omega t)]$$

Dispersion relation: $c = \frac{\omega}{k} = \left(\frac{E}{\rho}\right)^{1/2}$



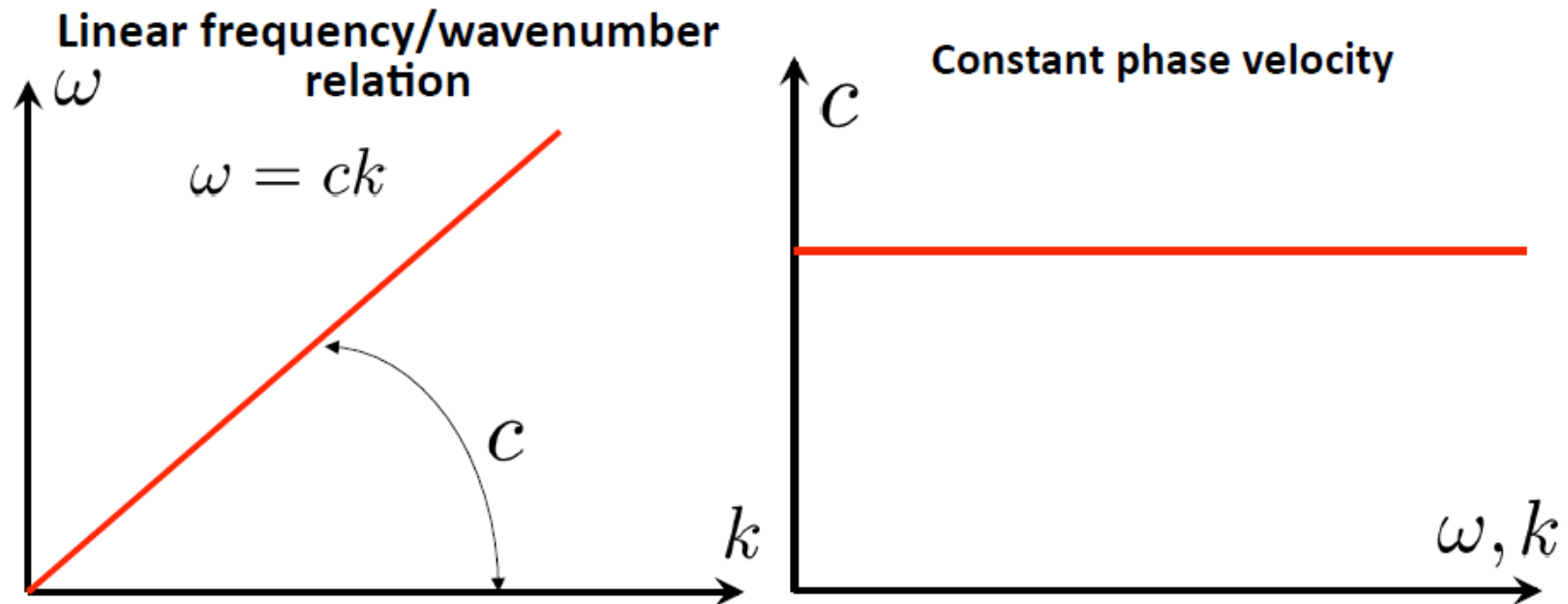
From M Ruzzene Courses



Dispersion relation: $\omega = \omega(k)$

Representation of the physical PDE operator

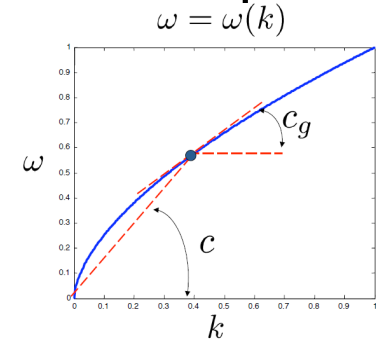
Non dispersive Media :



Let's consider 2 waves, of equal amplitude and at two frequencies :

$$u(x, t) = U_0[\sin(k_1x - \omega_1t) + \sin(k_2x - \omega_2t)]$$

$$u(x, t) = 2U_0 \underbrace{\cos(\Delta kx - \Delta\omega t)}_{\text{Modulation}} \underbrace{\sin(k_0x - \omega_0t)}_{\text{Carrier wave}}$$



Speed of wave packet velocity : $x = \frac{\Delta\omega}{\Delta k}t + Const$

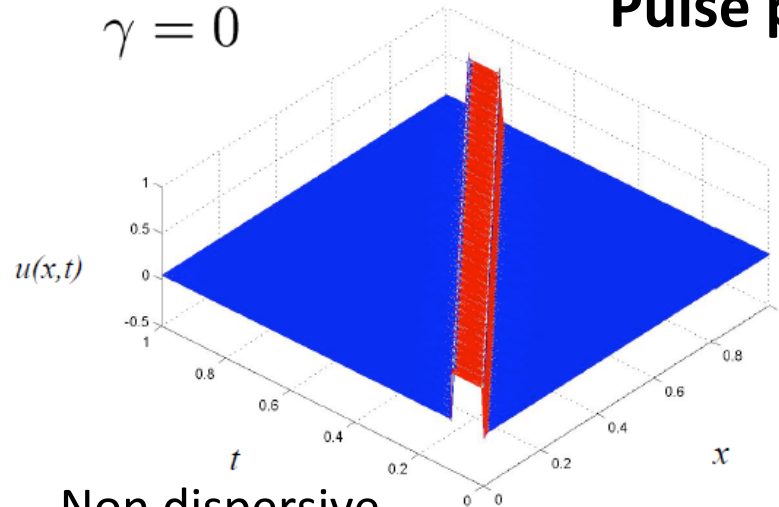
$$c_g = \frac{\Delta\omega}{\Delta k}$$

At the limit one obtains the Group Velocity : $c_g = \frac{d\omega}{dk}$

From M Ruzzene Courses

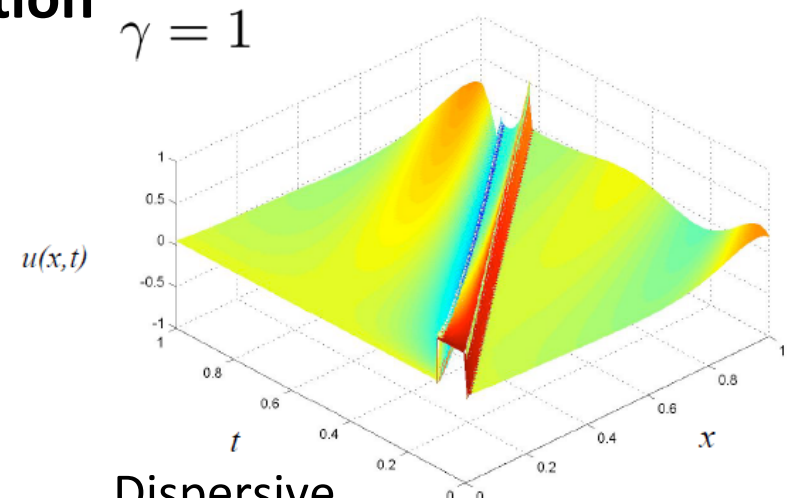
Pulse propagation

$\gamma = 0$



Non dispersive

$\gamma = 1$



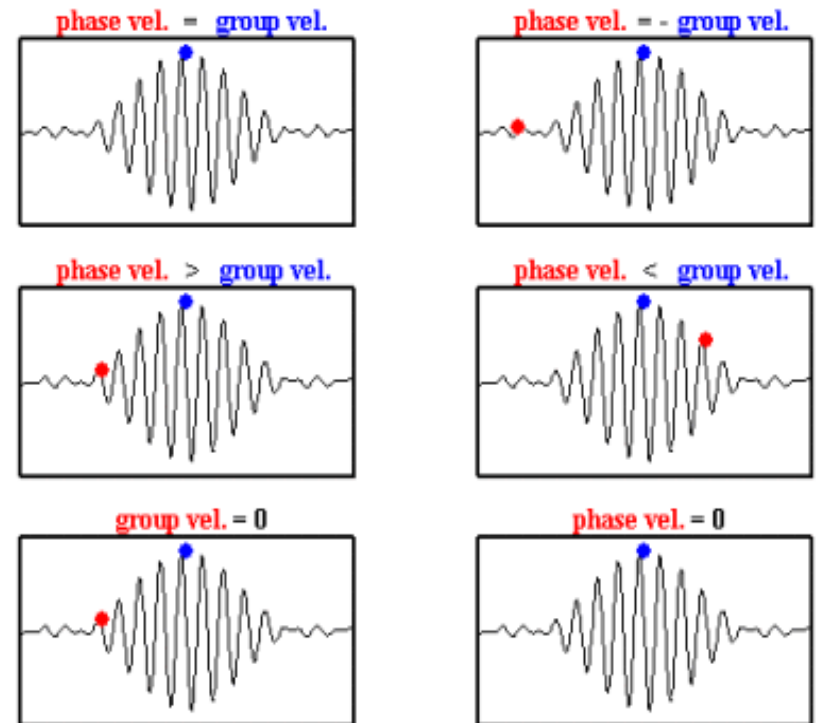
Dispersive

- Phase velocity
 - speed of the individual waves.: $c_{\downarrow\phi} = \omega/k$
- Group velocity
 - speed of the wavepacket: $c_{\downarrow g} = \partial\omega/\partial k$

 Non-dispersive wave $c_{\downarrow\phi} = c_{\downarrow g}$

 Dispersive wave $c_{\downarrow\phi} \neq c_{\downarrow g}$

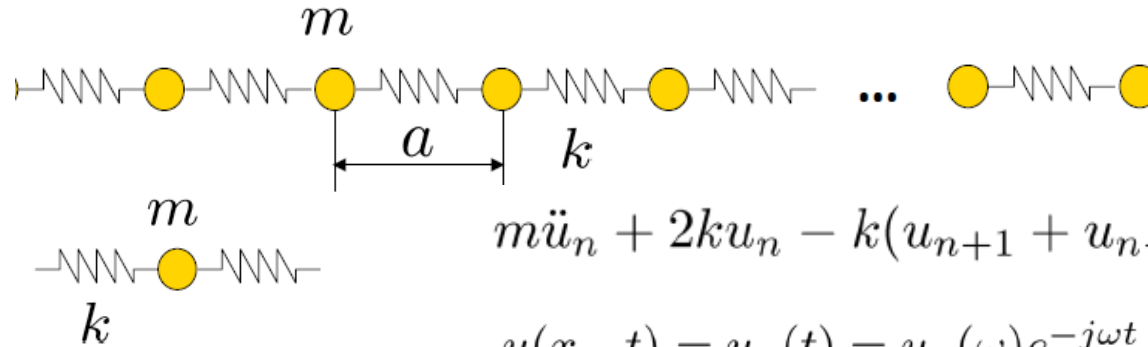
(undamped systems...)



isvr

Taken from http://resource.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial_files/Web-further-dispersive.htm

Let's consider a monoatomic lattice :



$$m\ddot{u}_n + 2ku_n - k(u_{n+1} + u_{n-1}) = 0$$

$$u(x_n, t) = u_n(t) = u_n(\omega)e^{-j\omega t}$$

$$u_n(\omega) = u_0[\kappa(\omega)]e^{j\kappa x_n}$$

$$u_n(\omega) = u_0[\mu(\omega)]e^{j\mu n}$$

From M Ruzzene Courses

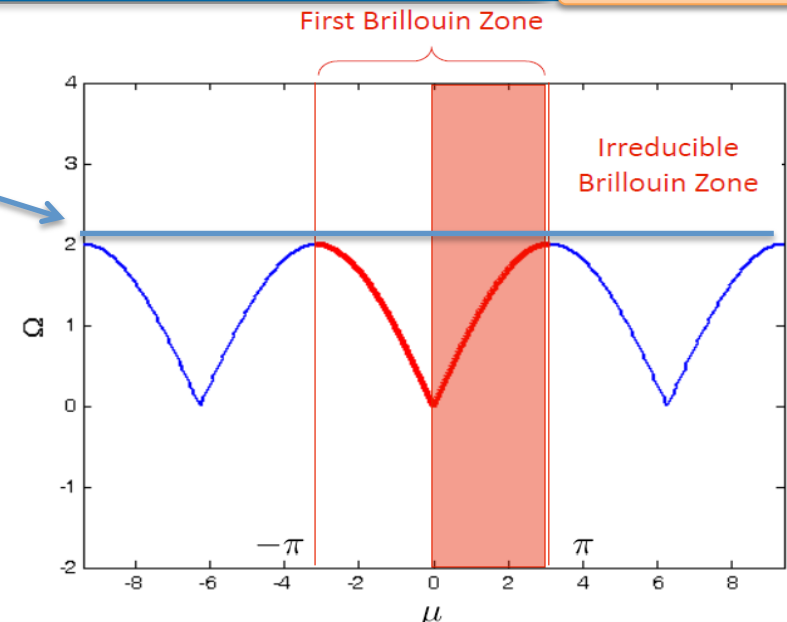
Dispersion – Bragg Scattering:

$$-\omega^2 m + 2k(1 - \cos \mu) = 0$$

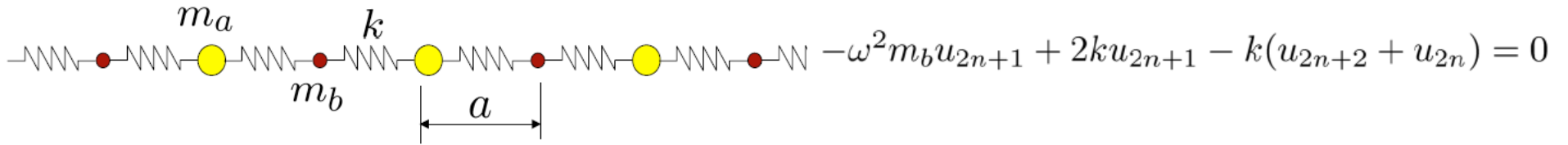
$$\omega_0^2 = \frac{k}{m}$$

$$\Omega^2 = 2(1 - \cos \mu)$$

$$\Omega = \frac{\omega}{\omega_0}$$



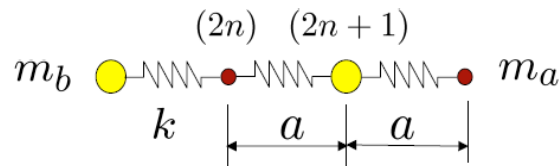
Let's consider a biatomic lattice : $-\omega^2 m_a u_{2n} + 2k u_{2n} - k(u_{2n+1} + u_{2n-1}) = 0$



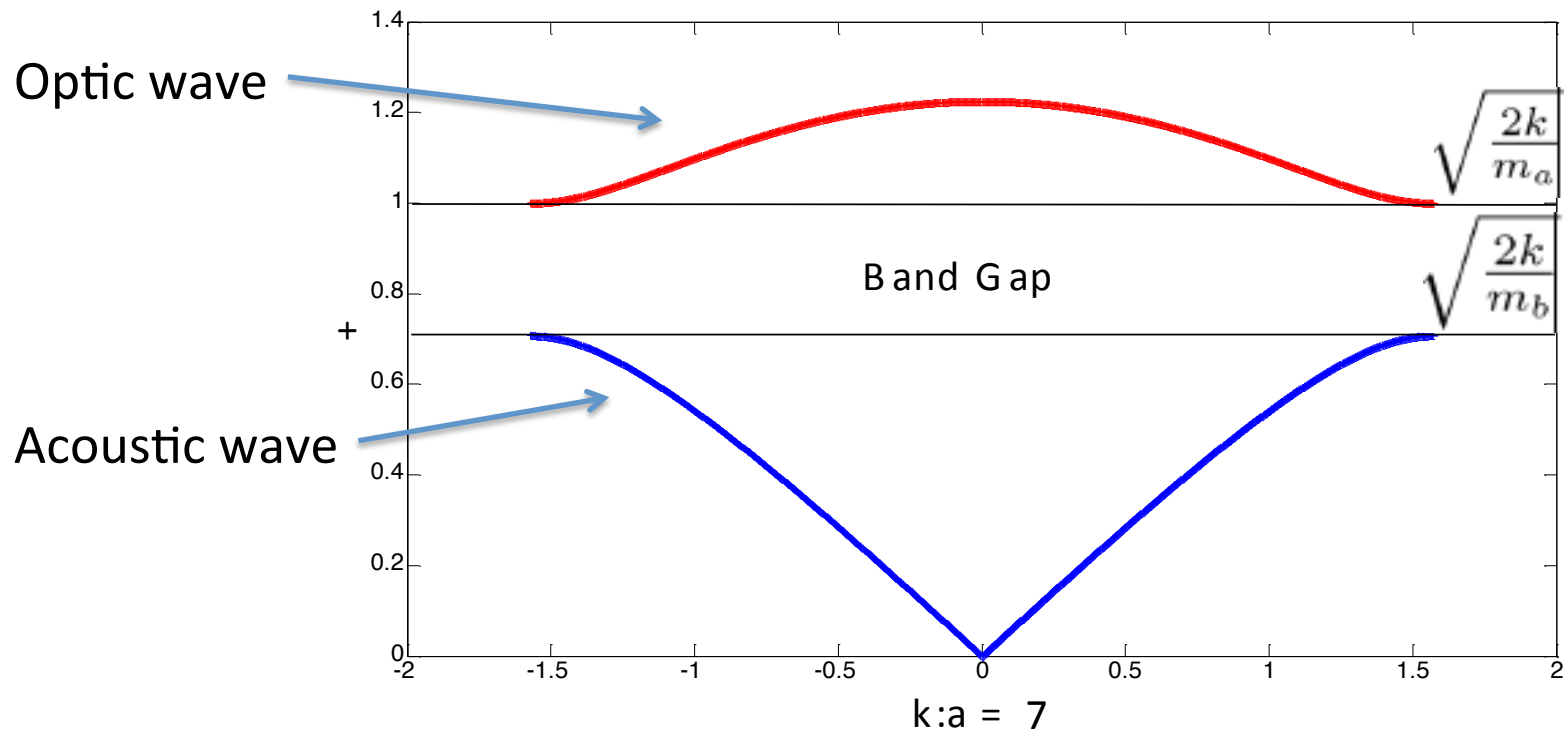
$$-\omega^2 m_b u_{2n+1} + 2k u_{2n+1} - k(u_{2n+2} + u_{2n}) = 0$$

$$u_{2n}(\omega) = u_a(\omega) e^{j\kappa x_{2n}} = u_a(\omega) e^{j2a\kappa n}$$

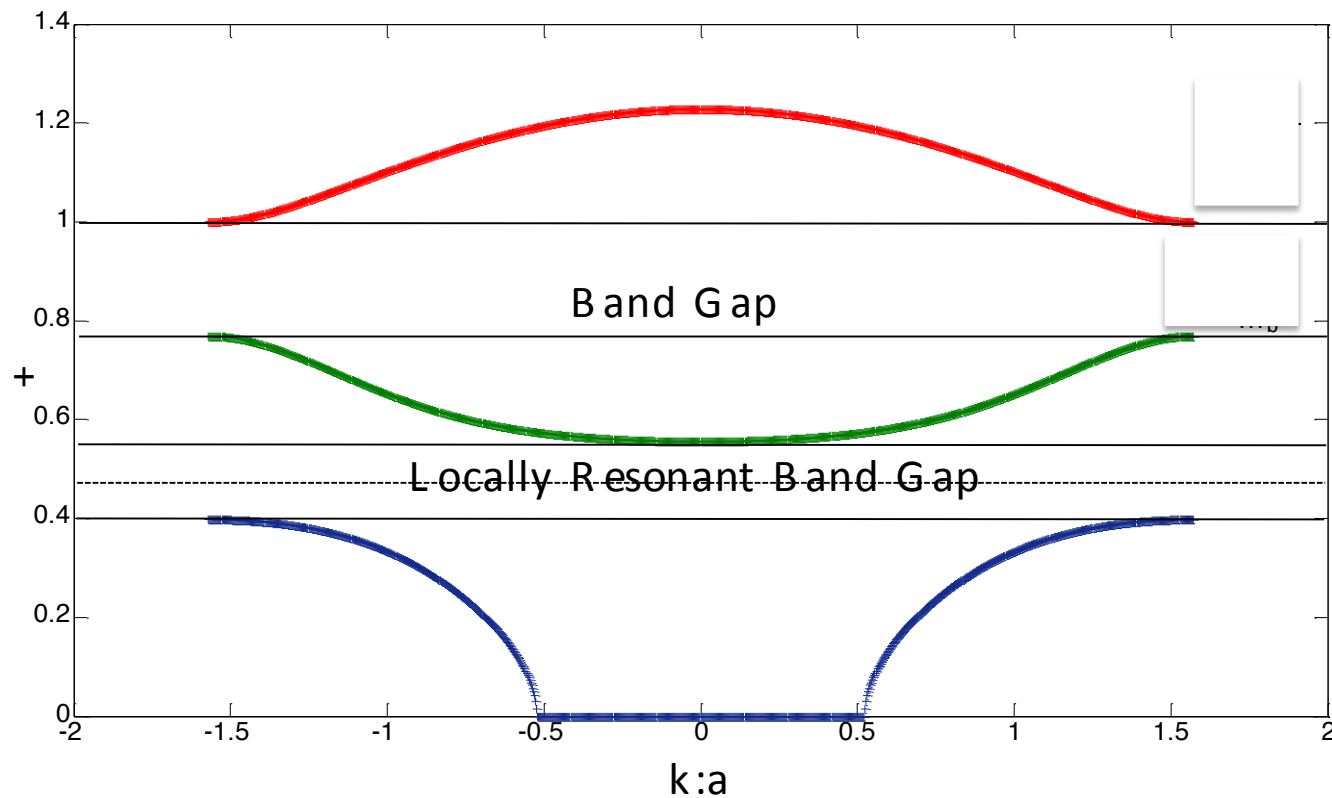
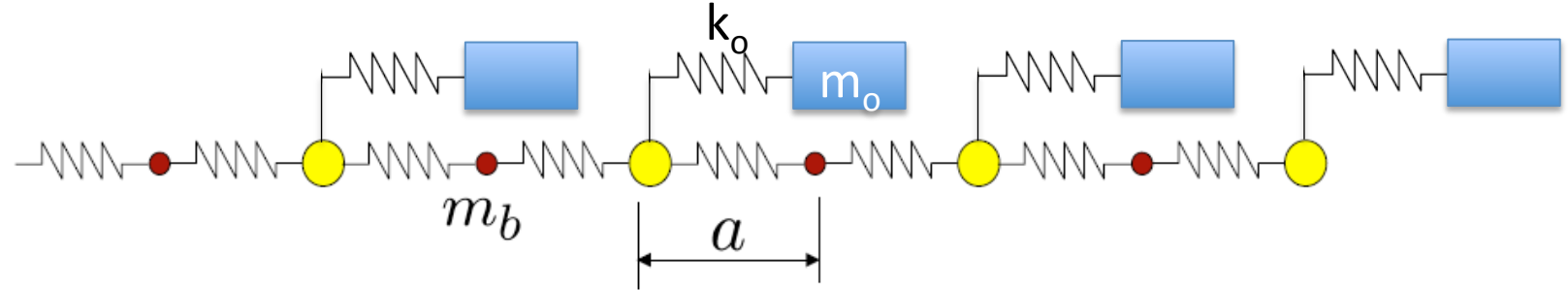
$$u_{2n+1}(\omega) = u_b(\omega) e^{j\kappa x_{2n+1}} = u_b(\omega) e^{j(2n+1)a\kappa}$$

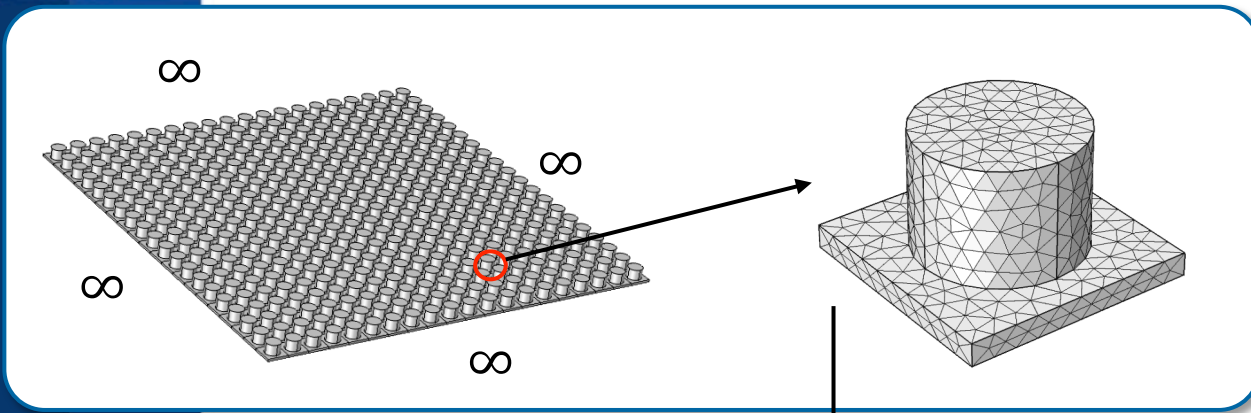


From M Ruzzene Courses

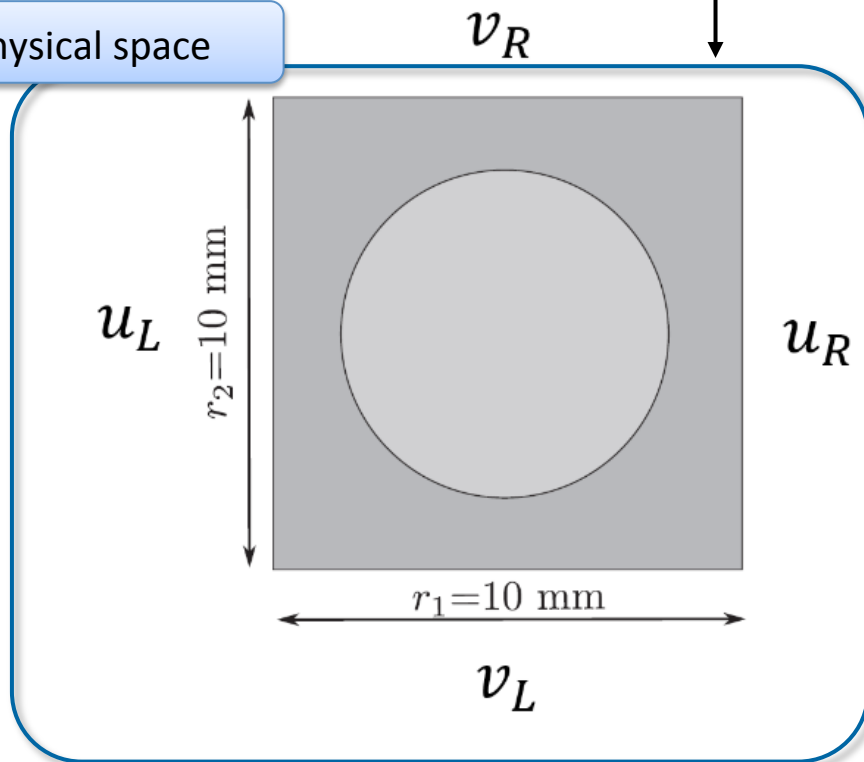


Let's consider a biatomic sonic lattice :

