Multiple Scattering Theory: Introduction and Practical tools

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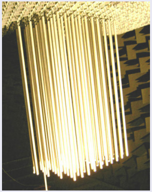
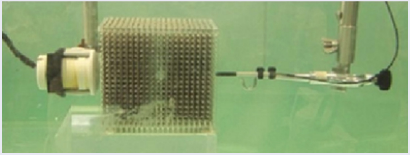
Context

- Cours détaillé introductif, *B. Djafari-Rouhani*
- Métamatériaux dans l'industrie de l'acoustique audible : Cas du métaporeux, *C. Lagarrigue*
- Métamateriaux acoustiques, *J. Sánchez-Dehesa*
- Relation de dispersion - PWE, EPWE, *J. Vasseur*
- Métamatériaux et aspects perceptifs, *N. Côté*
- Technique d'homogénéisation, *A. Maurel*

Sonic Crystals

- Particular case of phononic crystal with a fluid as host medium.
- Made of rigid, penetrable or resonance scatterers.





Bibliography

- P.A. Martin, *Multiple Scattering Interaction of Time-Harmonic Waves with N Obstacles*, Cambridge University Press, 2006
- L.C. Botten *et al.*, *Rayleigh multipole methods for photonic crystal calcualtaions*, PIER, 41:21-60, 2003
- G. Tayeb and D. Maystre, *Rigorous theorical study of finite-size two-dimensional photonic crystals doped by microcavity*, J. Opt. Soc. Am. A, 12:3323-3332, 1993.
- V. Twersky, *Elementary function representations of Schlömilch series*, Arch. Ration. Mech. An., 8:323-332, 1961
- Abramowitz & Stegun, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, 1964
- Barber and Hill, *Light Scattering by Particles: Computational Methods*, World Scientific Publishing, 1990
- D. Torrent, *Multiple Scattering Theory*, Training School: Sound waves in metamaterials and porous media, www.denorms.eu
- ...

Outline

- Part I. Introduction
  - What is scattering?
  - One dimensional scattering
- Part II. Scattering by circular rigid cylinders
  - General background
  - Scattering by a single circular cylinder
  - Scattering by N circular cylinders
- Part III. Scattering by a periodic arrangement of circular cylinders
  - Scattering of a plane incident by an array of rigid cylinders
  - Reflection and transmission coefficients by an array of rigid cylinders
  - Reflection and Transmission coefficients by a stack of gratings
  - Band diagram calculation

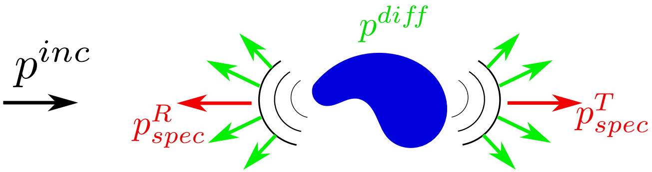
Part I. Introduction

- What is scattering?
- One dimensional scattering

What is scattering?

What wikipedia says?

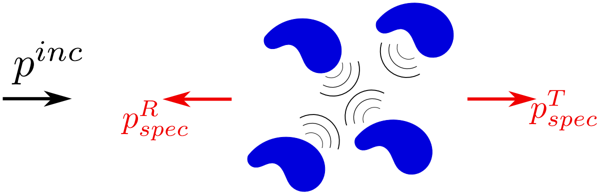
Scattering *is a general physical process where some forms of radiation, such as light, sound, or moving particles, are* **forced to deviate from a straight trajectory by one or more paths due to localized non-uniformities** *in the medium through which they pass. In conventional use, this also includes deviation of reflected radiation from the angle predicted by the law of reflection. Reflections that undergo scattering are often called* **diffuse reflections** *and unscattered reflections are called* **specular (mirror-like) reflections**.

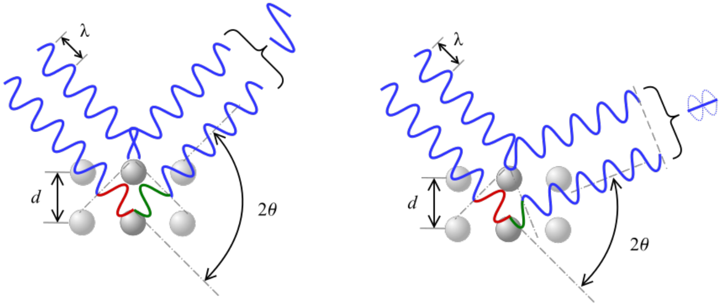


Is it more related to energy?