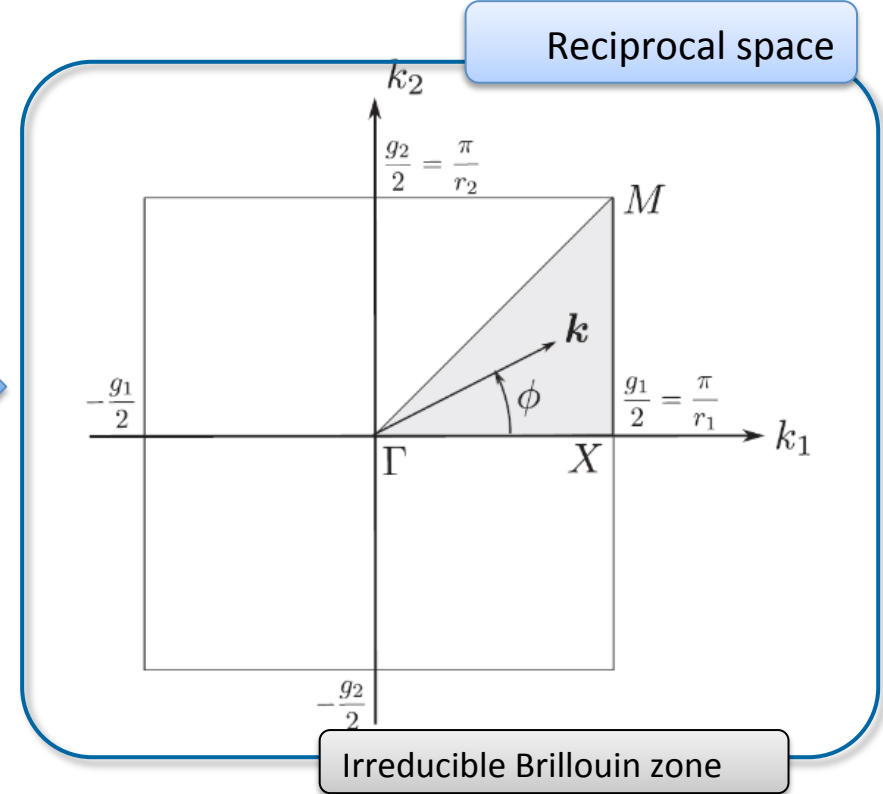




Physical space



Reciprocal space



Irreducible Brillouin zone

Wave propagation characteristics in the (infinite) periodic structure

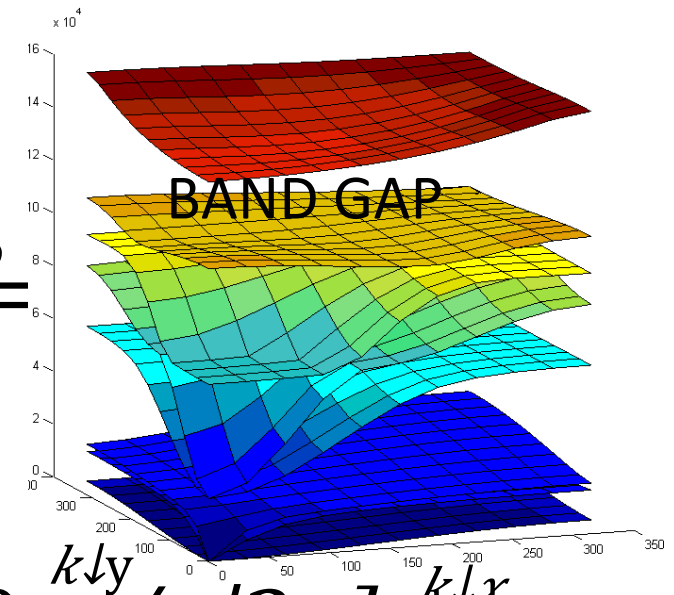
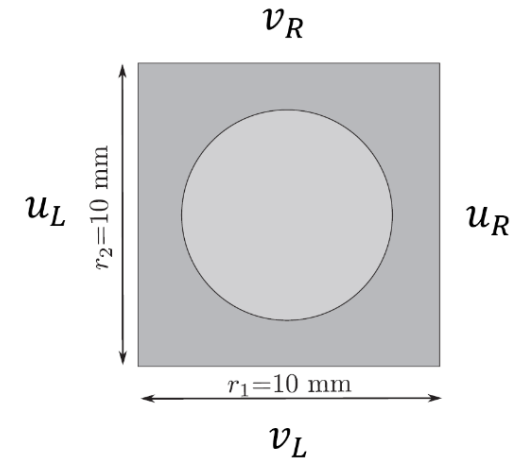
=> Solve the parametric eigenvalue problem

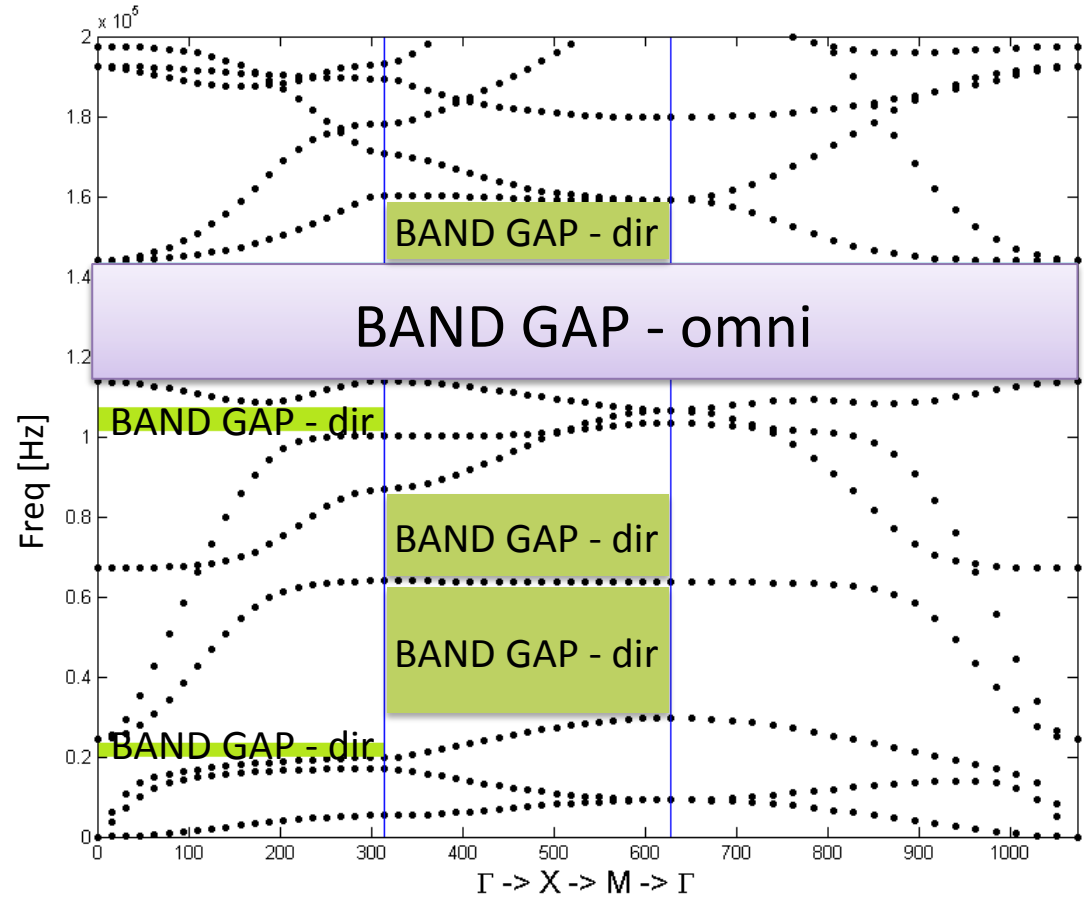
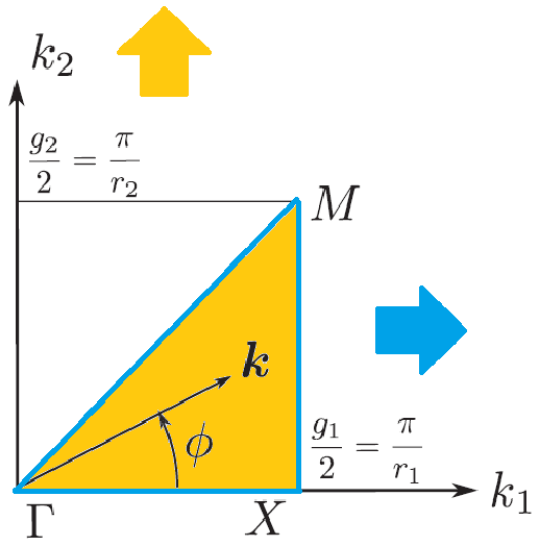
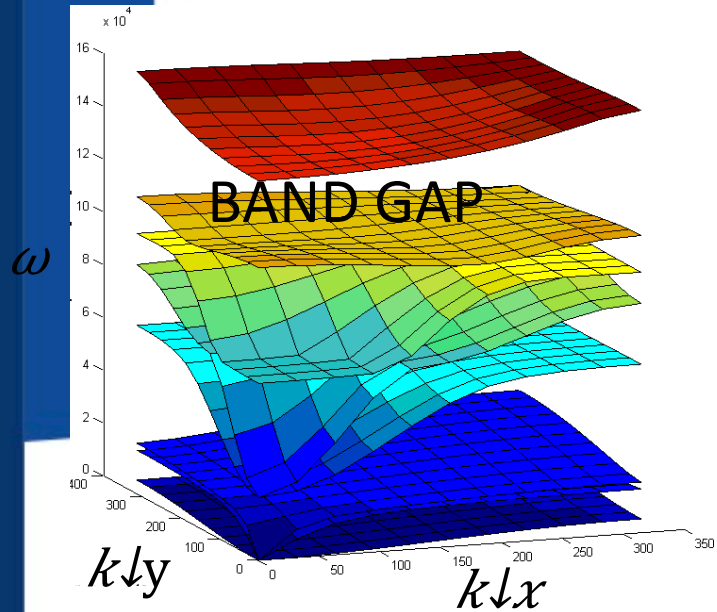
$$\left\{ \begin{array}{l} \rho \omega^2 u + \nabla \sigma = 0 \\ \sigma = C : \varepsilon \end{array} \right.$$

with « Floquet-Bloch » periodic conditions

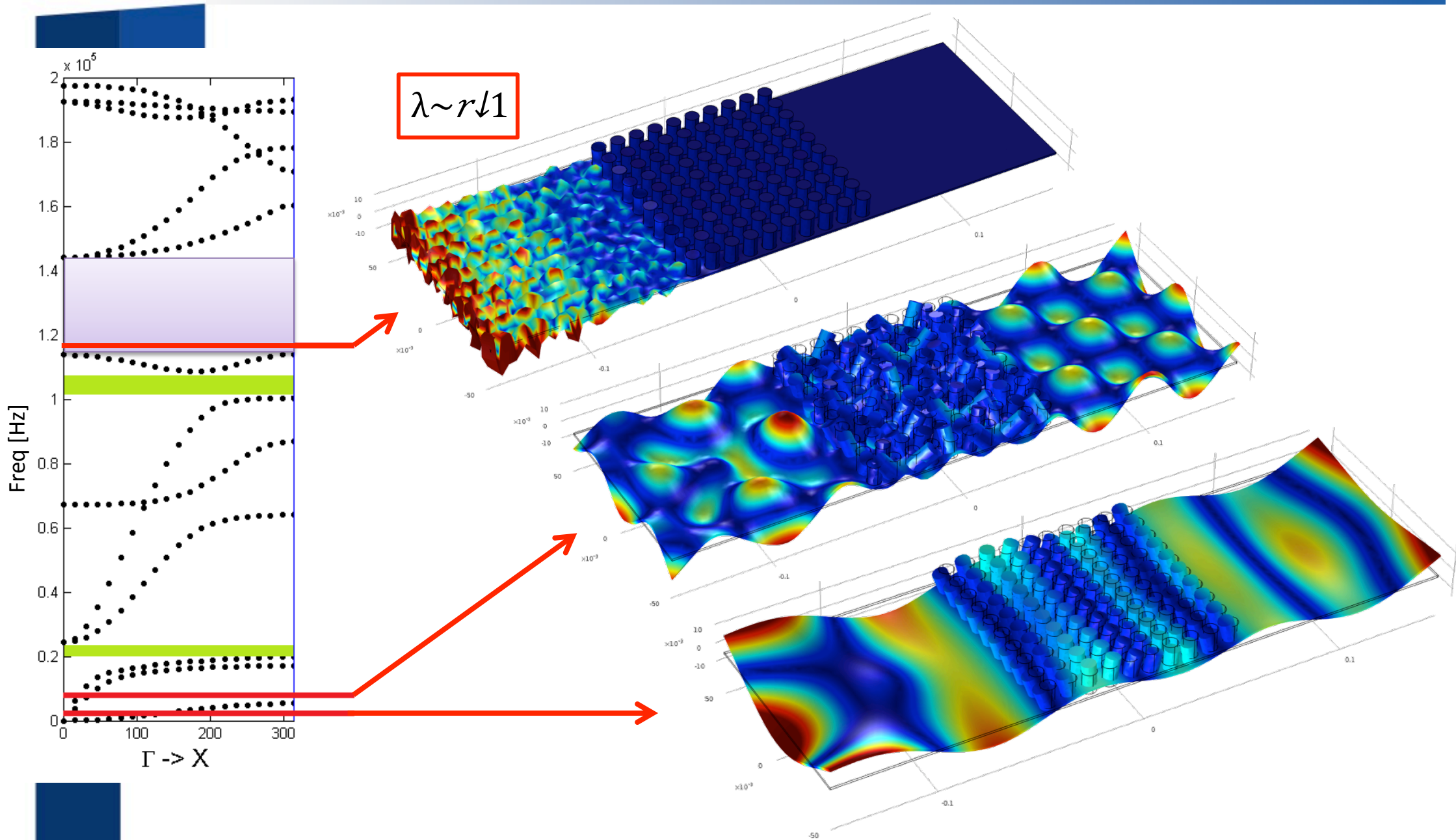
$$u \downarrow R = e^{i k \downarrow x} r \downarrow 1 u \downarrow L \quad \text{and} \quad v \downarrow R = e^{i \omega} r \downarrow 2 v \downarrow L$$

Parameters: $k \downarrow x = [0; \pi / r \downarrow 1]$ and $k \downarrow y = [0; \pi / r \downarrow 2]$.



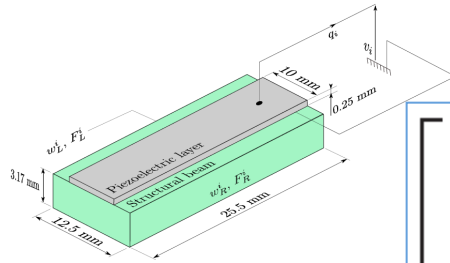
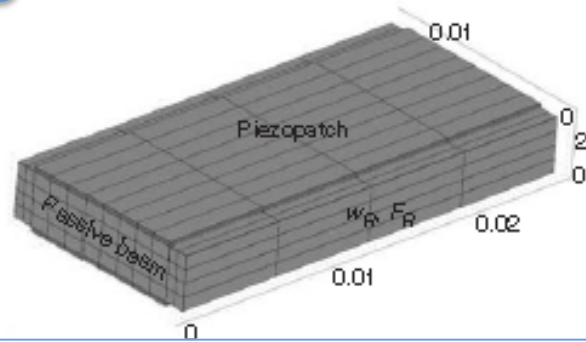
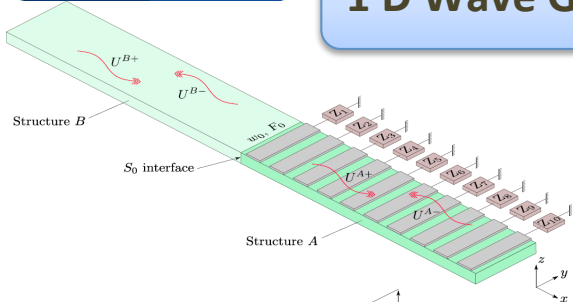


Band diagram: only for band gap analyses



Modeling aspects : Wave Finite Elements WFE

1 D Wave Guide



$$\begin{bmatrix} \bar{S}_{ll} & \bar{S}_{lr} & -B_{LI} \\ \bar{S}_{lr}^T & \bar{S}_{rr} & -B_{RI} \\ Z(i\omega)B_{LI}^T & Z(i\omega)B_{RI}^T & 1 + Z(i\omega)D_I \end{bmatrix} \begin{bmatrix} w_L \\ w_R \\ V_I \end{bmatrix} = \begin{bmatrix} F_L \\ F_R \\ 0 \end{bmatrix}$$

$$w(x, y, z, i\omega) = W(y, z)e^{-ikx} e^{i\omega t}$$

$$\lambda = e^{-ikd}$$

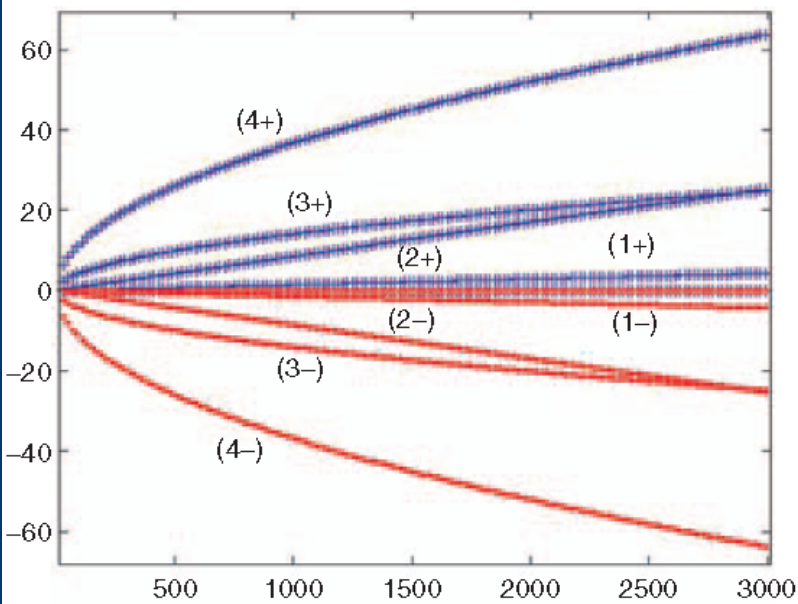
$$W_R(y, z) = \lambda W_L(y, z)$$

$$F_R^{(I)} = -\lambda F_L^{(I)}$$

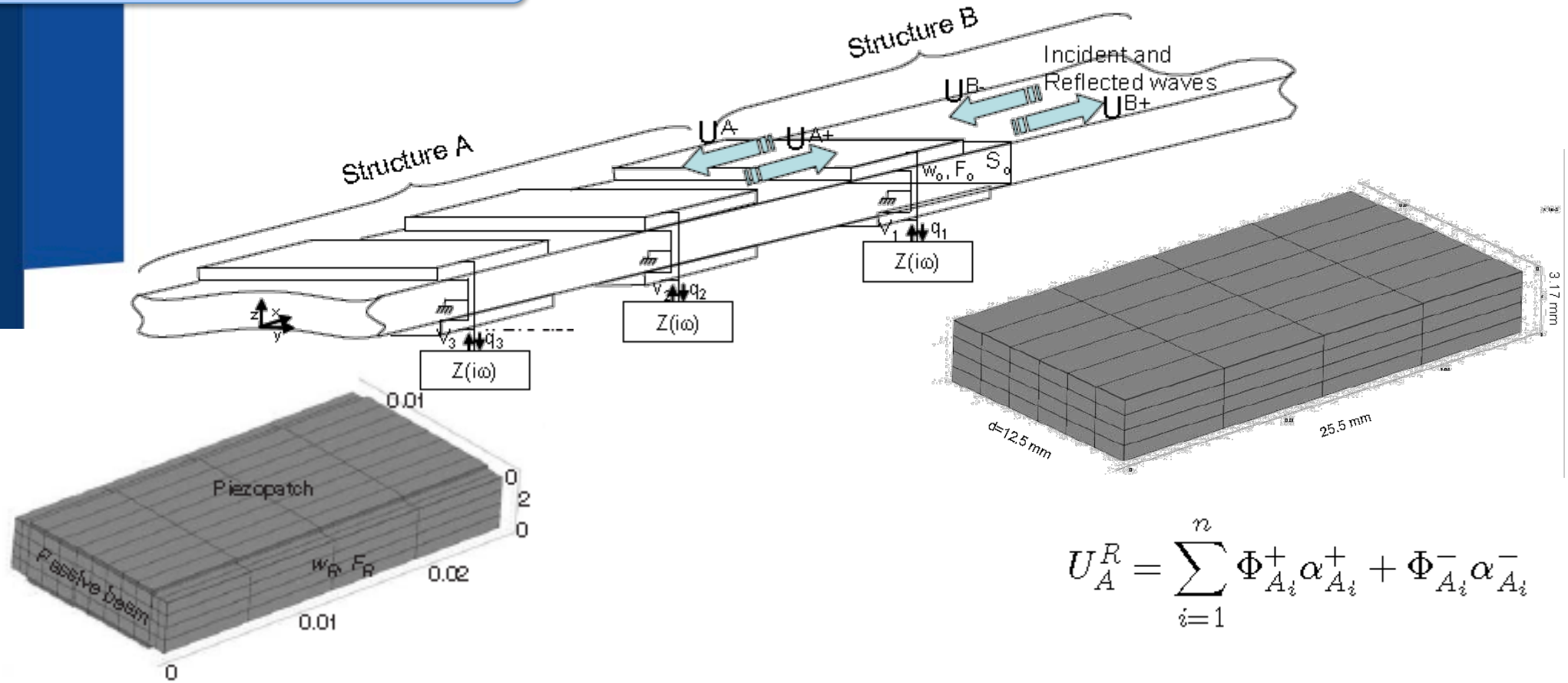
Floquet Bloch Conditions

$$\begin{bmatrix} -S_{ll}^a & -S_{lr}^a \\ S_{lr}^{aT} & S_{rr}^a \end{bmatrix} \begin{bmatrix} w_L \\ \lambda w_L \end{bmatrix} = \begin{bmatrix} F'_L \\ \lambda F'_L \end{bmatrix}$$

Symplectic Eigenvalue problem



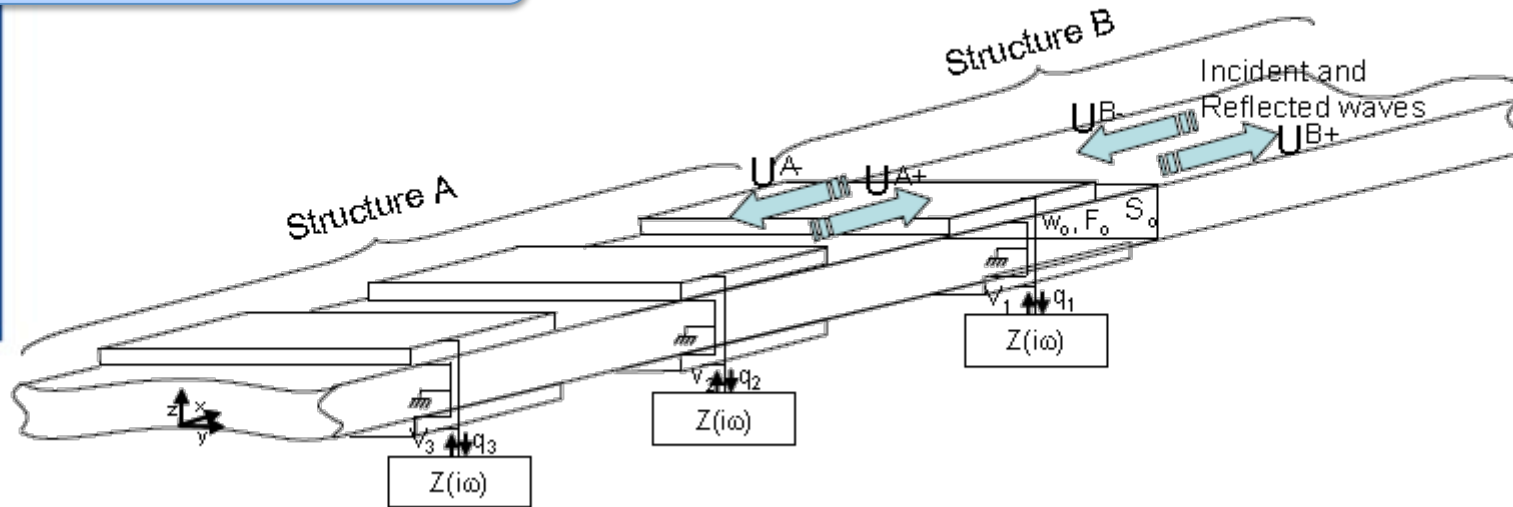
1D Wave Guide : Diffusion operator



$$U_A^R = \sum_{i=1}^n \Phi_{A_i}^+ \alpha_{A_i}^+ + \Phi_{A_i}^- \alpha_{A_i}^-$$

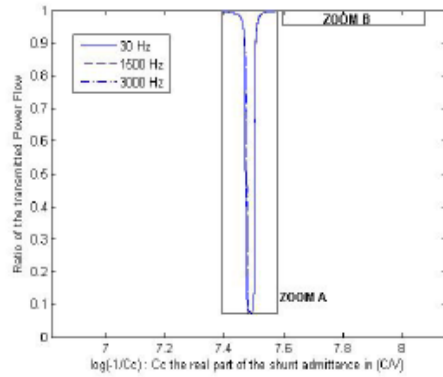
$$U_B^L = \sum_{i=1}^n \Phi_{B_i}^+ \alpha_{B_i}^+ + \Phi_{B_i}^- \alpha_{B_i}^-$$

1D Wave Guide : Diffusion operator

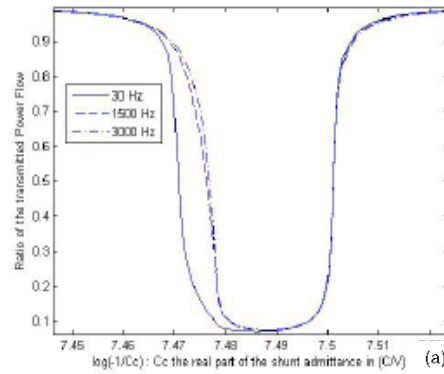


$$\begin{bmatrix}
 0 & \Phi_A^{+T} J_n \Phi_A^- & -\Phi_A^{+T} J_n \Phi_B^+ & -\Phi_A^{+T} J_n \Phi_B^- & N_A^{+T} \\
 \Phi_A^{-T} J_n \Phi_A^+ & 0 & -\Phi_A^{-T} J_n \Phi_B^+ & -\Phi_A^{-T} J_n \Phi_B^- & 0 \\
 \Phi_B^{+T} J_n \Phi_A^+ & \Phi_B^{+T} J_n \Phi_A^- & 0 & -\Phi_B^{+T} J_n \Phi_B^- & 0 \\
 \Phi_B^{-T} J_n \Phi_A^+ & \Phi_B^{-T} J_n \Phi_A^- & -\Phi_B^{-T} J_n \Phi_B^+ & 0 & N_B^{-T} \\
 N_A^{+T} & 0 & 0 & N_B^{-T} & 0
 \end{bmatrix}
 \begin{bmatrix}
 \alpha_A^+ \\
 \alpha_A^- \\
 \alpha_B^+ \\
 \alpha_B^- \\
 \xi
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 0 \\
 U_0
 \end{bmatrix}$$

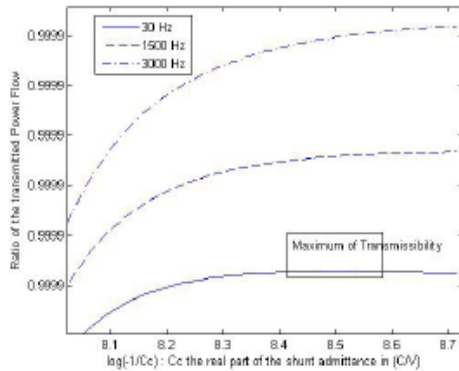
1D Wave Guide : Diffusion operator



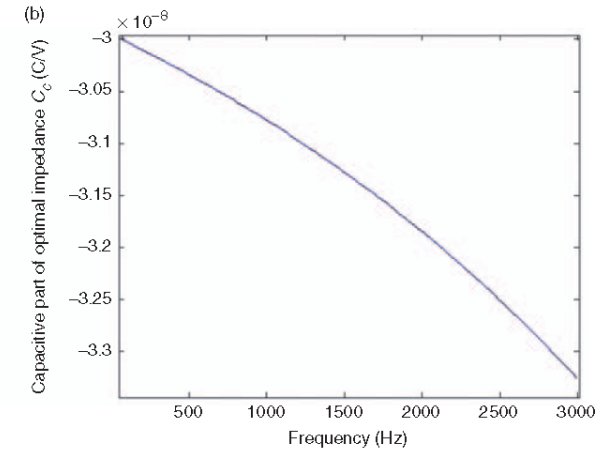
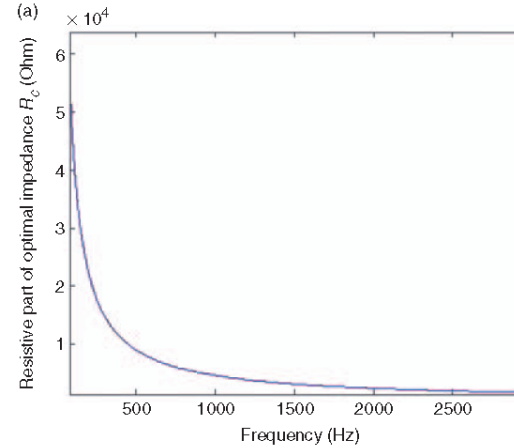
(a) General view



(b) Zoom A



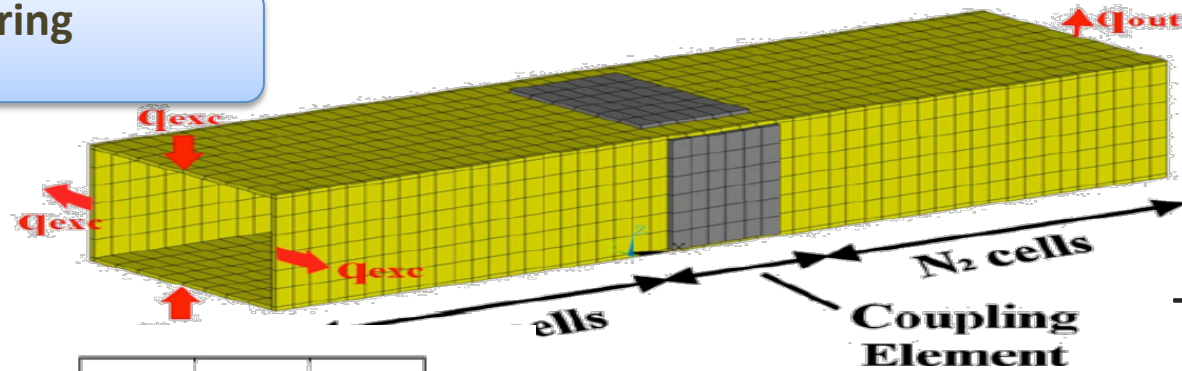
(c) Zoom B



quasi C negative + resistance

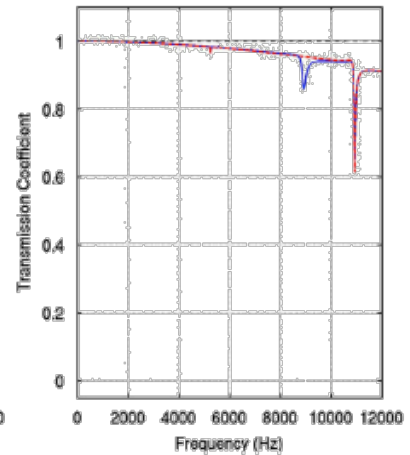
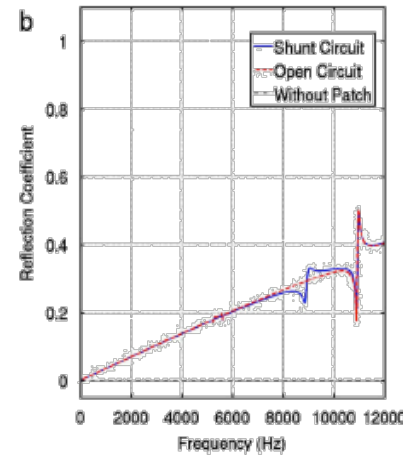
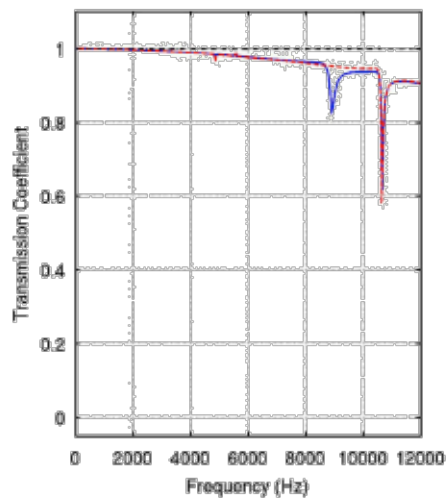
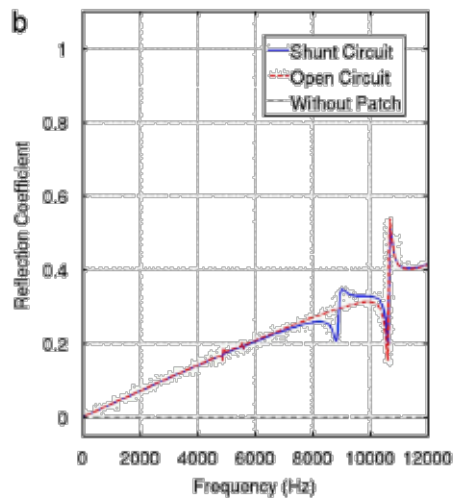
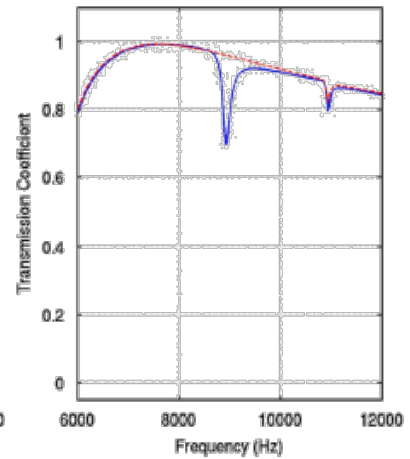
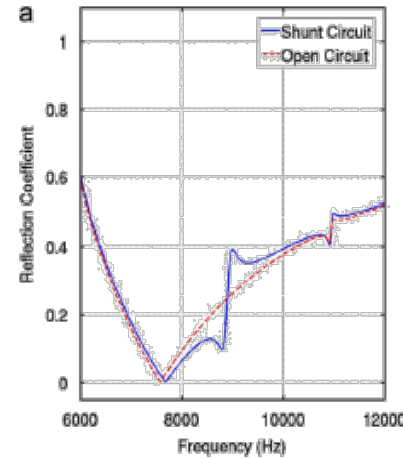
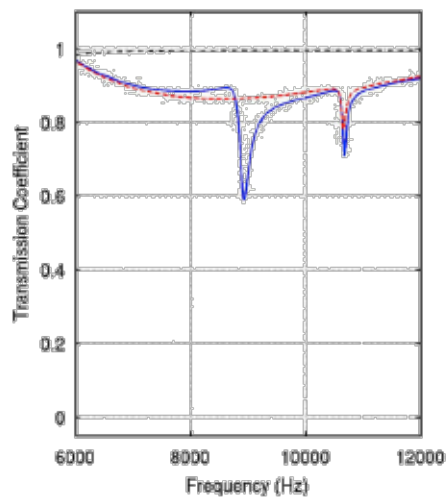
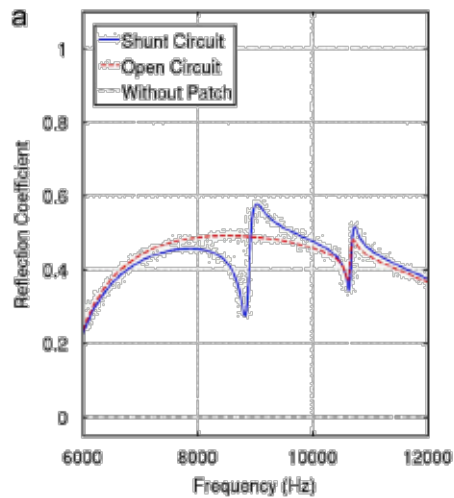
Figure III.11: Ratio of the transmitted flexural power flux as a function of negative capacitance shunt at 30 Hz, 1500 Hz and 3000 Hz

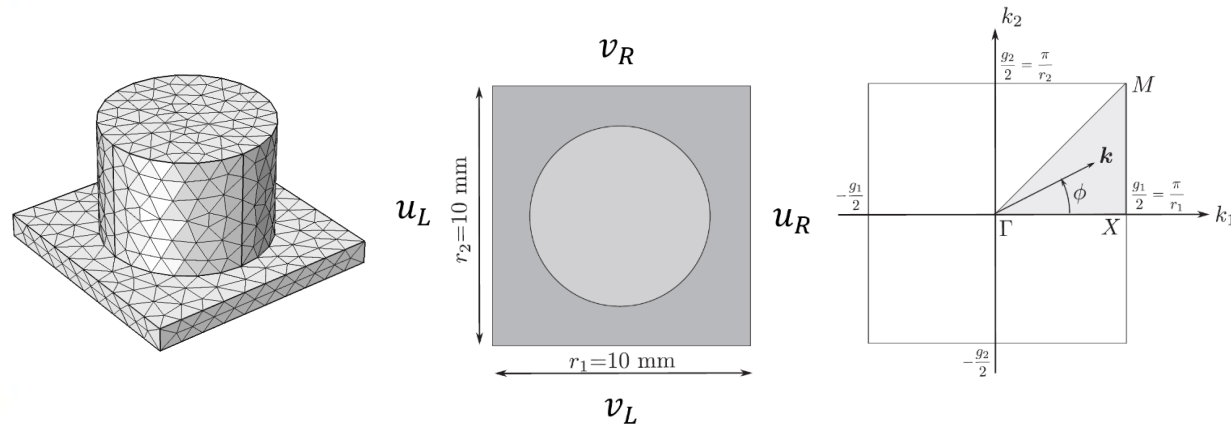
1D Wave Guide : Scattering computation



Reflection

Transmission





The physics

$$\rho(\mathbf{x})\omega^2 \mathbf{w}(\mathbf{x}) + \nabla C(\mathbf{x}) \nabla_{sym}(\mathbf{w}(\mathbf{x})) = 0 \quad \forall \mathbf{x} \in \mathbb{R}^3$$

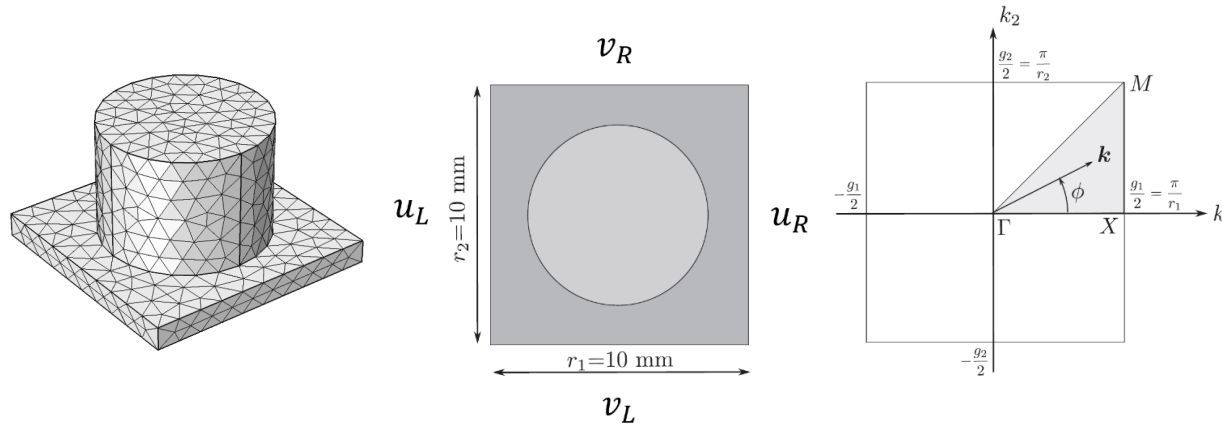
Floquet-Bloch modes

$$\mathbf{w}(\mathbf{x}) = \mathbf{w}_{n,k}(\mathbf{x}, \mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}} \quad + \text{« Floquet-Bloch » periodic conditions}$$

The "shifted-cell" formulation

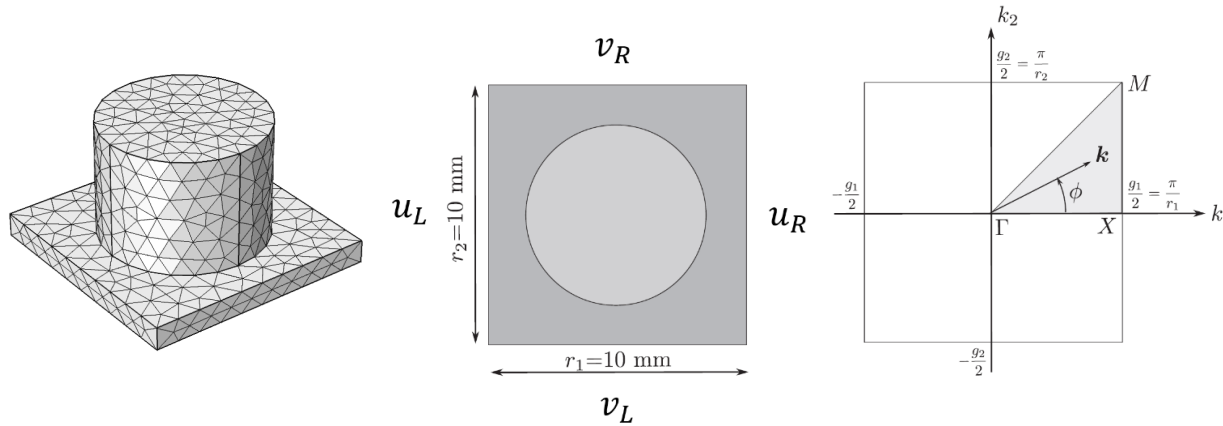
$$\frac{\partial}{\partial \mathbf{x}} \rightarrow \frac{\partial}{\partial \mathbf{x}} + i\mathbf{k} \quad \text{in the cell} \quad + \text{symmetric boundary conditions}$$

A. Bensoussan, J.L. Lions, and G. Pananicolaou. Asymptotic Analysis for Periodic Structures. North Holland, 1978.
C. Wilcox. Theory of bloch waves. Journal d'Analyse Mathématique, 33:146–167, 1978. ISSN 0021-7670



The (shifted) physics

$$\begin{aligned}
 & \rho(\mathbf{x})\omega_n(\mathbf{k})^2 \mathbf{w}_{n,k}(\mathbf{x}) + \nabla C(\mathbf{x}) \nabla_{sym}(\mathbf{w}_{n,k}(\mathbf{x})) \\
 & -iC(\mathbf{x}) \nabla_{sym}(\mathbf{w}_{n,k}(\mathbf{x})) \cdot \mathbf{k} - i \nabla C(\mathbf{x}) \frac{1}{2} (\mathbf{w}_{n,k}(\mathbf{x}) \cdot \mathbf{k}^T + \mathbf{k} \cdot \mathbf{w}_{n,k}^T(\mathbf{x})) \\
 & + C(\mathbf{x}) \frac{1}{2} (\mathbf{w}_{n,k}(\mathbf{x}) \cdot \mathbf{k}^T + \mathbf{k} \cdot \mathbf{w}_{n,k}^T(\mathbf{x})) \cdot \mathbf{k} = 0 \quad \forall \mathbf{x} \in \Omega_R, \\
 & \mathbf{w}_{n,k}(\mathbf{x} - \mathbf{R} \cdot \mathbf{n}) - \mathbf{w}_{n,k}(\mathbf{x}) = 0 \quad \forall \mathbf{x} \in \Gamma_R.
 \end{aligned}$$



Weak form => FE formulation

$$(\mathbf{K} + \lambda \mathbf{L}(\Phi) - \lambda^2 \mathbf{H}(\Phi) - \omega_n^2(\lambda, \Phi) \mathbf{M}) \mathbf{w}_{n,k}(\Phi) = 0$$

$$\lambda = ik$$

$$\mathbf{k} = k \begin{bmatrix} \sin(\theta) \cos(\phi) \\ \sin(\theta) \sin(\phi) \\ \cos(\theta) \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \sin(\theta) \cos(\phi) \\ \sin(\theta) \sin(\phi) \\ \cos(\theta) \end{bmatrix}$$

$$\mathbf{M} \rightarrow \int_{\Omega_R} \rho(\mathbf{x}) \omega_n^2(\mathbf{k}) \tilde{\mathbf{w}}_{n,k}(\mathbf{x}) \mathbf{w}_{n,k}(\mathbf{x}) d\Omega,$$

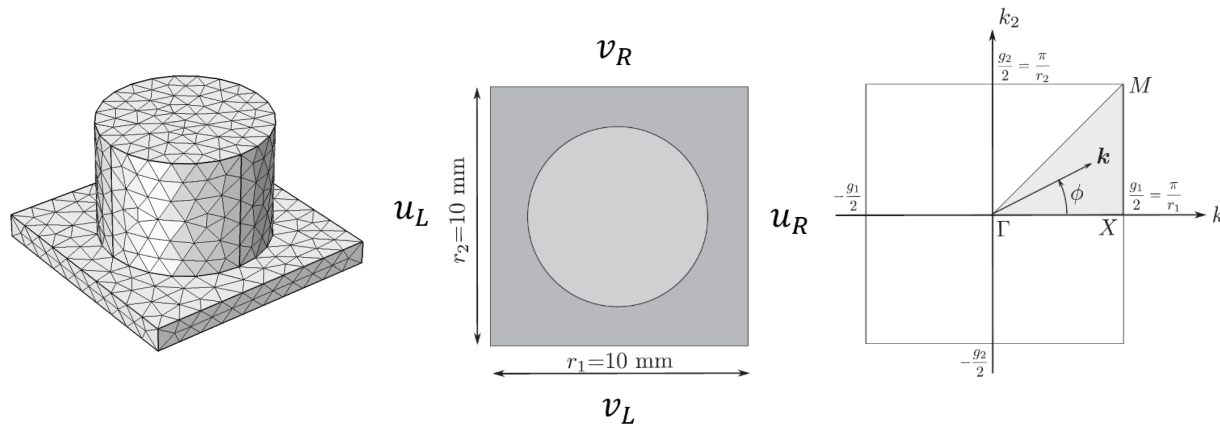
$$\mathbf{K} \rightarrow \int_{\Omega_R} \tilde{\boldsymbol{\varepsilon}}_{n,k}(\mathbf{x}) \mathbf{C}(\mathbf{x}) \boldsymbol{\varepsilon}_{n,k}(\mathbf{x}) d\Omega,$$

$$\mathbf{L} \rightarrow \int_{\Omega_R} -\tilde{\boldsymbol{\kappa}}_{n,k}(\mathbf{x}) \mathbf{C}(\mathbf{x}) \boldsymbol{\varepsilon}_{n,k}(\mathbf{x}) + \tilde{\boldsymbol{\varepsilon}}_{n,k}(\mathbf{x}) \mathbf{C}(\mathbf{x}) \boldsymbol{\kappa}_{n,k}(\mathbf{x}) d\Omega,$$

skew-symmetric

$$\mathbf{H} \rightarrow \int_{\Omega_R} \tilde{\boldsymbol{\kappa}}_{n,k}(\mathbf{x}) \mathbf{C}(\mathbf{x}) \boldsymbol{\kappa}_{n,k}(\mathbf{x}) d\Omega.$$

symmetric semi-definite positive



Weak form => FE formulation

$$(K + \lambda L(\Phi) - \lambda^2 H(\Phi) - \omega_n^2(\lambda, \Phi) M) w_{n,k}(\Phi) = 0$$

fix $k \downarrow x$ and $k \downarrow y$, find ω and w

Rearrangement

$$(K - \omega^2 M) + \lambda_n(\omega, \Phi) L(\Phi) - \lambda_n^2(\omega, \Phi) H(\Phi) w_{n,k}(\Phi) = 0$$

fix ϕ and ω , find k and w

⇒ Solve a **QEP** to characterize **wave propagation** in any **frequency-dependent** (visco, poro, piezo...) **periodic structure**

Some practical issues

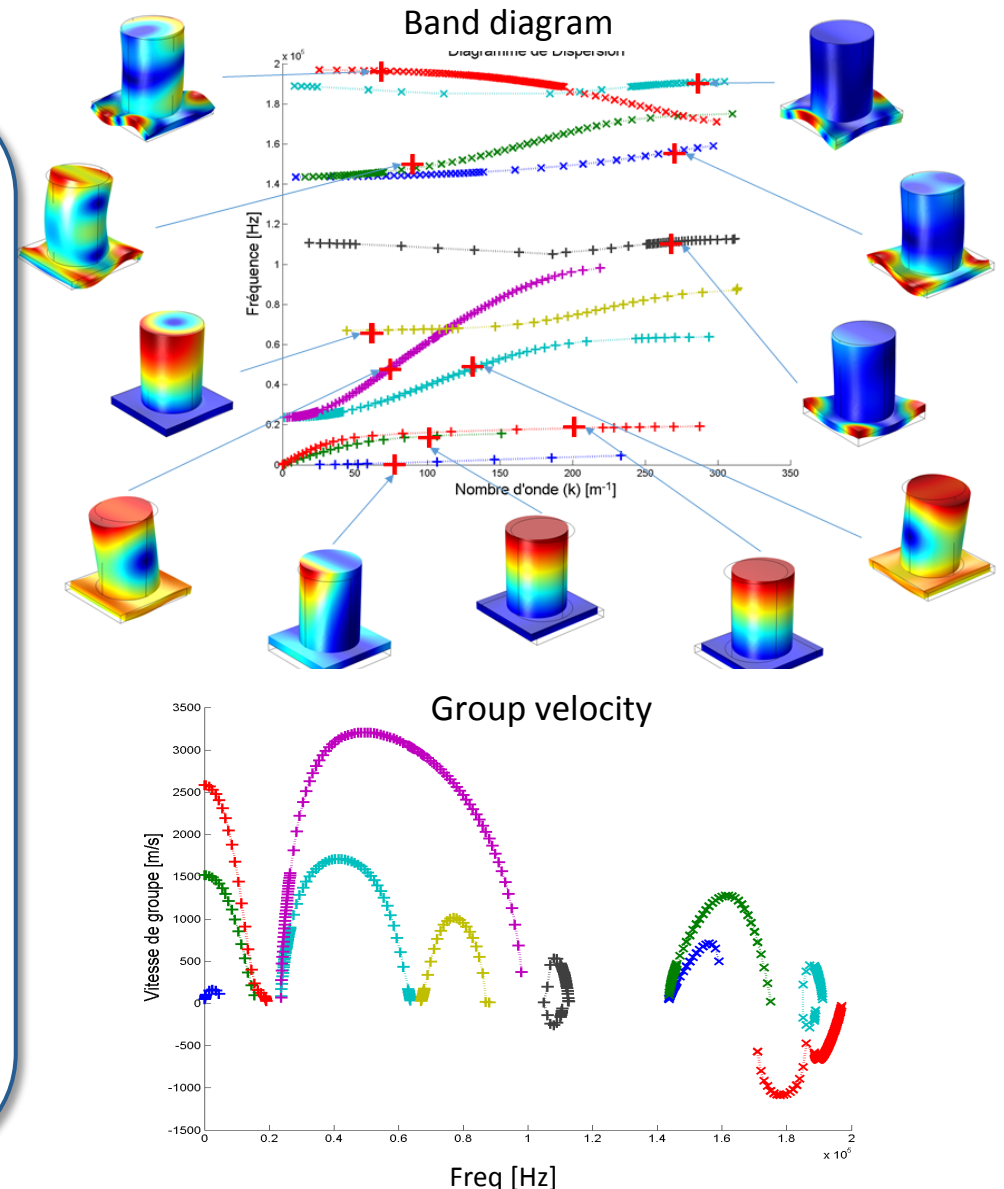
- All wave numbers are **complex**
=> need for suitable criteria to distinguish “**propagative**” and “**evanescent**” waves
- How to **track** a given wave when parameters change?
=> need for **correlation criteria**
- Computation of the **group velocity** for periodic **damped** structures?