What is Single and Multiple Scattering?

When radiation is only scattered by **one localized scattering center***, this is called* **single scattering***. It is very common that scattering centers are grouped together; in such cases,* **radiation may scatter** *many times, in what is known as multiple scattering. The* **main difference** *between the effects of single and multiple scattering is that single scattering can usually be treated as a random phenomenon, whereas* **multiple scattering***, somewhat counterintuitively, can be modeled as a* **more deterministic** *process because the combined results of a large number of scattering events tend to average out. Multiple scattering can thus often be modeled well with* **diffusion theory***.*



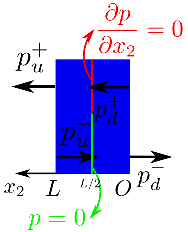


$$2d \sin\theta = n\lambda.$$

In particular, only specularly reflected and transmitted waves are propagative in the surrounding medium for finite depth sonic crystals within the first Bragg bandgap.

One dimensional scattering

One dimensional scattering



Pressure field is splitted into upward and downward going waves:

$$\begin{cases} p_u^+ = R p_u^- + T p_d^+, \\ p_d^- = R p_d^+ + T p_u^-, \end{cases}$$

in case of reciproque and symetric scattering.

$$\overbrace{\begin{bmatrix} p_u^+ \\ p_d^- \end{bmatrix}}^{\text{Outgoing waves}} = \underbrace{\begin{bmatrix} R & T \\ T & R \end{bmatrix}}_{\text{Scattering matrix } \mathcal{SC}} \overbrace{\begin{bmatrix} p_u^- \\ p_d^+ \end{bmatrix}}^{\text{Ingoing waves}} .$$

$\mathcal{SC}$ eigenvalues are $\lambda = (R \pm T)$: symetric and antisymetric problem.

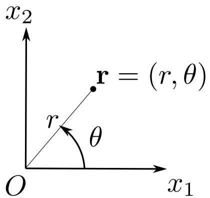
- $|\lambda_S|^2$ ($|\lambda_A|^2$) reflected energy in the (anti)symetric problem
- $\alpha_S = 1 - |\lambda_S|^2$ ($\alpha_A = 1 - |\lambda_A|^2$) absorbed energy in the (anti)symetric problem
- $|R|^2 = \left|\frac{\lambda_S + \lambda_A}{2}\right|^2$ and $|T|^2 = \left|\frac{-\lambda_S + \lambda_A}{2}\right|^2$ reflected and transmitted energy by the global system
- $\alpha = \frac{\alpha_S + \alpha_A}{2}$ absorbed energy by the global system

Part II. Scattering by circular rigid cylinders

- General background
- Scattering by a single circular cylinder
- Scattering by N circular cylinders

General background

Helmholtz equation in cylindrical coordinates



Helmholtz equation ($e^{-\mathrm{i}\omega t}$ time convention)

$$\left(\triangle + k^2\right) p(\mathbf{r}) = 0, \forall \mathbf{r} \in \mathbb{R}^2.$$

In cylindrical coordinate system, $\triangle = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$ and the Helmholtz equation reads as

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + k^2\right) p(\mathbf{r}) = 0, \forall \mathbf{r} \in \mathbb{R}^2.$$

Separation of variables: $p(\mathbf{r}) = F(\theta)G(r)$, with
$F(\theta) = F(\theta + 2n\pi)$, $\forall n \in \mathbb{Z}$ (θ periodic) + geometry

$$\underbrace{\frac{1}{F(\theta)}\frac{\partial F(\theta)^2}{\partial \theta^2}}_{\text{Function of } \theta} = \underbrace{-k^2 r^2 - \left(\frac{r^2}{G(r)}\frac{\partial^2 G(r)}{\partial r} + \frac{r}{G(r)}\frac{\partial G(r)}{\partial r}\right)}_{\text{Function of } r}, \forall \mathbf{r} \in \mathbb{R}^2.$$