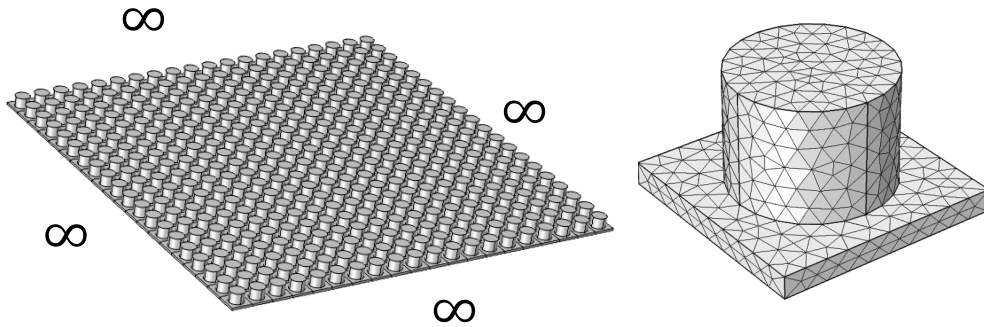
$$C \downarrow g = (\partial \omega / \partial k) ???$$

- For an **homogeneous material** with

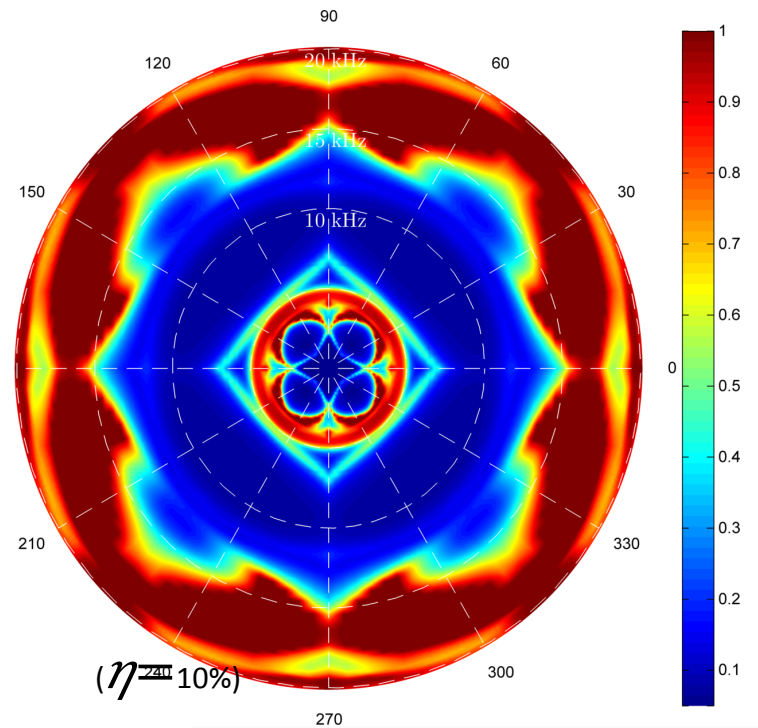
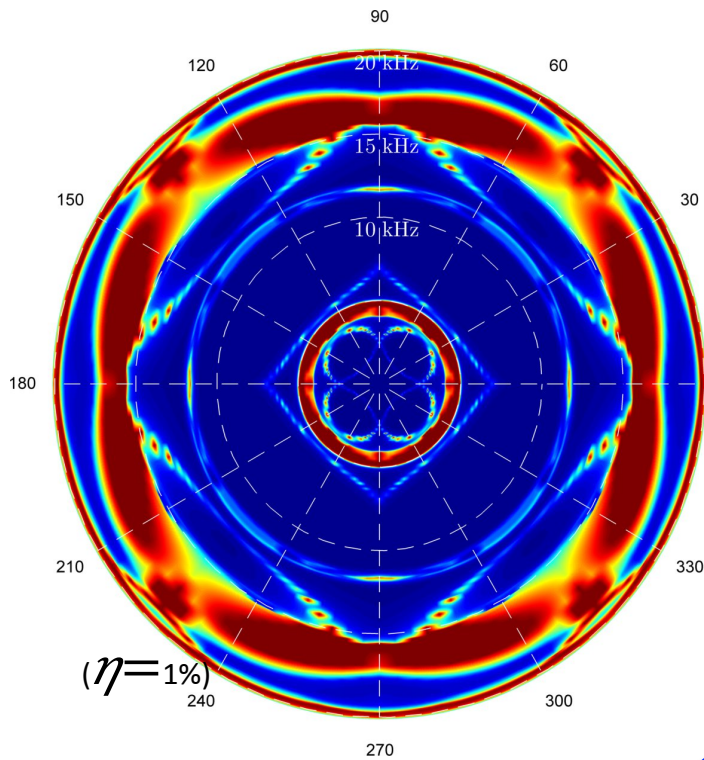
$$E(\omega) = f(\omega) E \downarrow 0 \quad \text{one has}$$

$$C \downarrow g = \text{real} (j \phi \downarrow i \uparrow l \uparrow T [f(\omega) (-L \downarrow 0 + L \downarrow 0 \uparrow T + 2 \lambda \downarrow i H \downarrow 0)] \phi \downarrow i \uparrow r / \phi \downarrow i \uparrow l \uparrow T [\omega \uparrow 2 (\partial f / \partial \omega / f(\omega)) - 2 \omega] M \phi \downarrow i \uparrow r)$$





All waves are highly evanescent



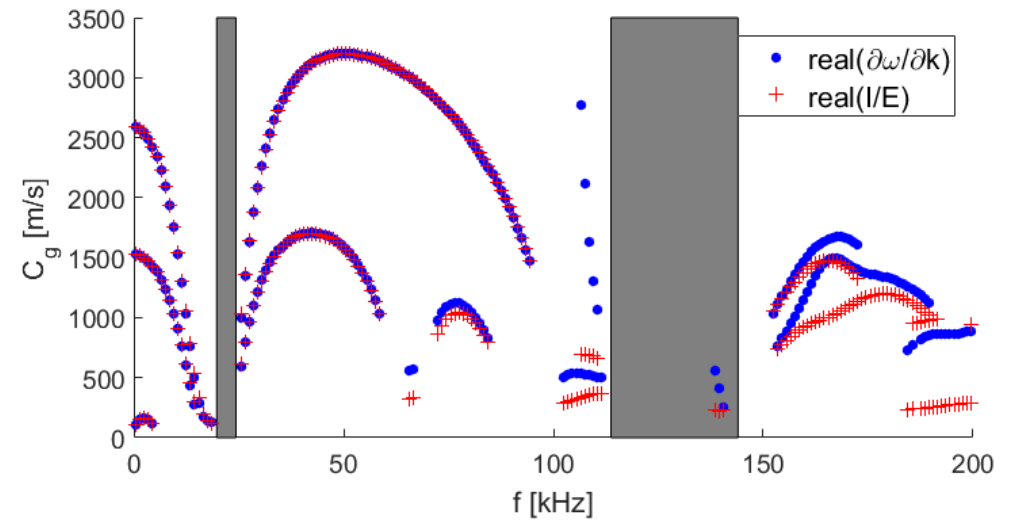
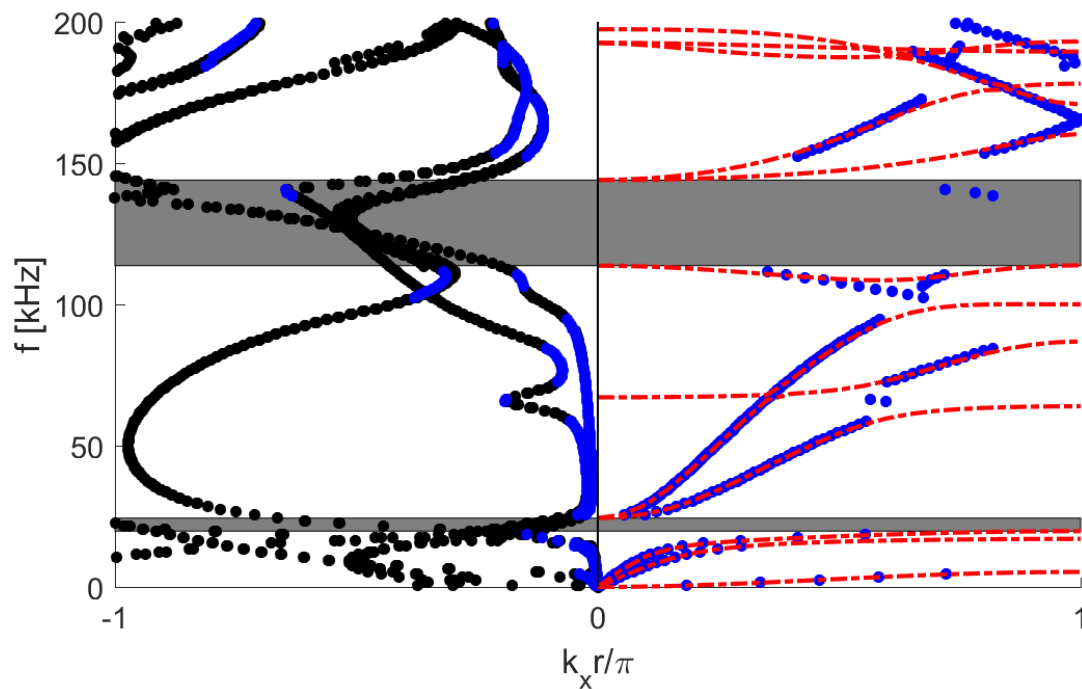
Directivity in terms of evanescence

Some waves are highly propagative

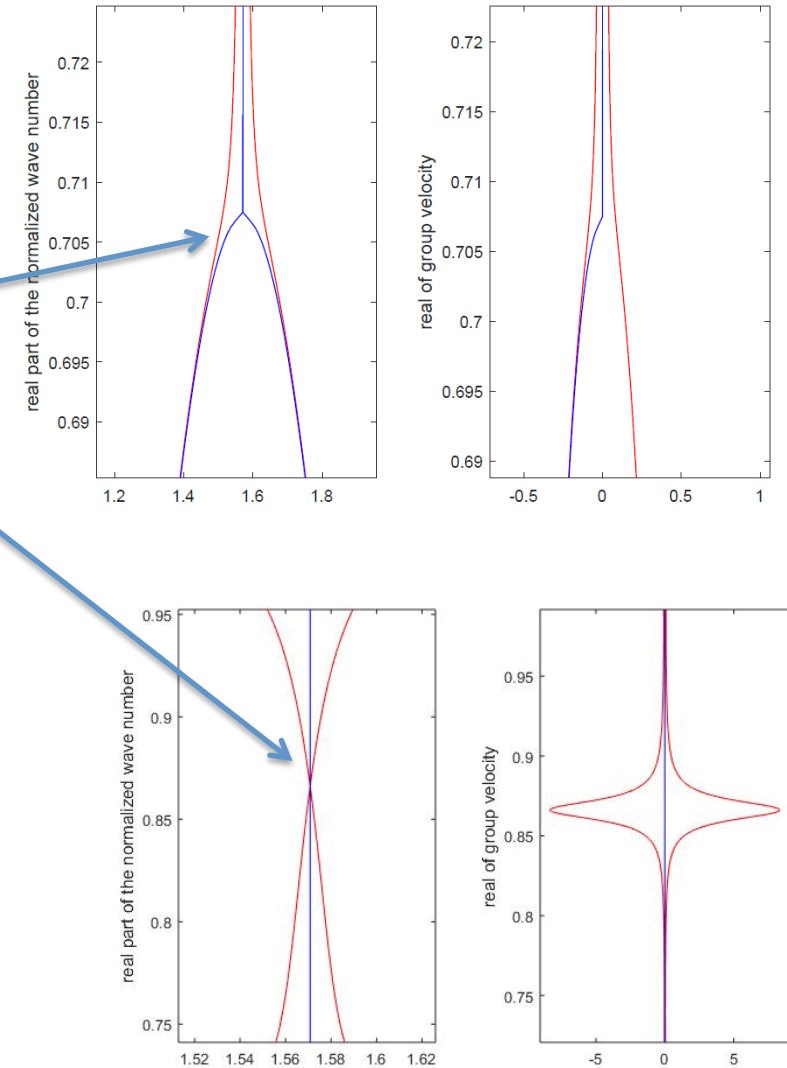
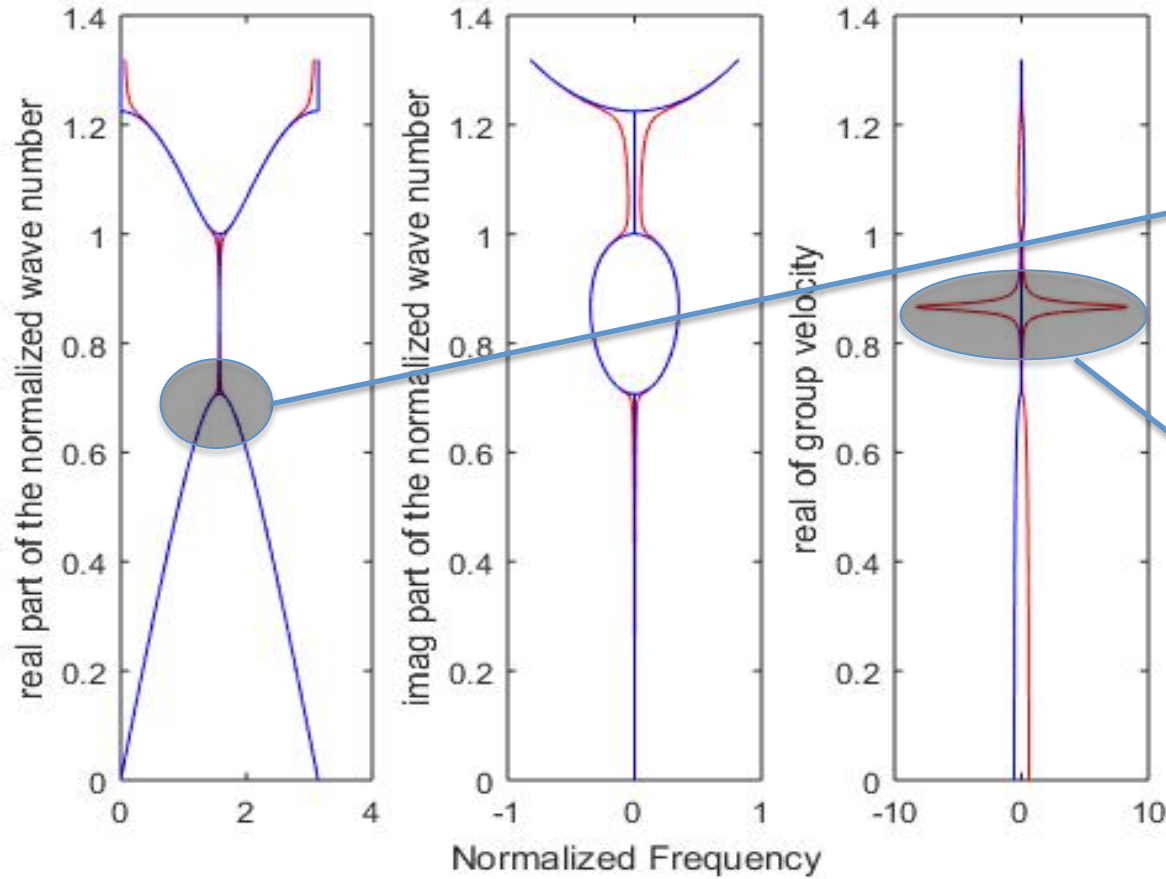
Dampen Medium

$\eta=10\%$

- Shift Cell $\eta=10\%$ $\phi=0^\circ$ Real(k_x)
- Shift Cell $\eta=10\%$ $\phi=0^\circ$ Imag(k_x)
- Floquet-Bloch $\eta=0\%$ $\phi=0^\circ$



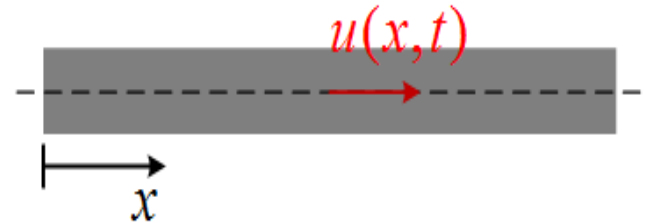
Group velocity in dampen medium



$$C_g = \text{Re}\left(\frac{d\omega}{dk}\bigg|_{k_m}\right) - \text{Im}\left(\frac{d\omega}{dk}\bigg|_{k_m}\right) \frac{\Lambda_i}{\Lambda_r}$$

Muschietti L and Dumb C T 1993 *Phys. Fluids B* 5 1383–1397

Example : Time varying beam system



$$E(x,t) = E(x + \lambda_m, t + T_m) \quad \rho(x,t) = \rho(x + \lambda_m, t + T_m)$$

$$\lambda_m = \frac{2\pi}{k_m}, \quad T_m = \frac{2\pi}{\omega_m}$$

Time varying longitudinal motion

$$\frac{\partial}{\partial x} \left[E(x,t) \frac{\partial u(x,t)}{\partial x} \right] - \frac{\partial}{\partial t} \left[\rho(x,t) \frac{\partial u(x,t)}{\partial t} \right] = 0$$

Outline of Plane Wave Expansion method

- Fourier expansion of material parameters:

$$E(x,t) = \sum_{p=-\infty}^{p=+\infty} \hat{E}_p e^{ip(\omega_m t - k_m x)} \quad \rho(x,t) = \sum_{p=-\infty}^{p=+\infty} \hat{\rho}_p e^{ip(\omega_m t - k_m x)}$$

$\hat{E}_p, \hat{\rho}_p$: Fourier coefficients

- Bloch expansion of the solution:

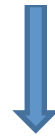
$$u(x,t) = e^{i(\omega t - k x)} \sum_{q=-N}^{+N} U_q e^{iq(\omega_m t - k_m x)}$$

N : Truncation order

Outline of Plane Wave Expansion method

- Quadratic Eigenvalue Problem (QEP):

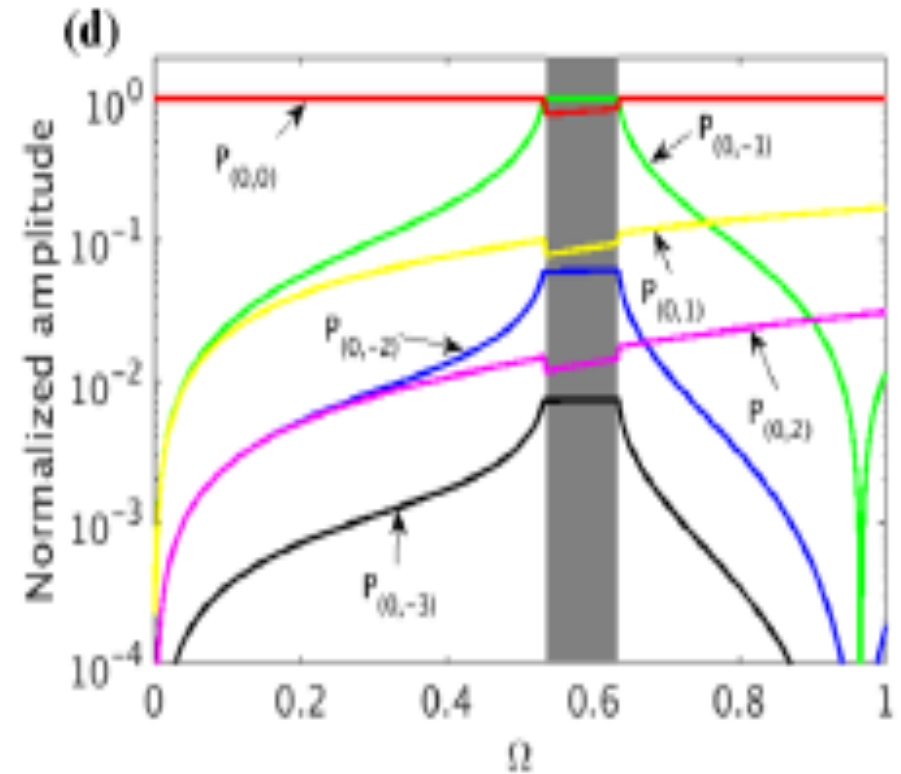
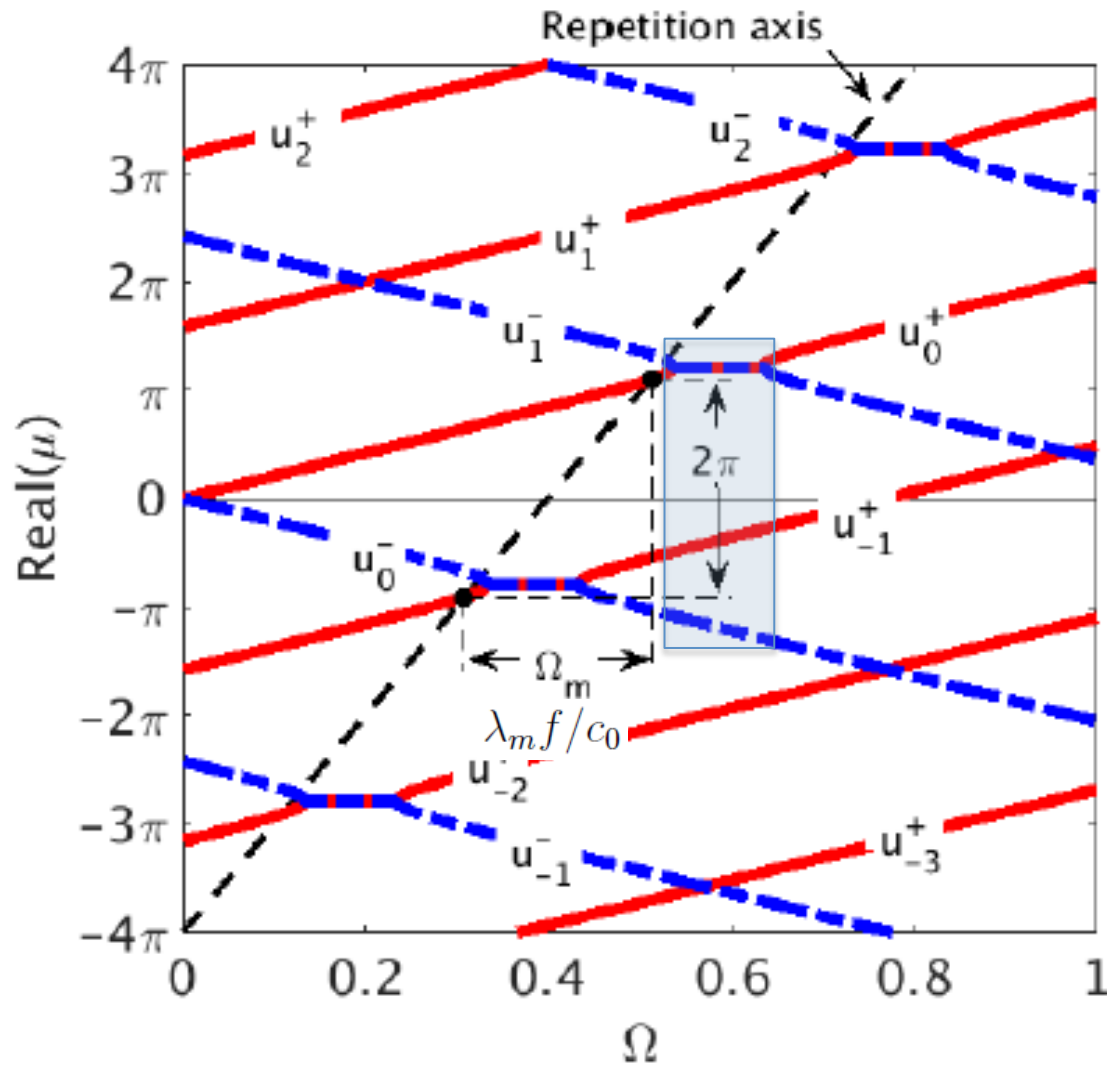
$$\sum_{q=-\infty}^{\infty} (k + qk_m)(k + nk_m) \hat{E}_{n-q} U_q = (\omega + n\omega_m)^2 \rho_0 U_n$$



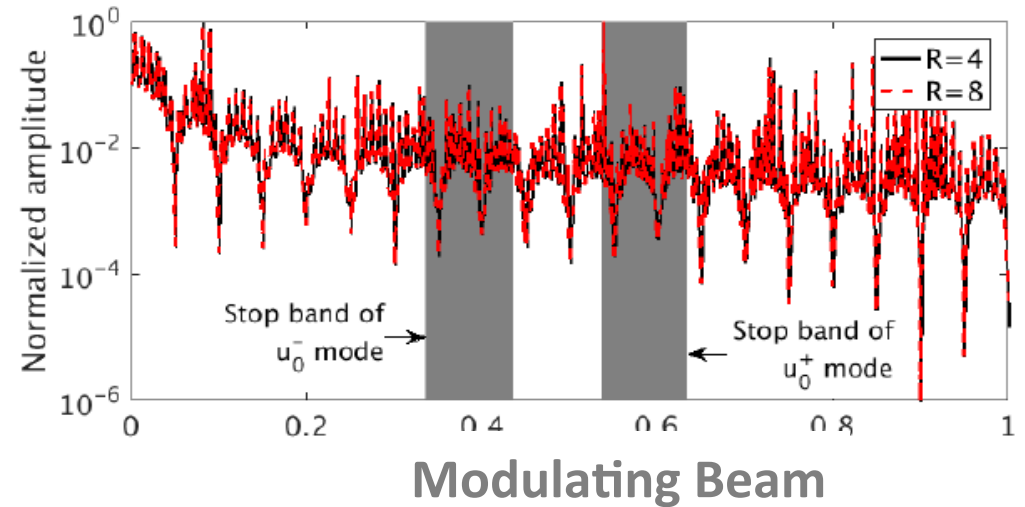
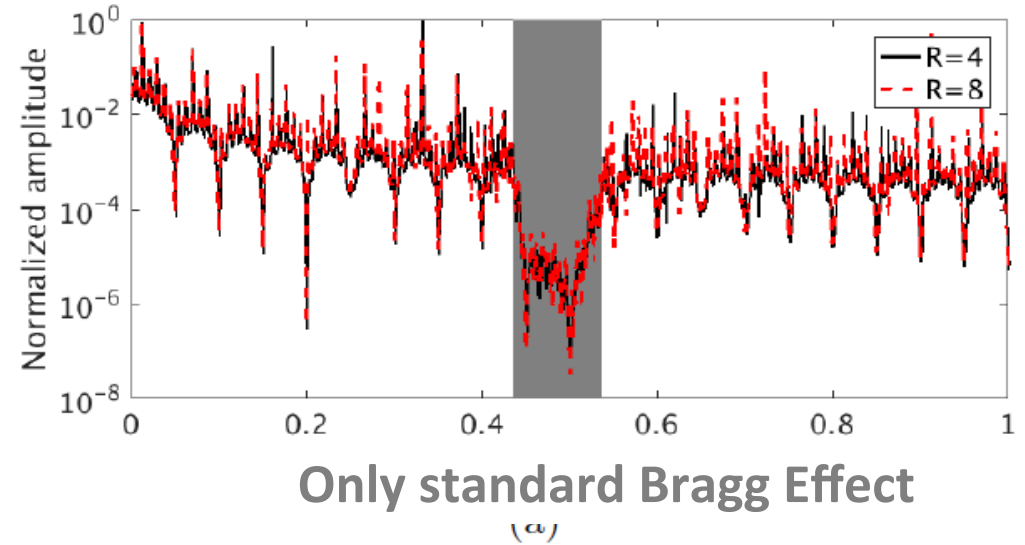
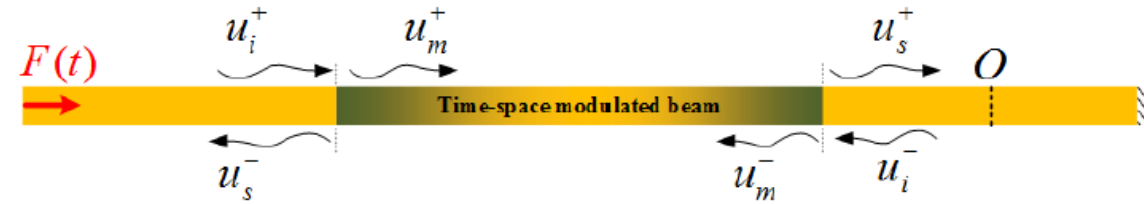
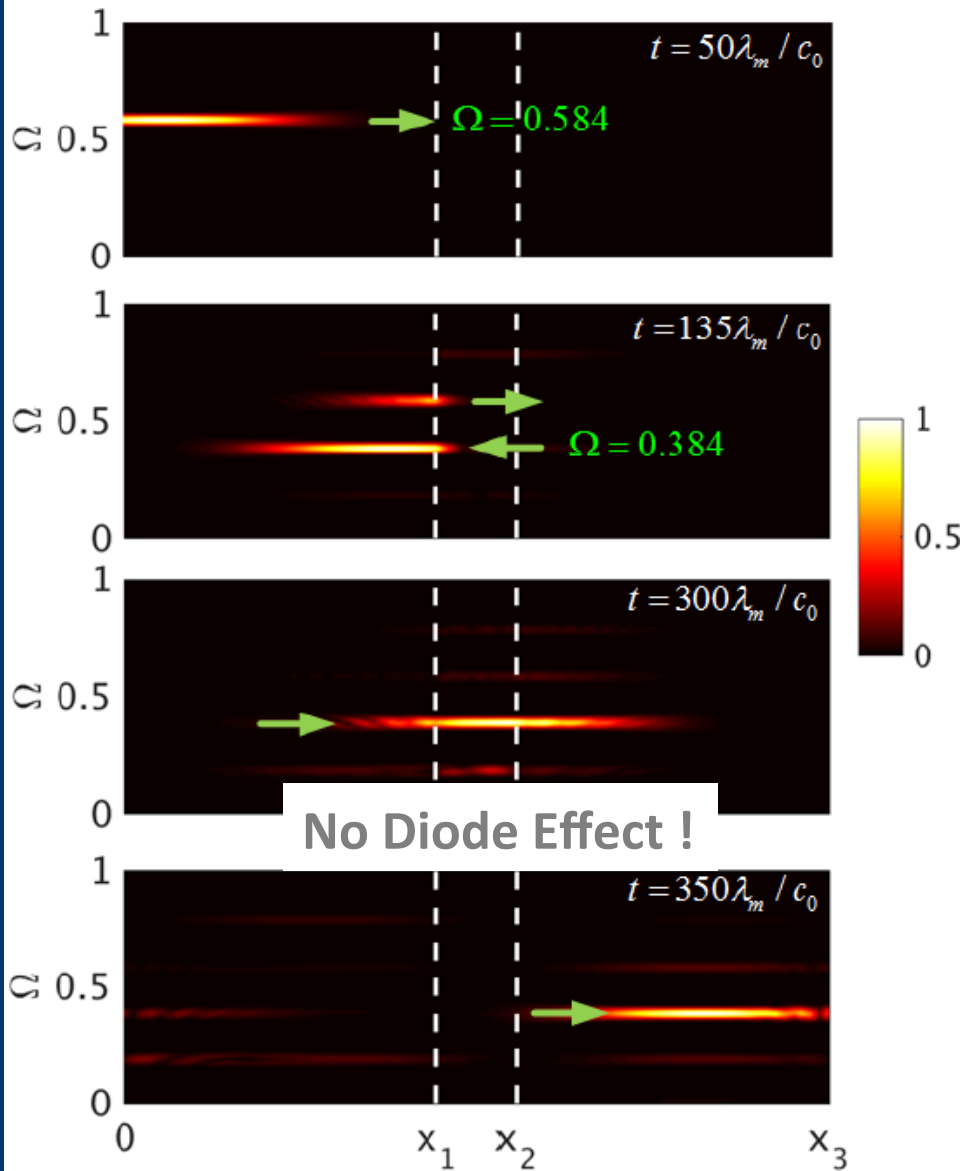
$$u_n^+(x, t, k_n^+, \omega) = \sum_{q=-N}^{+N} U_{(n,q)}^+ e^{i[(\omega + q\omega_m)t - (k_n^+ + qk_m)x]}$$

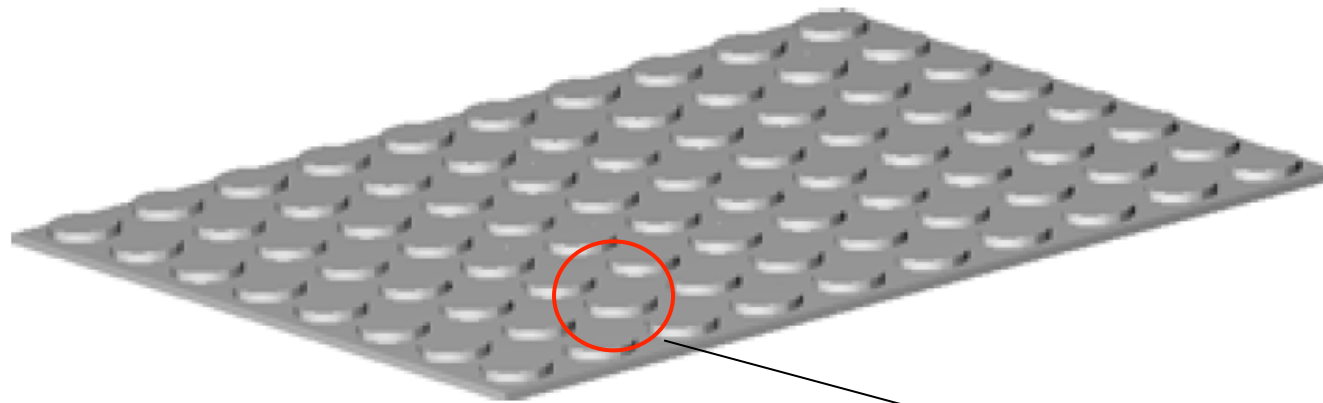
$$u_n^-(x, t, k_n^-, \omega) = \sum_{q=-N}^{+N} U_{(n,q)}^- e^{i[(\omega + q\omega_m)t - (k_n^- + qk_m)x]}$$

Example of obtained wave dispersion curves :



Example of obtained response :

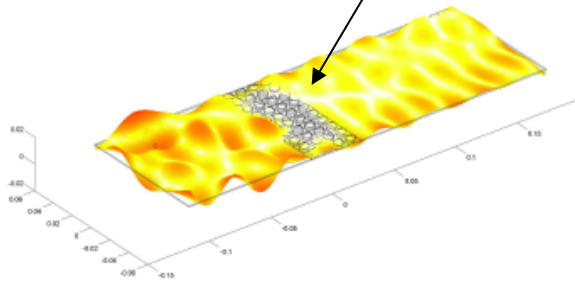
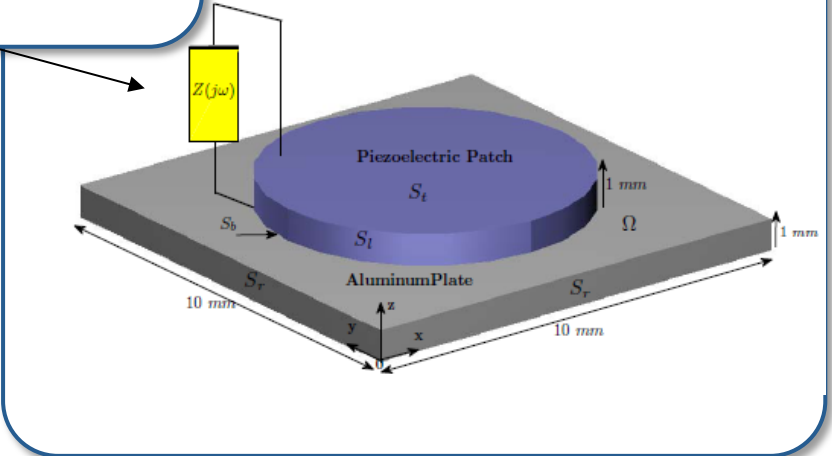




The metacomposite

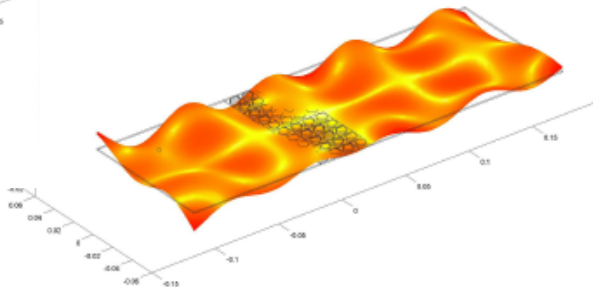
PZT-Aluminum Composite

Interface with two configurations

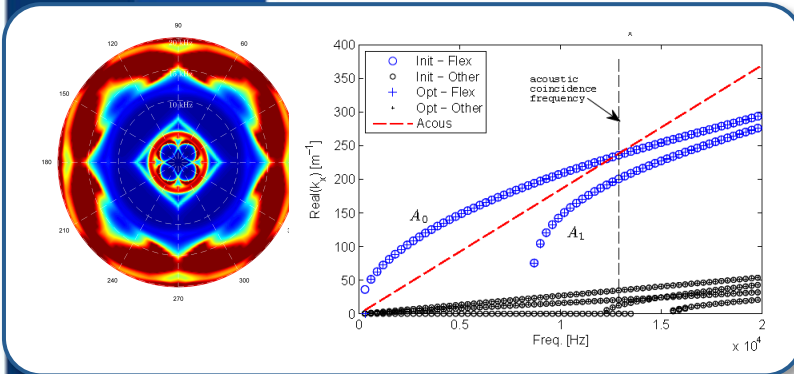


Vibration isolation

Vibration damping



...a distributed control device with no computation: combination of analog structural/electrical loops communicating through the structure

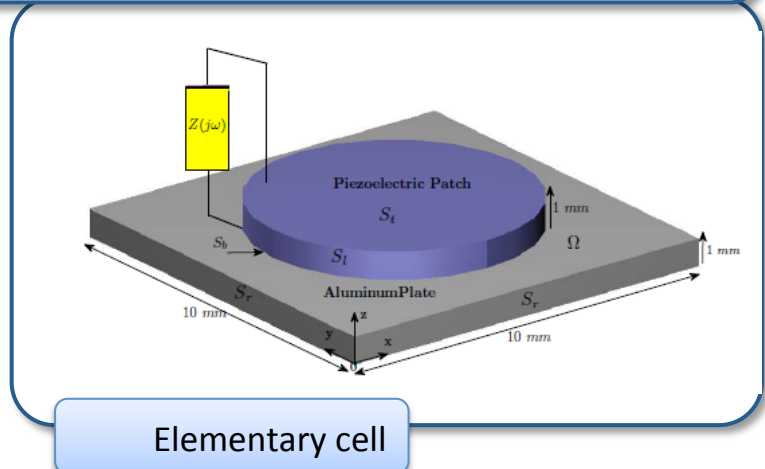


Efficient tool for computation of dispersion diagrams
 Multiphysics damped system

How to choose $Z(w)$ for specific fonctionnalités?

Minimize group velocity of flexural waves: vibration & acoustics limitation

Case REFL: Stop propagation of flexural waves

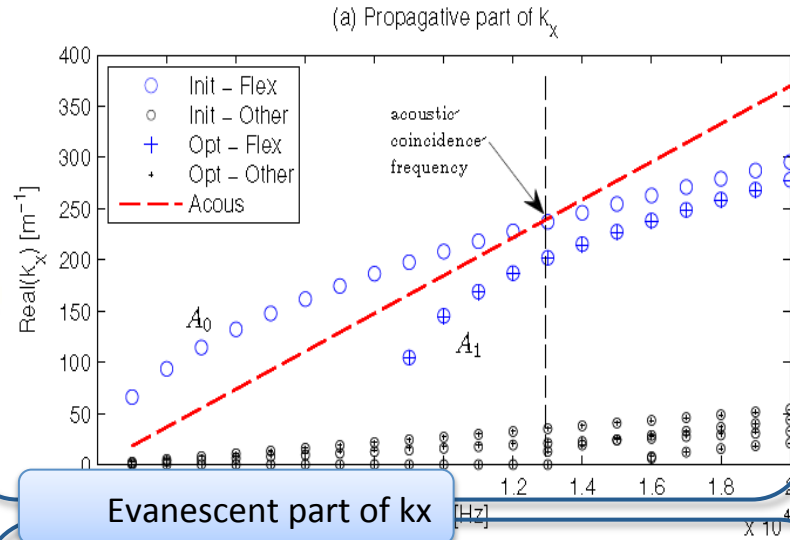


Maximize electric energy dissipation in shunt

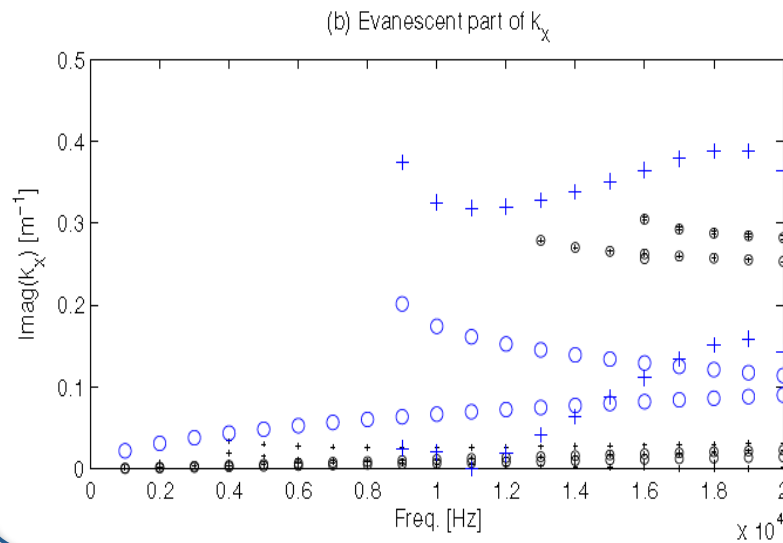
Case ABS: Maximize dissipation

Optimization procedure: find optimal $Z(w)$

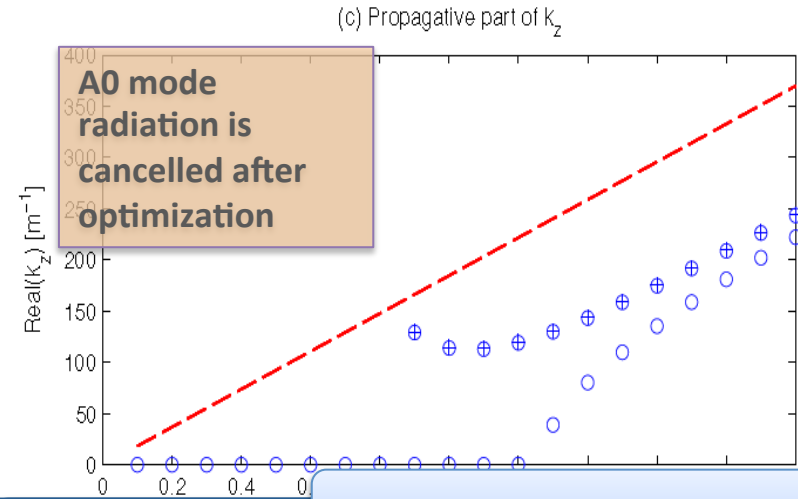
Propagative part of k_x



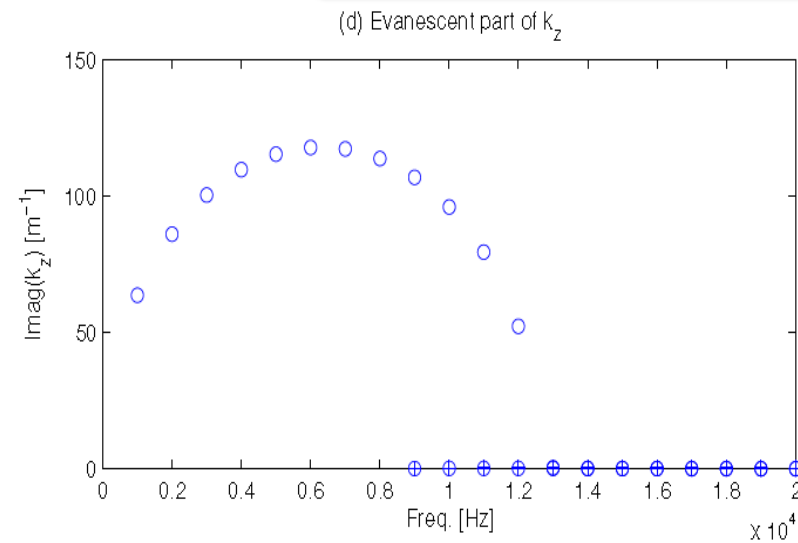
Evanescent part of k_x

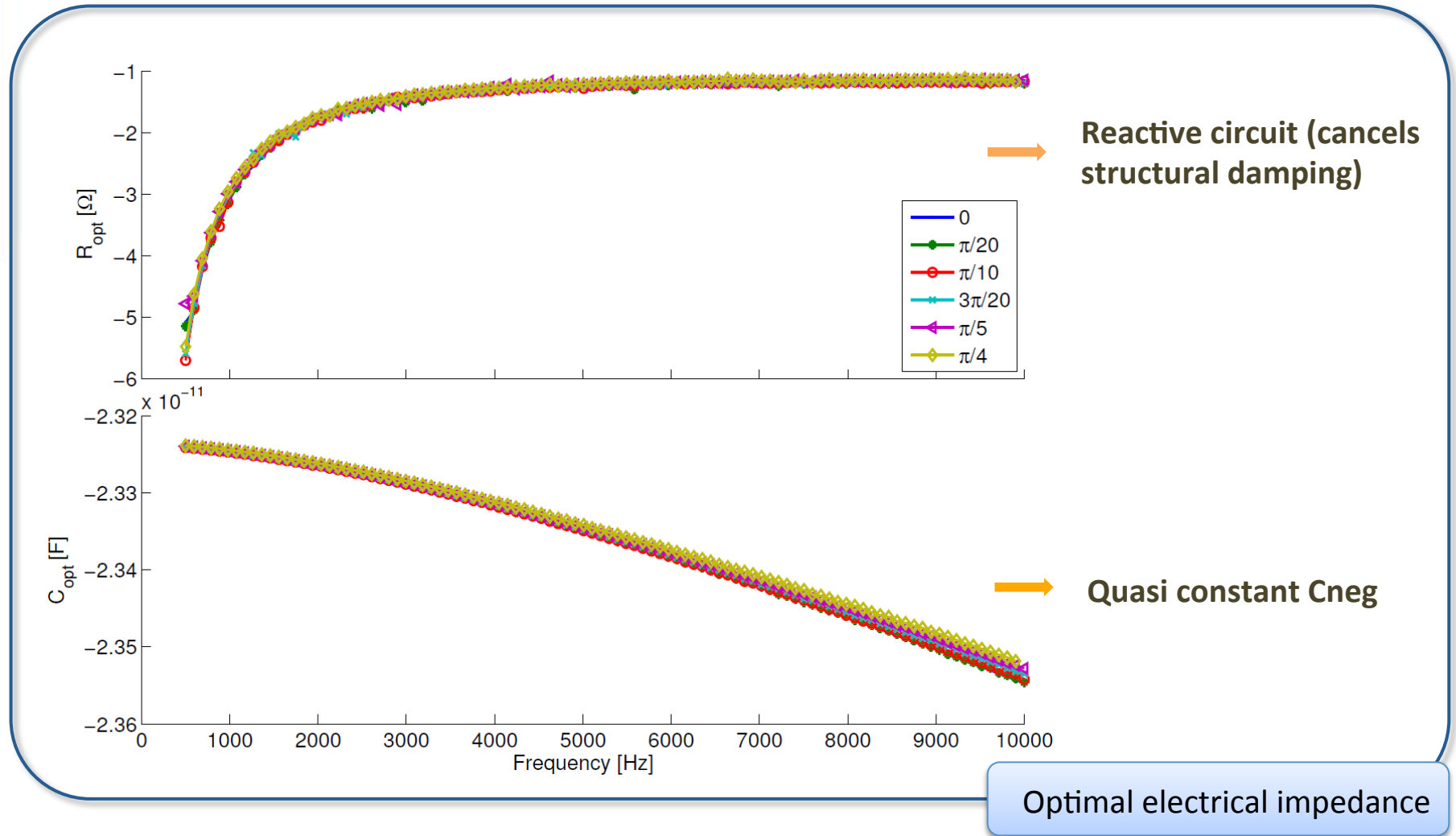


Propagative part of k_z

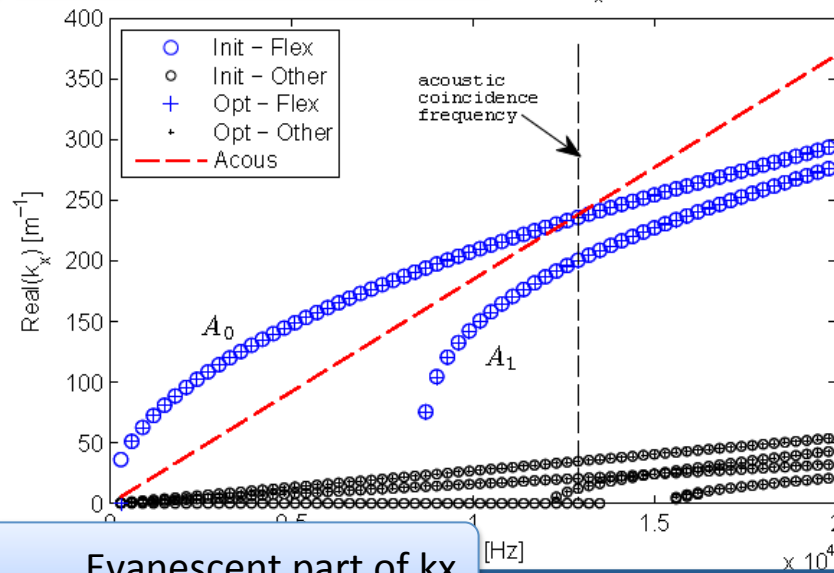


Evanescent part of k_z

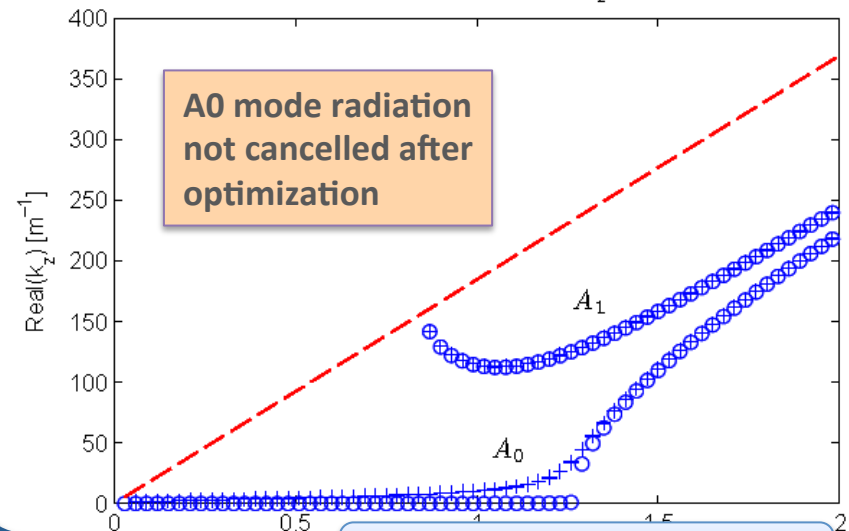




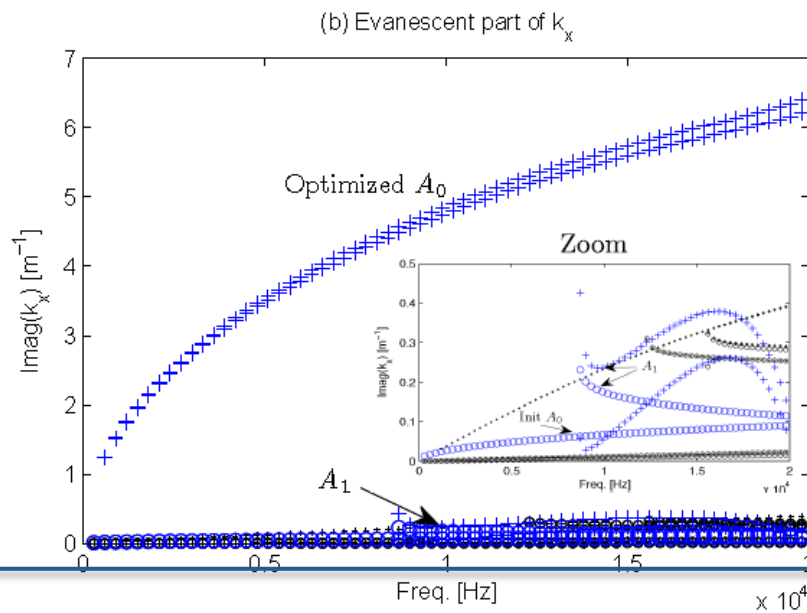
Propagative part of k_x



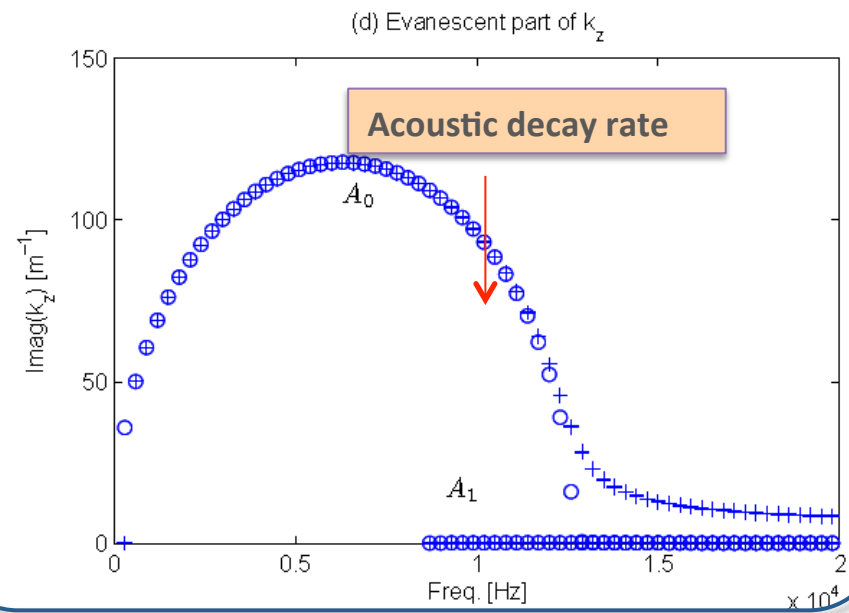
Propagative part of k_z

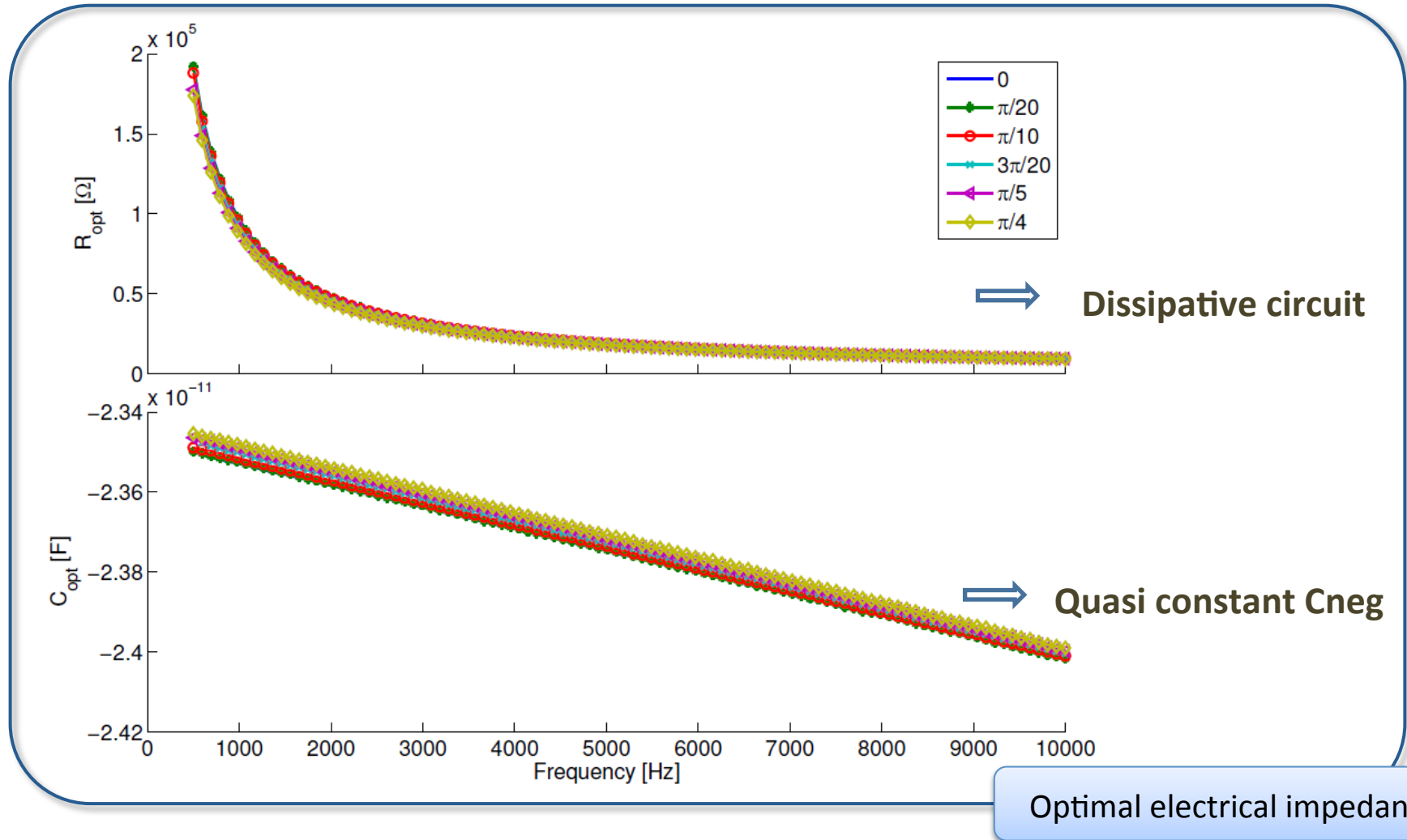


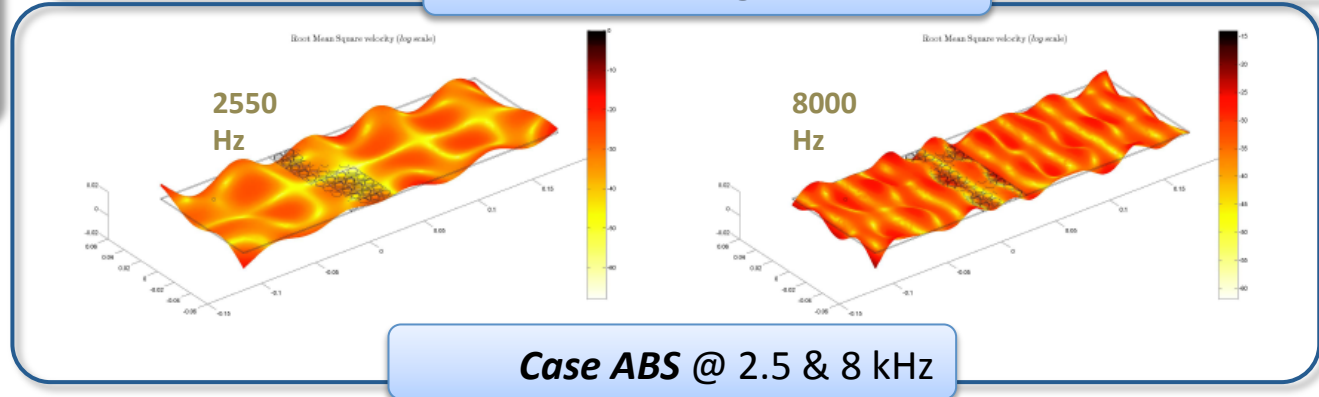
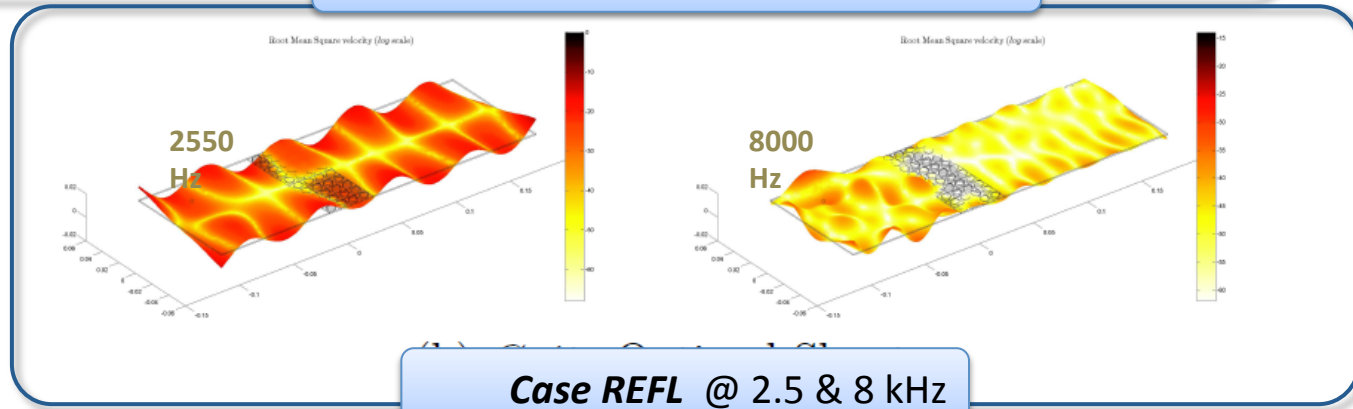
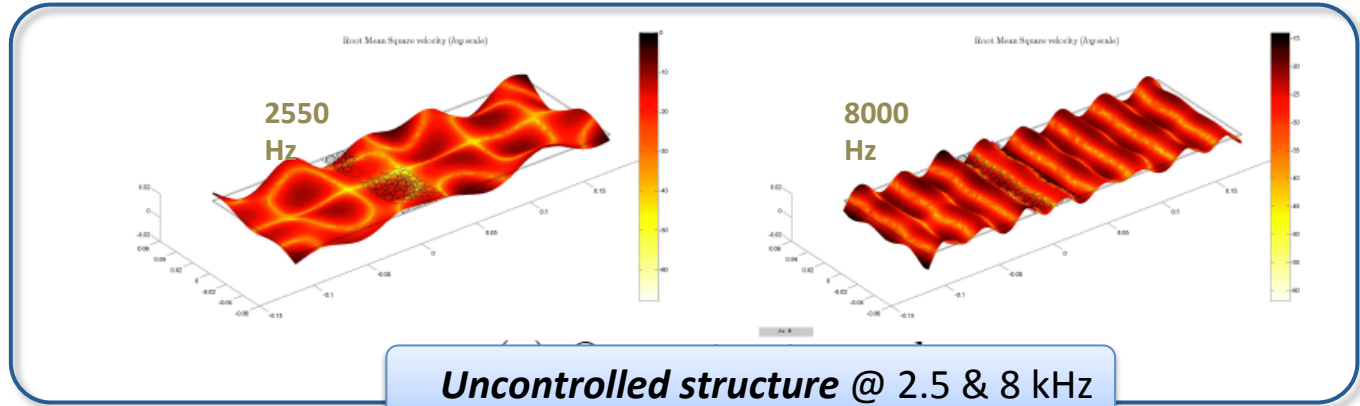
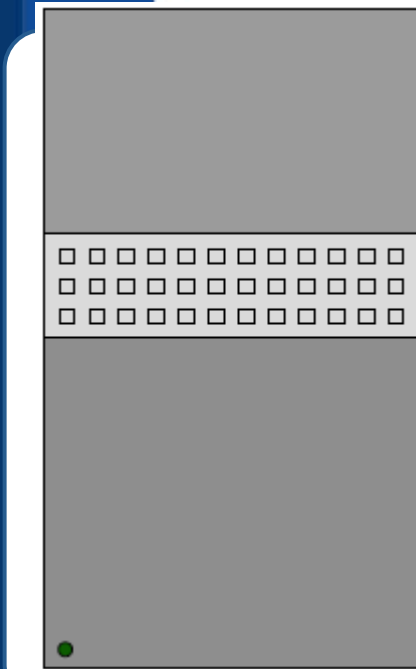
Evanescent part of k_x



Evanescent part of k_z

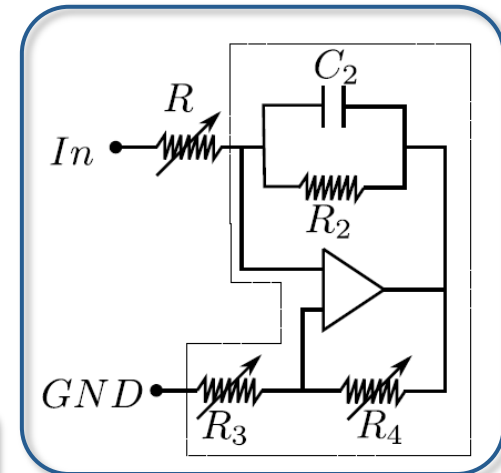
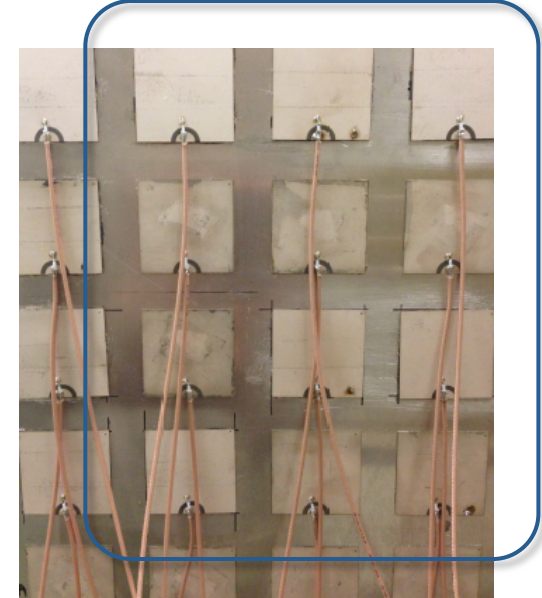
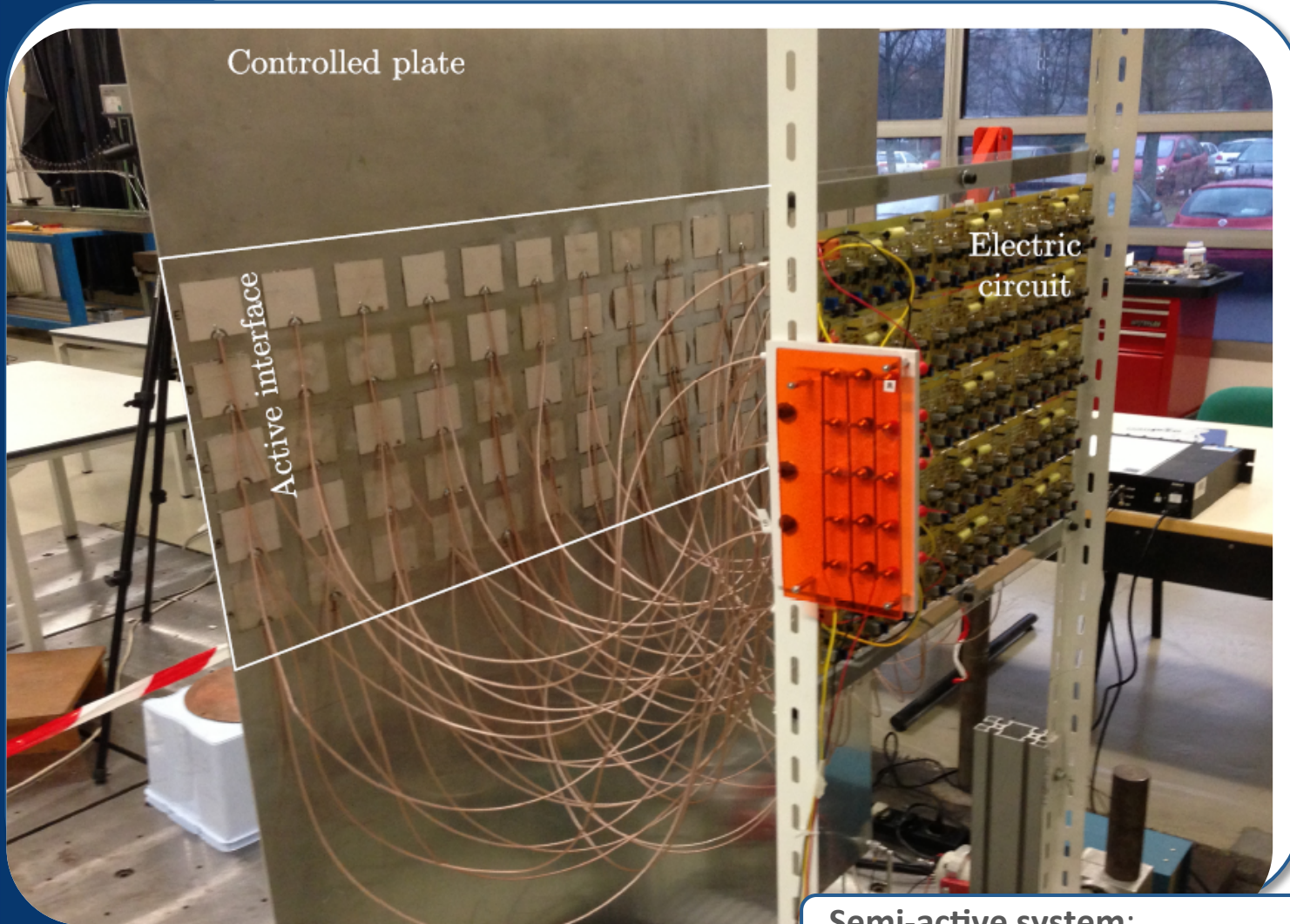






- Non-resonant shunt
- A0 wavelength @ 5 kHz = 30 cm
- Length of active interface : 35 cm

- Reconfiguration: only change the value of resistance in shunt circuit

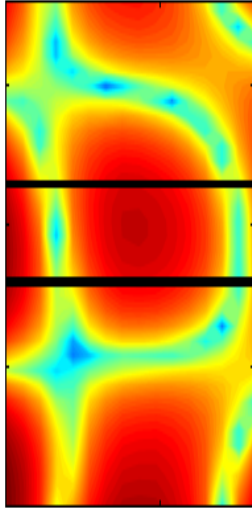


Semi-active system:

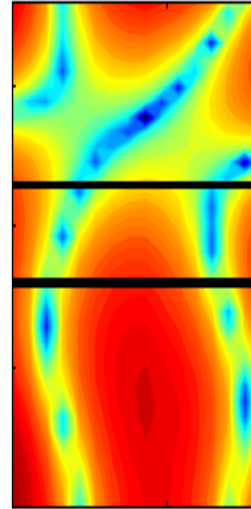
- No control loop => robustness
- Need only to power Op-Amps

Measurement @ 25 Hz

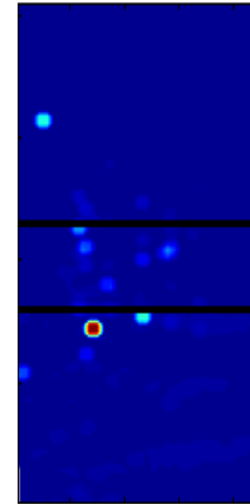
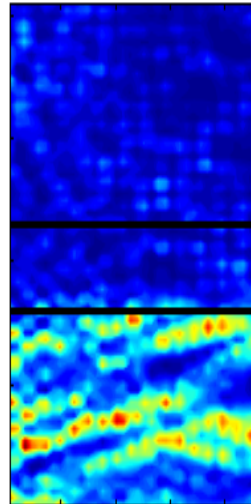
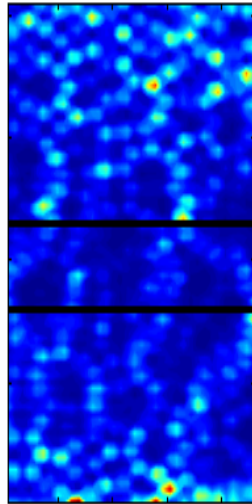
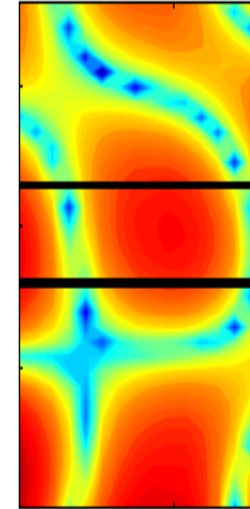
Ctrl off



Ctrl on *Case REFL*

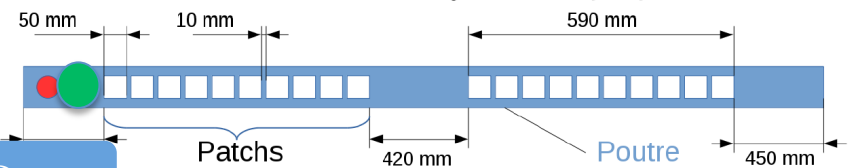
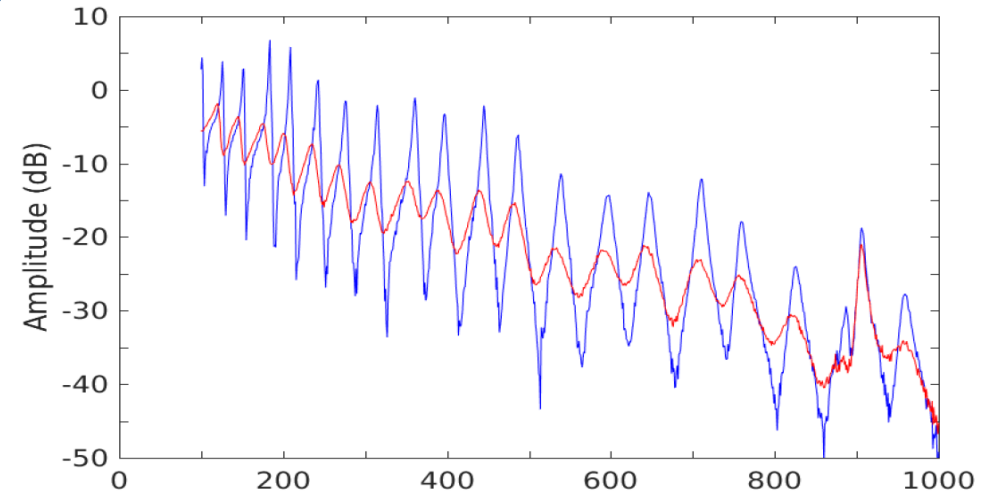
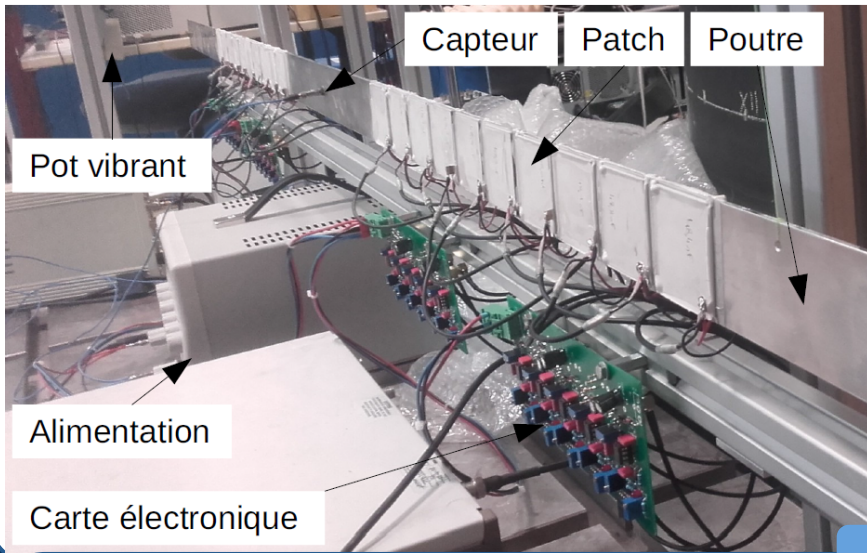


Ctrl on *Case ABS*

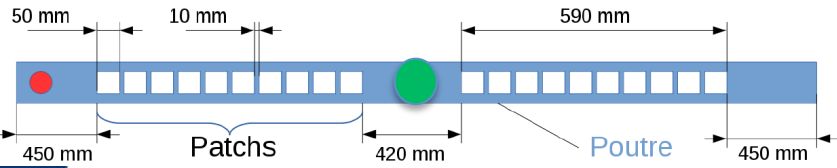
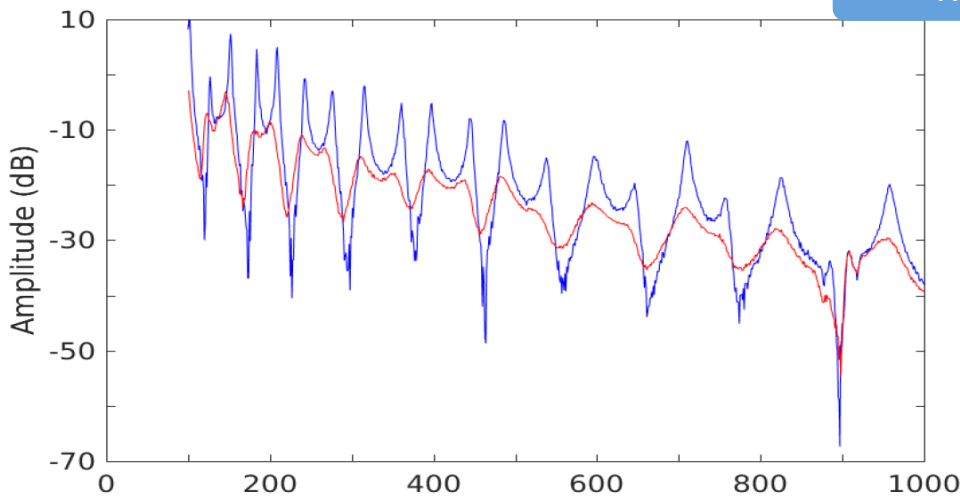
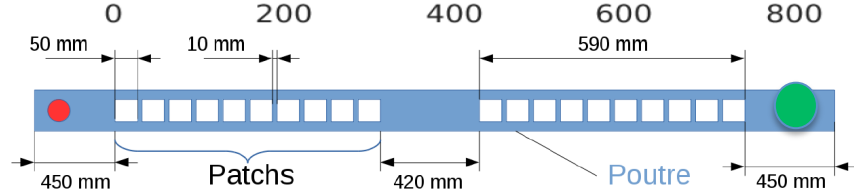
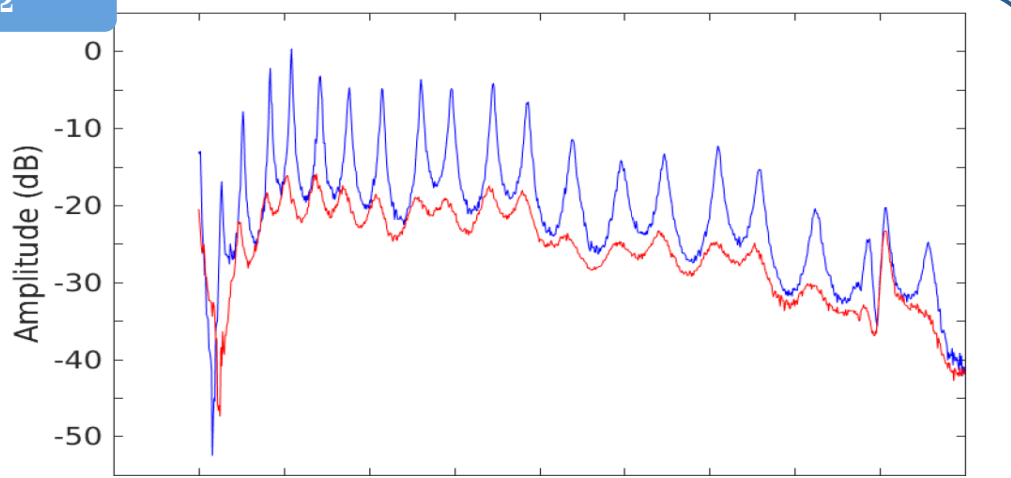


Measurement @ 3000 Hz

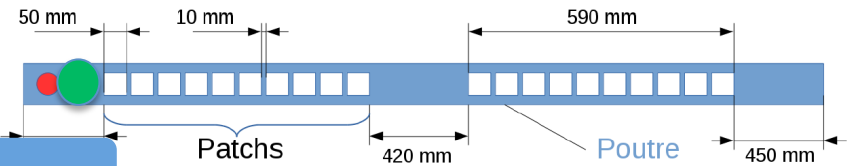
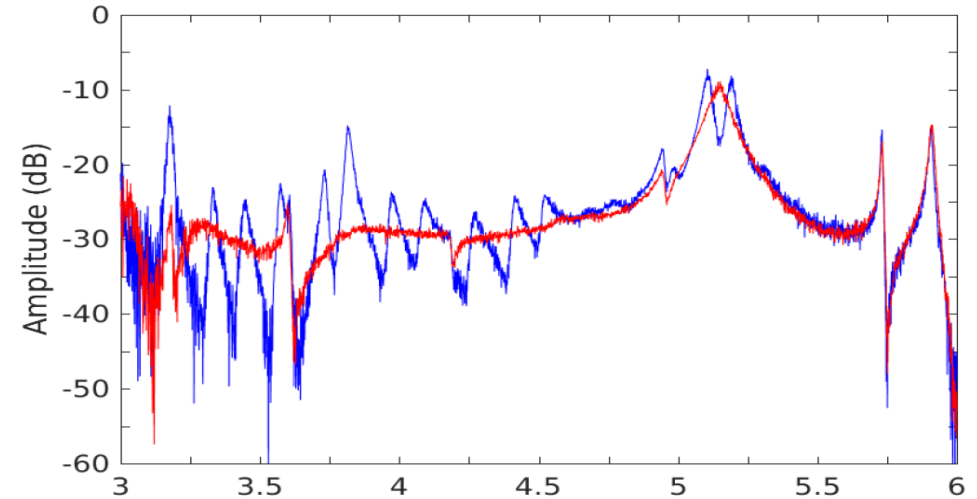
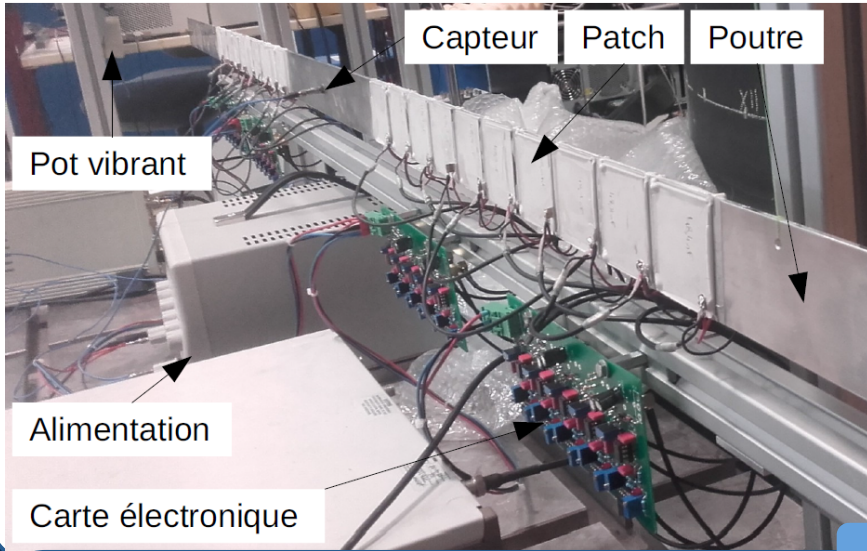
A beam with two interfaces: Low Frequency



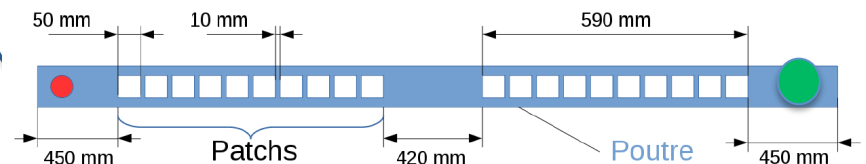
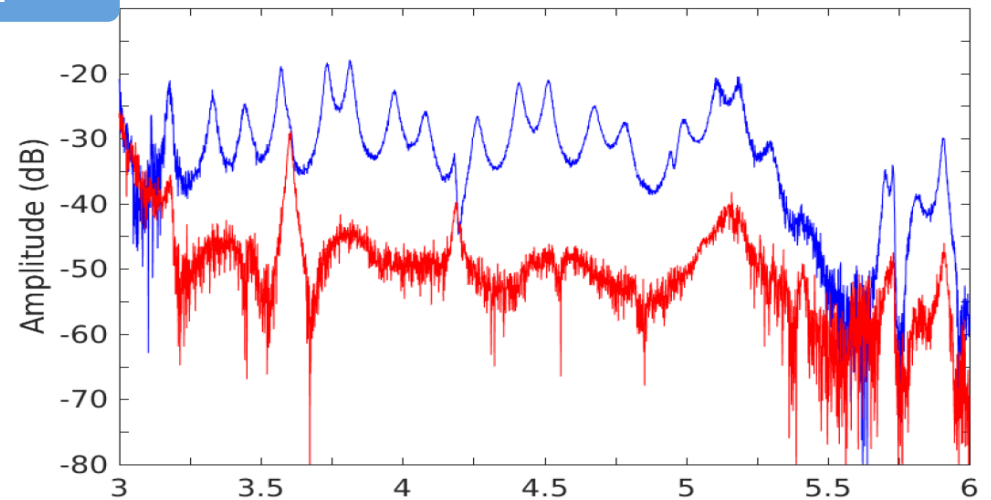
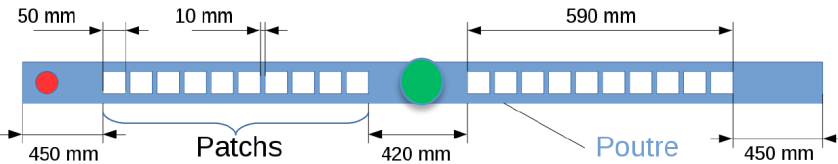
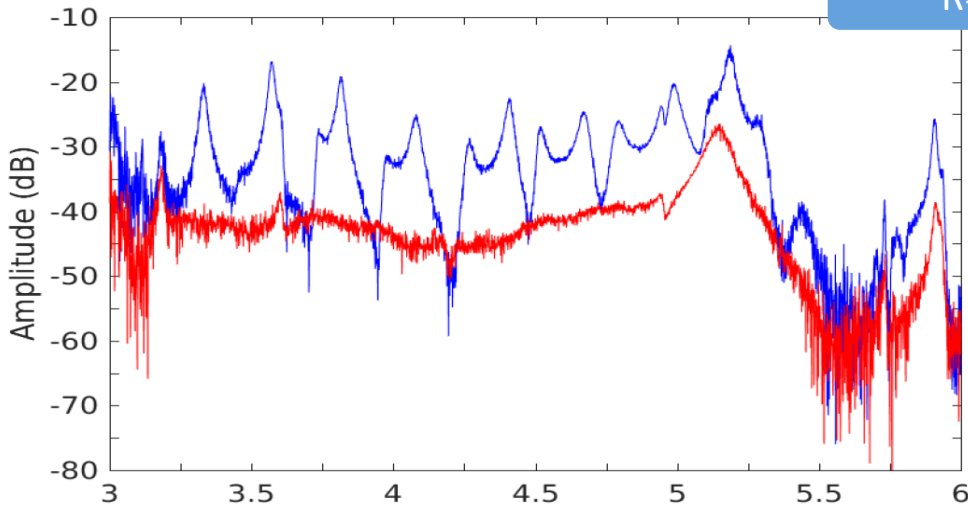
$R=500 \Omega$



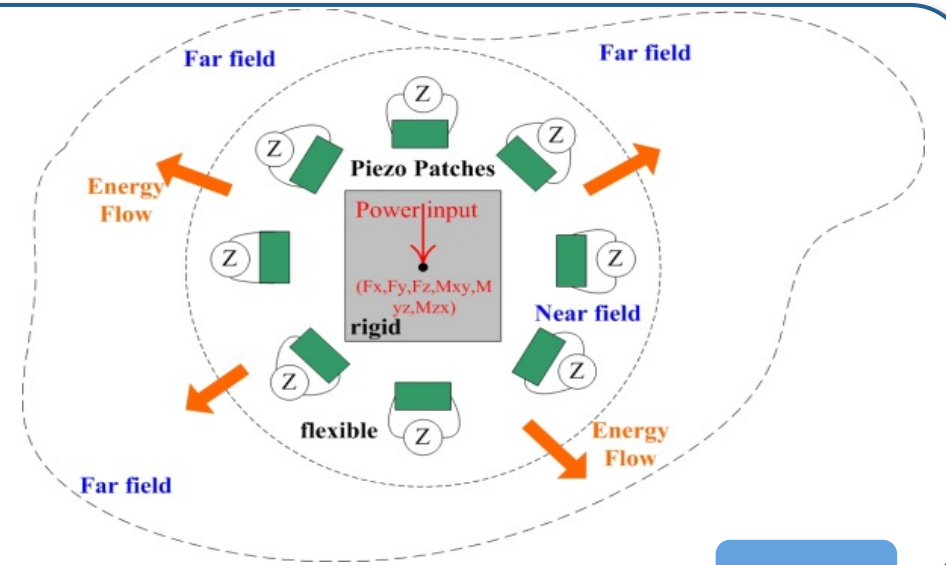
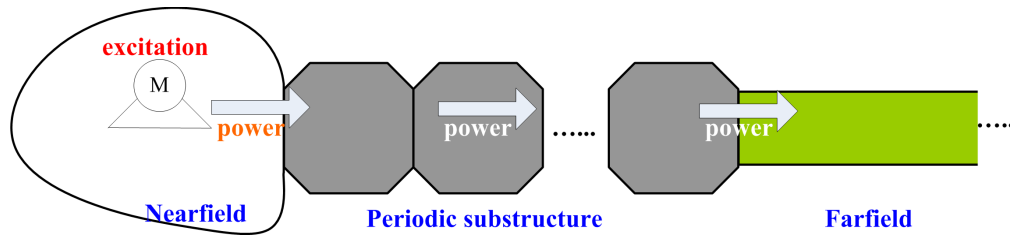
A beam with two interfaces: High Frequency



$R=40 \Omega$

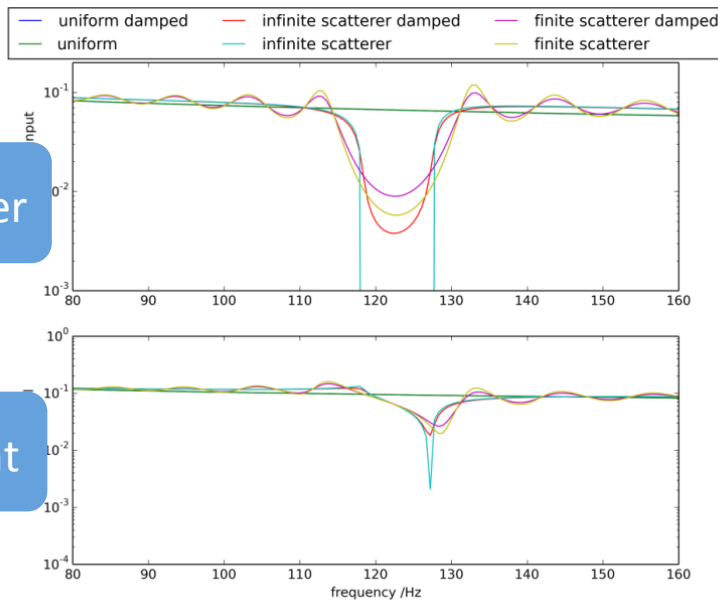


1D

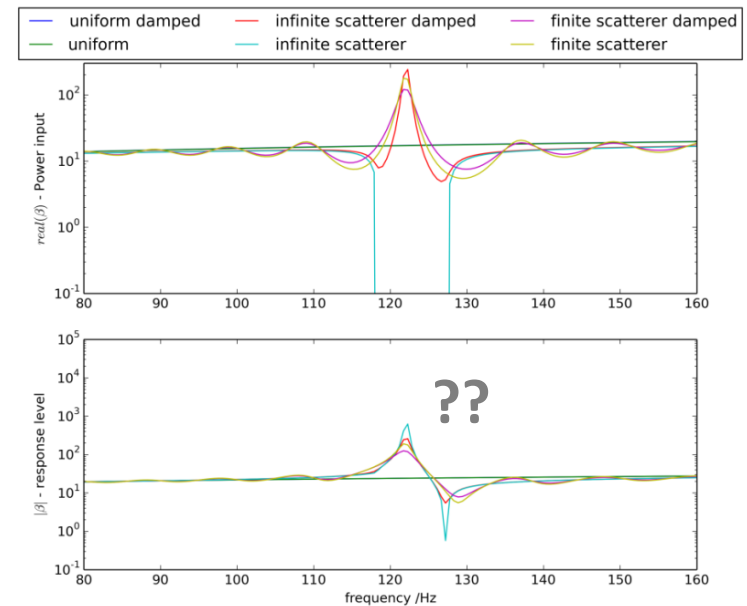


2D

Local modes not excited



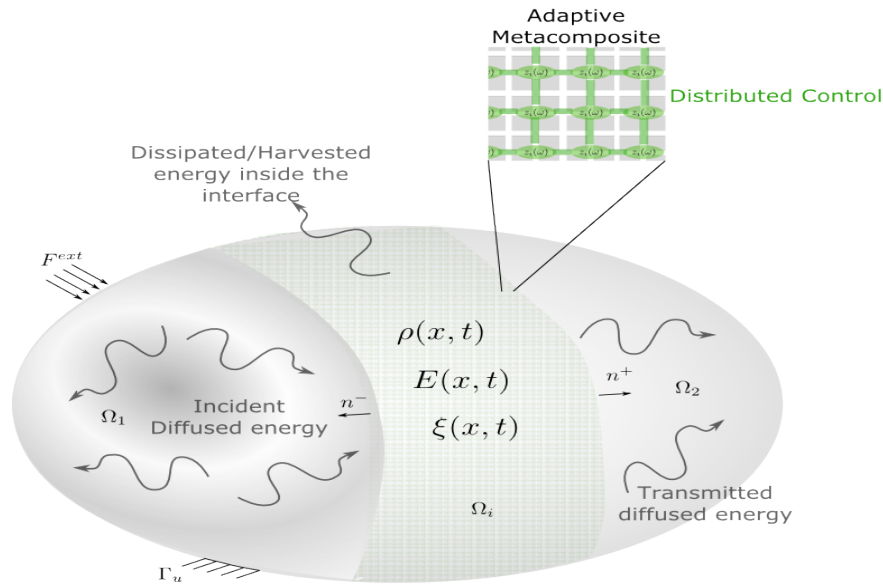
Local modes excited



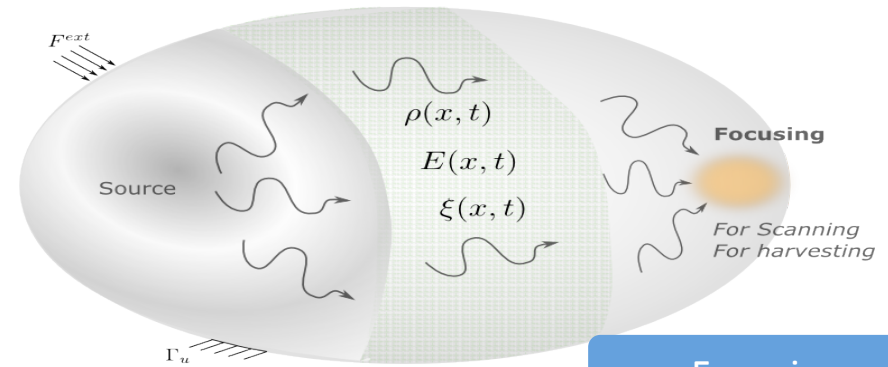
Output Power

Displacement

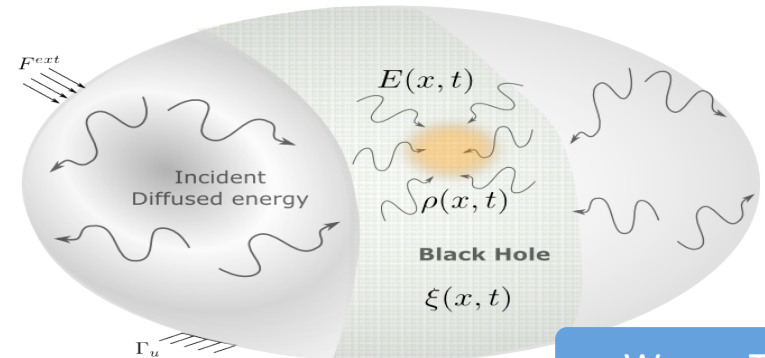
Design Graded Time Variant and Non-Linear architected Materials for Finite structure



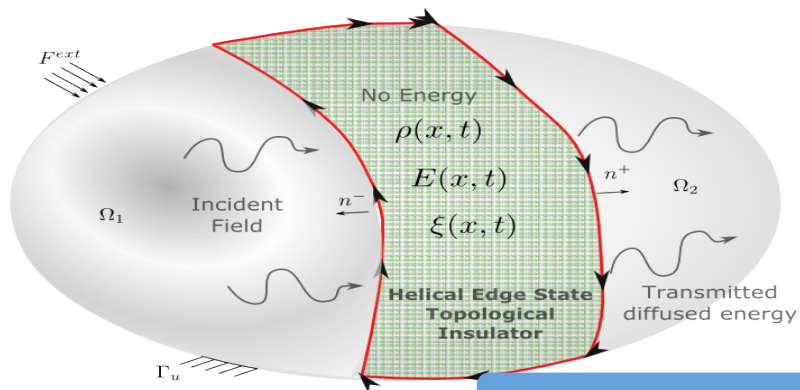
Energy diffusion



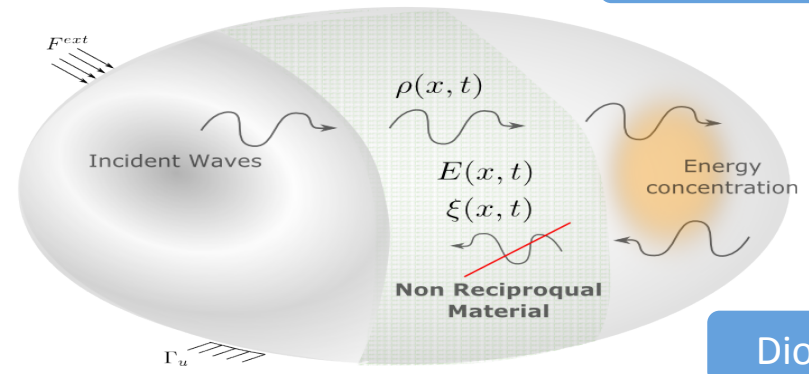
Focusing



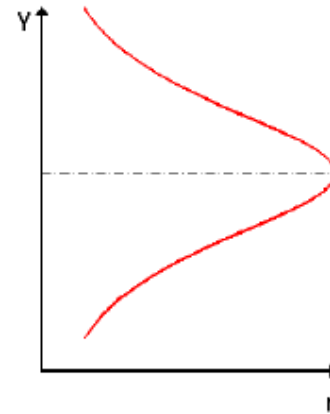
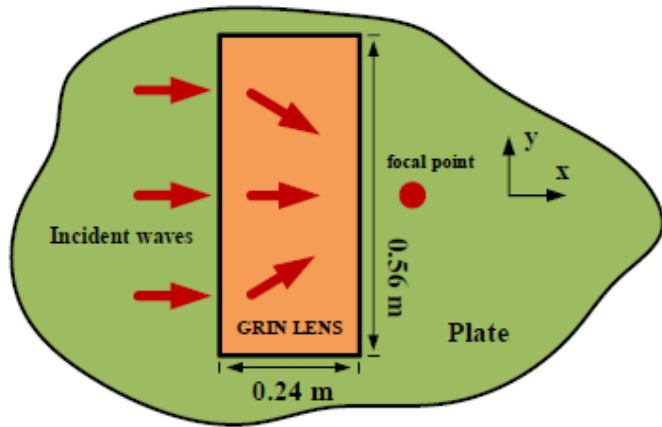
Waves Traps



Insulator

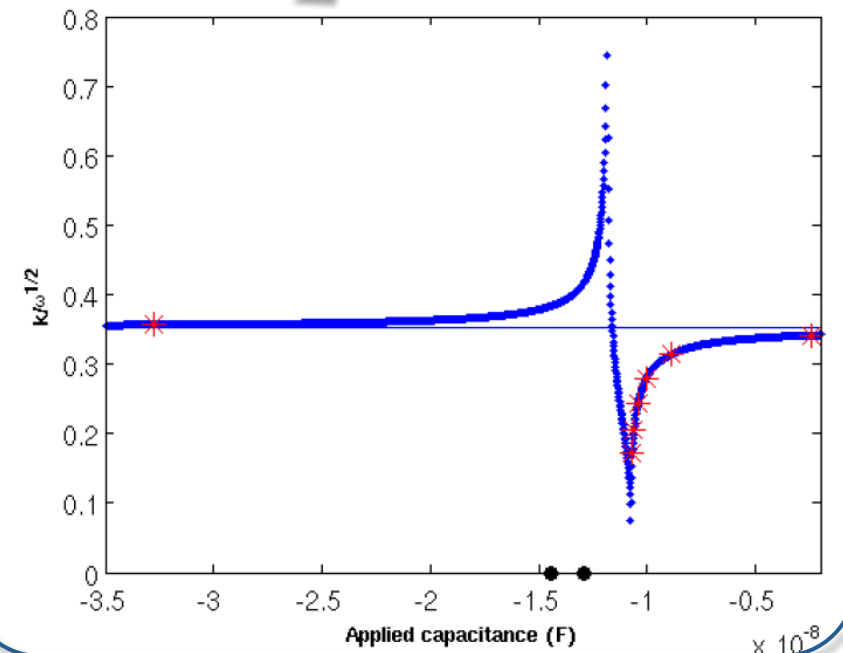
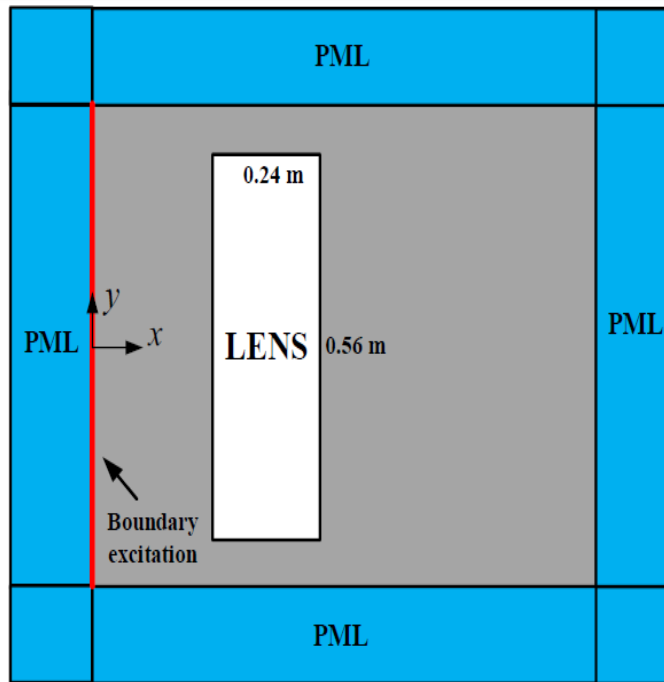


Diodes

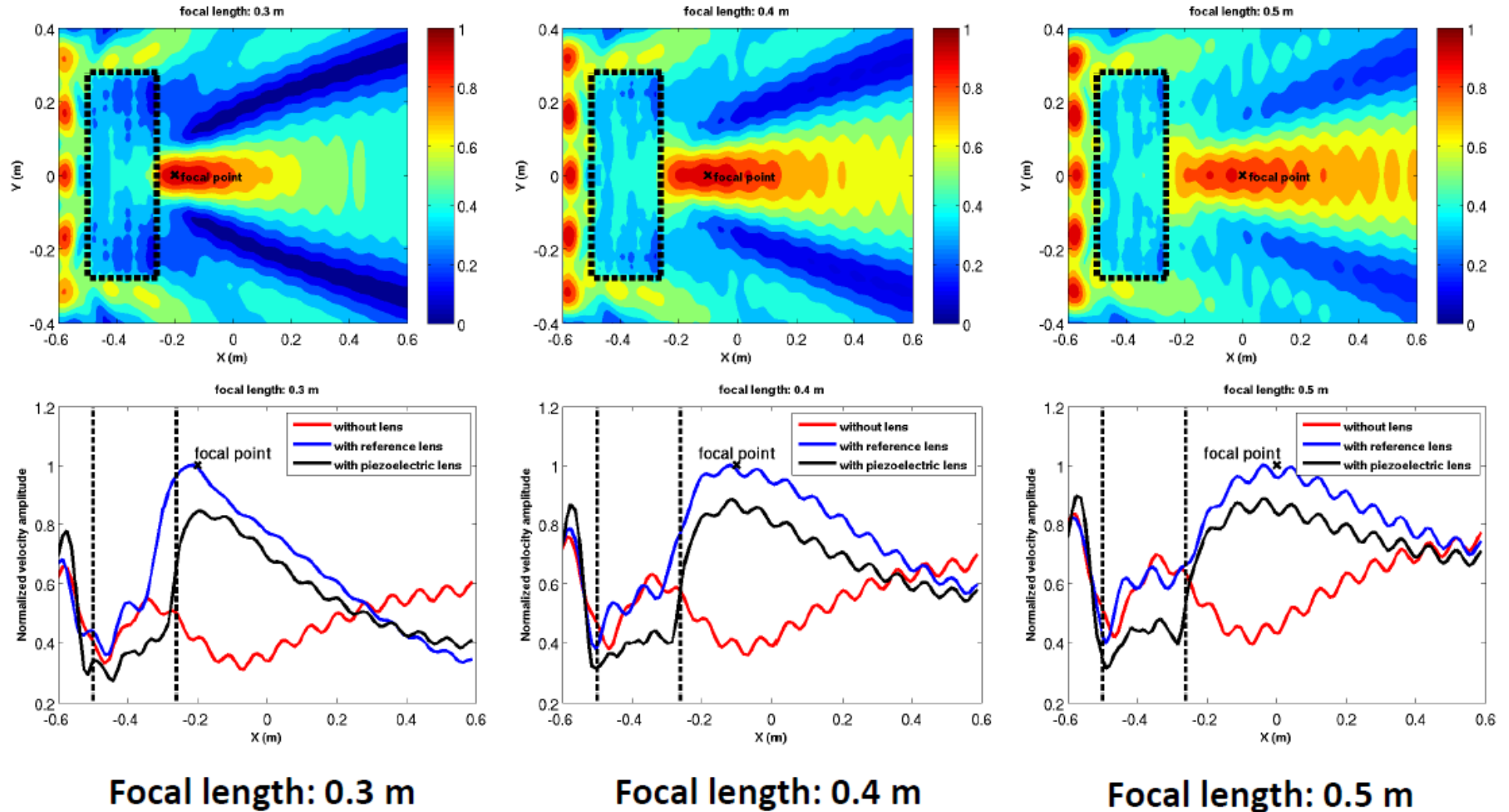


Refractive index

$$n(y) = \text{sech}[\alpha(y - y_0)]$$

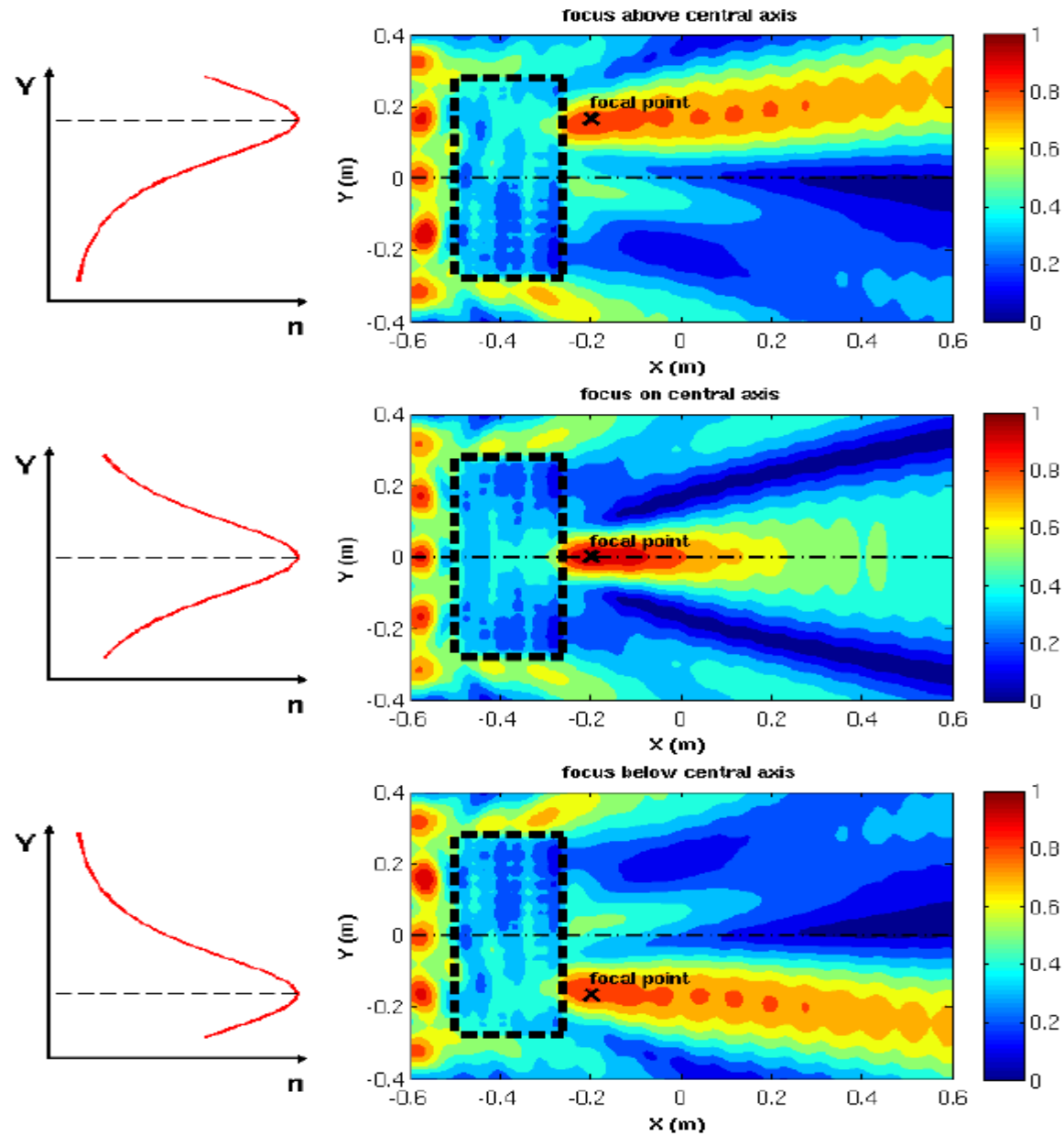


Tunability of the focal point

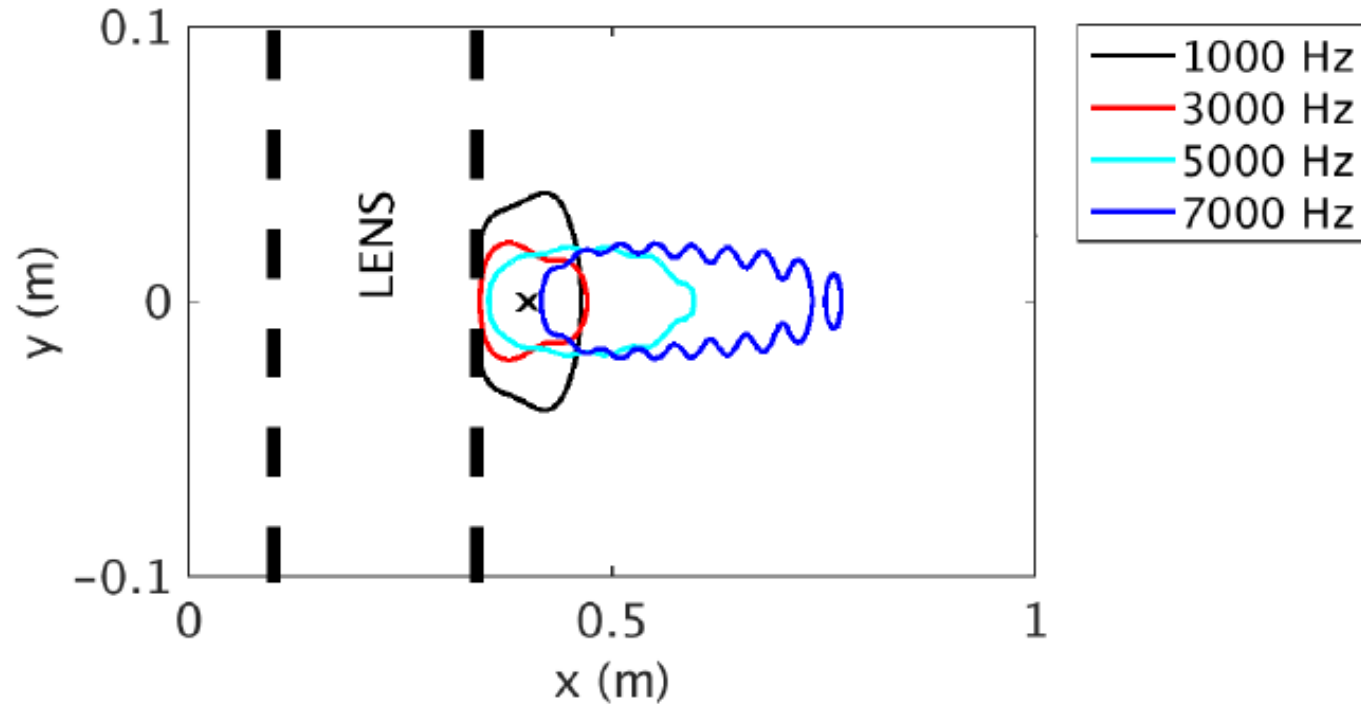


Tunability of the focal point

At 2000 Hz

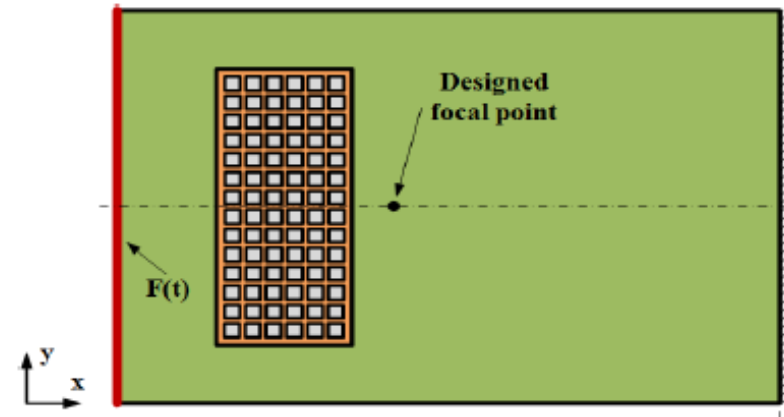


Energy Concentration zones at different frequencies

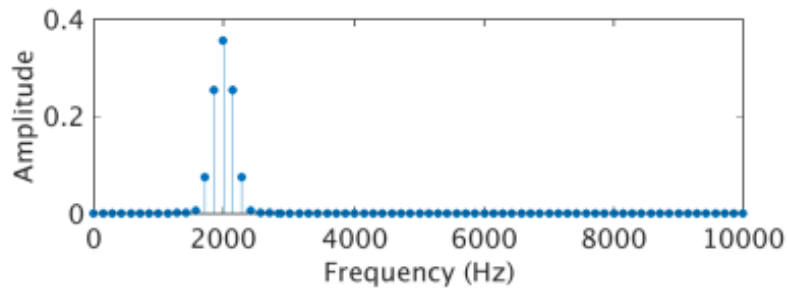
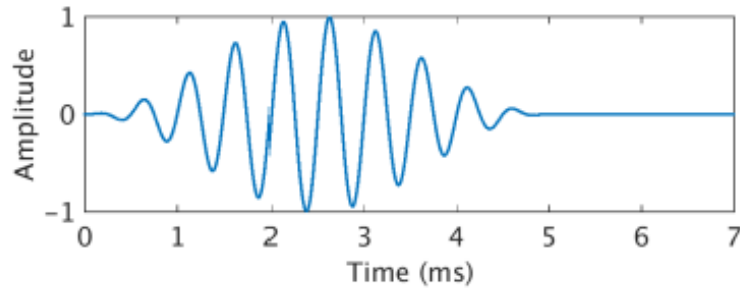


- ❑ At higher frequencies, the energy concentration zone shifts to the right.
- ❑ An overlapping zone after the designed focal point can be observed.

Model: focal_length=0.3 m



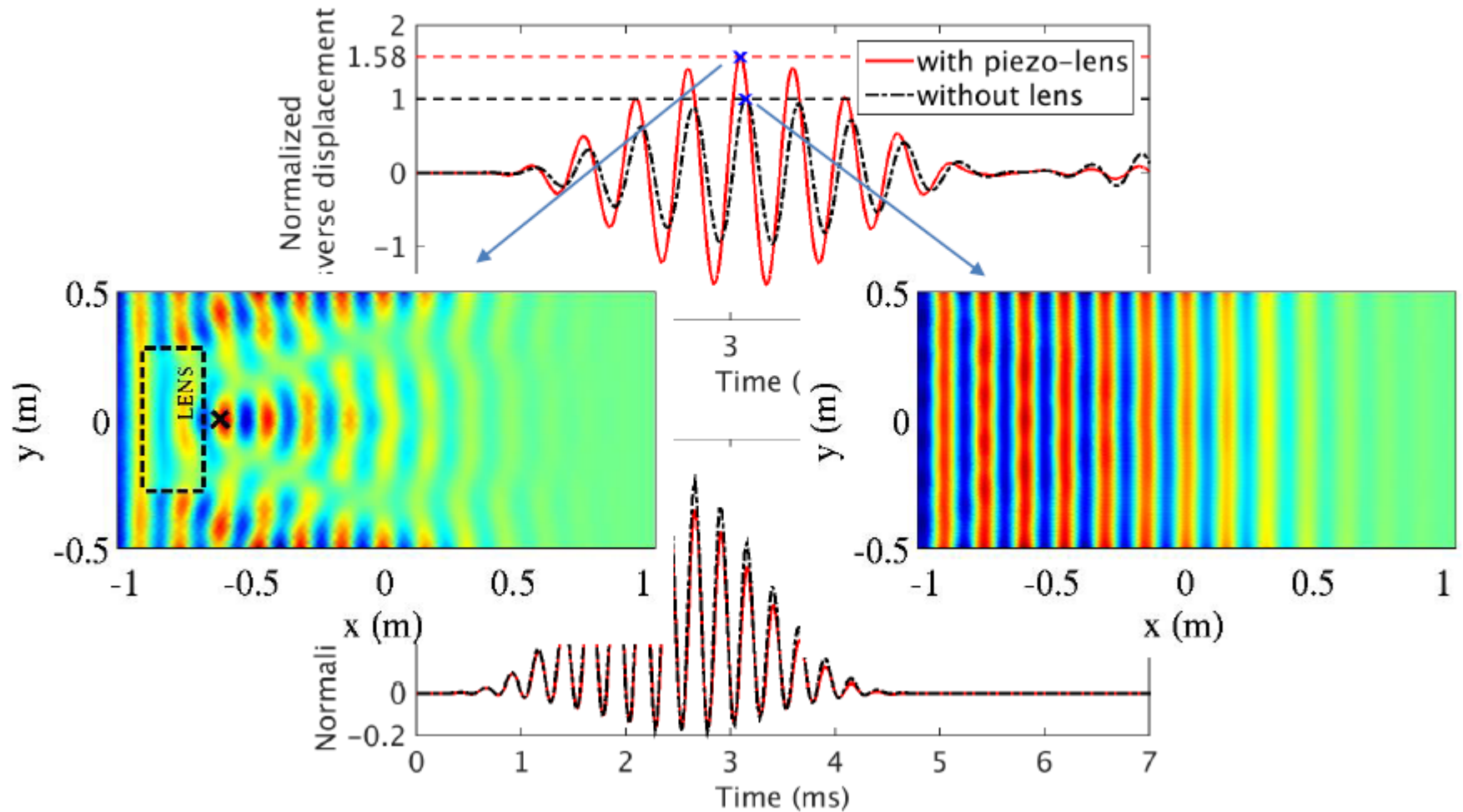
Tone burst excitation:



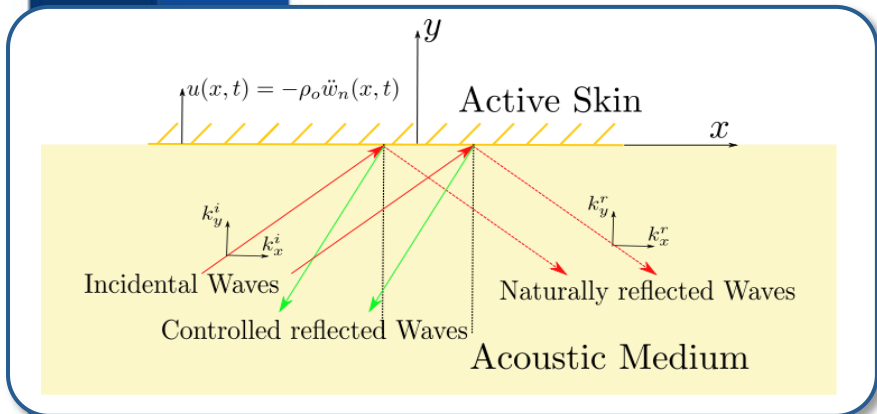
$$f_c = 2000Hz$$

$$f_{\max} = 2286Hz$$

- Transverse response at focal point and input power:



Beyond Band-Gap : Reciprocity breaking and diode



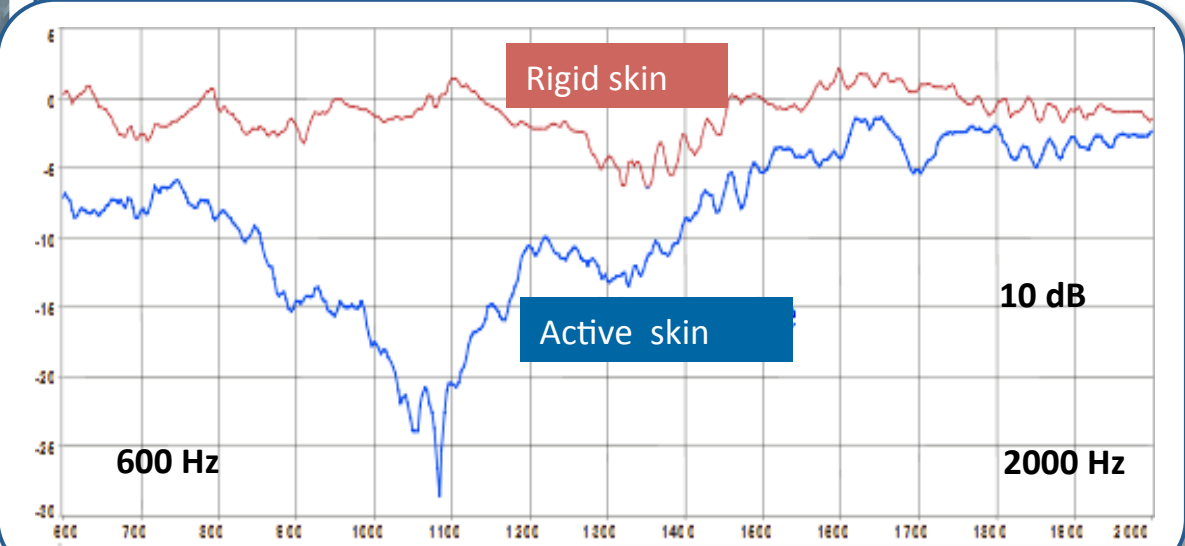
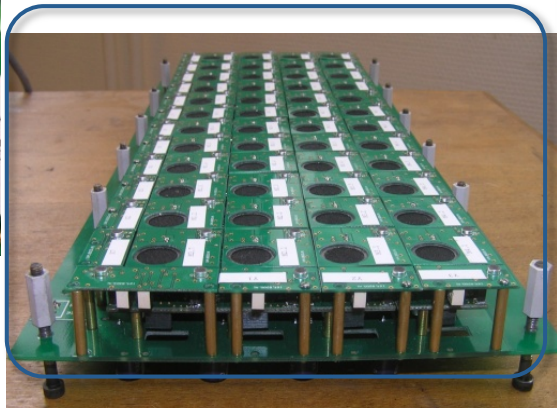
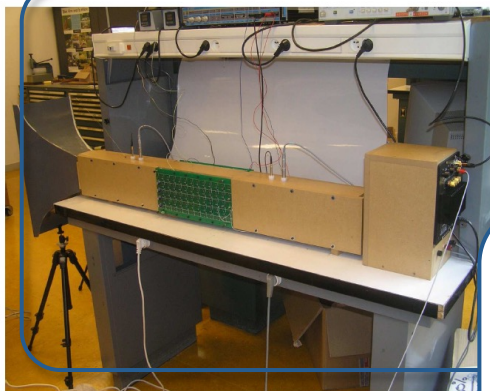
$$\begin{cases} \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0 & \text{on } \mathcal{R}_y^{-*} \times \mathcal{R}_x \times \mathcal{R}_t^{*+} \\ \frac{\partial p(x,0,t)}{\partial y} = u(x,t) \\ y(x,t) = p(x,0,t) \end{cases}$$

The physics

Control law that guarantees $\mathbf{kx} < \mathbf{0}$:

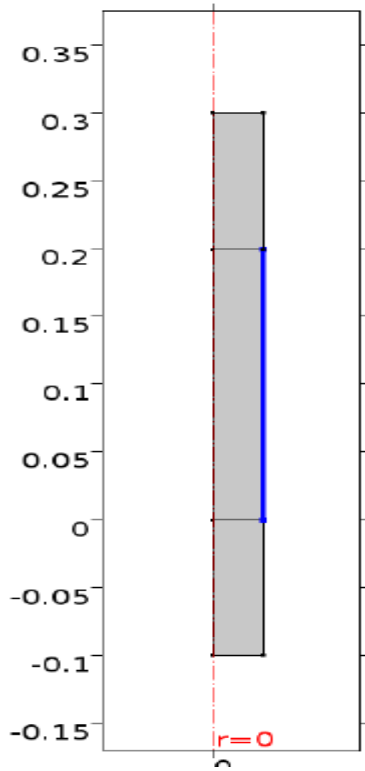
$$u(x,t) = - \left(\frac{1}{c_a} \frac{\partial p(x,0,t)}{\partial t} - \frac{\partial p(x,0,t)}{\partial x} \right)$$

Finite difference estimation of 1st-order derivatives



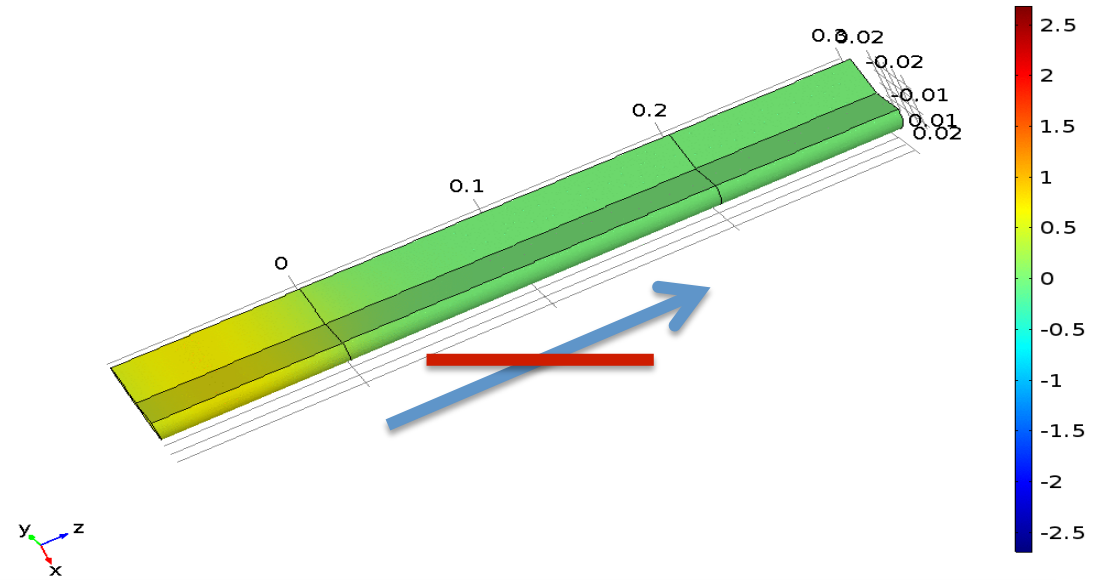
P. David, M Collet et al., SMS, 19(3), 2012

Beyond Band-Gap : Reciprocity breaking and diode

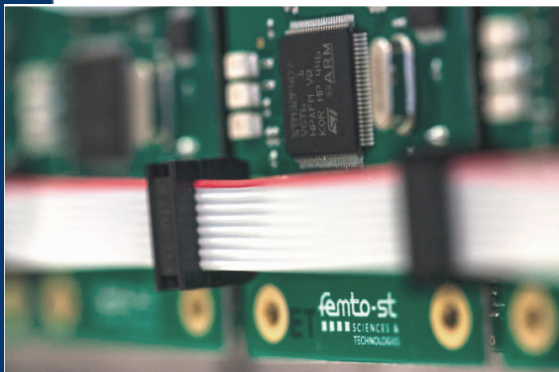
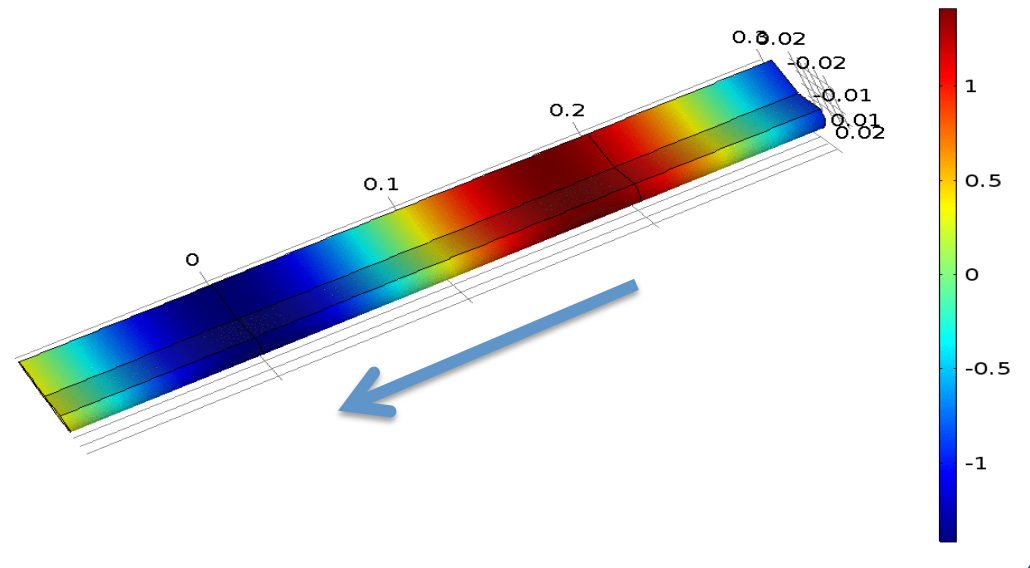


$$a_n = -\alpha(j\omega p / (\rho c) - \partial_z p / \rho)$$

alpha(2)=1 freq(19)=1000 Hz Surface: Champ de pression acoustique total (Pa)



freq(19)=1000 Hz Surface: Champ de pression acoustique total (Pa)



need more details?

M. Collet, M. Ouisse, F. Tateo

Adaptive Metacomposites for Vibroacoustic Control Applications

Cover of IEEE Sensors Journal 14(7), 2014

<http://dx.doi.org/10.1109/JSEN.2014.2300052>



F. Tateo, M. Collet, M. Ouisse, M. Ichchou, K.A. Cunefare, P. Abbe

Experimental characterization of a bi-dimensional array of negative capacitance piezo-patches for vibroacoustic control

Journal of Intelligent Material Systems and Structures, 2014

<http://dx.doi.org/10.1177/1045389X14536006>

manuel.collet@ec-lyon.fr // morvan.ouisse@femto-st.fr //