Solution of the Helholtz equation

- $$\begin{cases} \frac{1}{F(\theta)}\frac{\partial F(\theta)^2}{\partial \theta^2} = -\nu^2, \\ F(\theta) = F(\theta + 2n\pi),\ \forall n \in \mathbb{Z}, \\ + \text{ geometry} \end{cases} \Rightarrow F(\theta) = \sum_{n\in\mathbb{Z}} A e^{\mathrm{i}n\theta} + B e^{-\mathrm{i}n\theta}.$$
- For fixed n, introducing $\alpha = kr$, $G_n(\alpha)$ satisfies the Bessel's equation

$$\frac{\partial^2 G_n(\alpha)}{\partial \alpha^2} + \frac{1}{\alpha}\frac{\partial G_n(\alpha)}{\partial \alpha} + \left(1 - \frac{n^2}{\alpha^2}\right) G_n(\alpha) = 0,$$

whose solution is

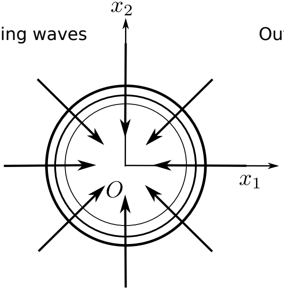
$$G_n(\alpha) = C \underbrace{\mathrm{J}_n(\alpha)}_{\text{Bessel function of } 1^{st} \text{ kind}} + D \overbrace{\mathrm{H}_n^{(1)}(\alpha)}^{\text{Hankel function of } 1^{st} \text{ kind}}.$$

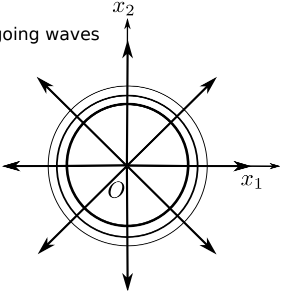
$$p(\mathbf{r}) = \sum_{n\in\mathbb{Z}} \left(\mathcal{A}_n \mathrm{J}_n(kr) + \mathcal{B}_n \mathrm{H}_n^{(1)}(kr)\right) e^{\mathrm{i}n\theta},$$

because $\mathrm{J}_{-n}(kr) = (-1)^n \mathrm{J}_n(kr)$ and $\mathrm{H}_{-n}^{(1)}(kr) = (-1)^n \mathrm{H}_n^{(1)}(kr)$.

Physical meaning of the solution

$$p(\mathbf{r}) = \sum_{n\in\mathbb{Z}} \mathcal{A}_n \mathrm{J}_n(kr)e^{in\theta} \qquad p(\mathbf{r}) = \sum_{n\in\mathbb{Z}} \mathcal{B}_n \mathrm{H}_n^{(1)}(kr)e^{in\theta}$$





Remark: the solution could alternatively be sought in the form

$$p(\mathbf{r}) = \sum_{n\in\mathbb{Z}} \left(\mathcal{A}'_n \mathrm{J}_n(kr) + \mathcal{B}'_n \underbrace{\mathrm{Y}_n(kr)}_{\text{Bessel function of } 2^{nd} \text{ kind}}\right) e^{in\theta}.$$

The scattering problem also implies to relate $\mathcal{B}_n$ to $\mathcal{A}_n$, $\forall n \in \mathbb{Z}$:

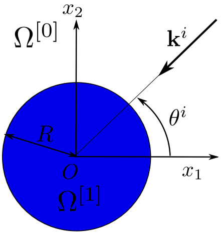
$$\mathcal{B} = \mathcal{SCA}.$$

Scattering by a single cylinder

Scattering of a plane incident wave by a rigid cylinder

Look for $p^{[0]}(\mathbf{r}),\ \forall \mathbf{r} \in \Omega^{[0]}$,

$$\begin{cases} (\nabla + k^2) p^{[0]}(\mathbf{r}) = 0, \\ + \\ p^{[0]}(\mathbf{r}) - p^i(\mathbf{r}) \sim \text{ outgoing waves,} \end{cases}$$



wherein $p^i(\mathbf{r}) = e^{\mathrm{i}k_1^i x_1 - \mathrm{i}k_2^i x_2}$, with $k_1^i = -k\cos\left(\theta^i\right)$ and $k_2^i = \sqrt{k^2 - (k_1^i)^2}$, with $\mathrm{Re}(k_2^i) \geq 0$.

Boundary conditions:

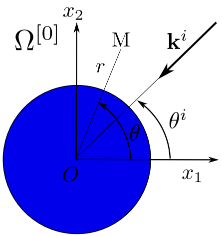
$$V_r^{[0]}(R) = 0 \ \Rightarrow \ \left.\frac{\partial p^{[0]}}{\partial r}\right|_{r=R} = 0.$$

The pressure field in $\Omega^{[0]}$ takes the following form

$$p(\mathbf{r}) = \underbrace{\sum_{m\in\mathbb{Z}} \mathcal{A}_m \mathrm{J}_m(kr) e^{\mathrm{i}m\theta}}_{\text{Incident field}} + \underbrace{\sum_{n\in\mathbb{Z}} \mathcal{B}_n \mathrm{H}_n^{(1)}(kr) e^{\mathrm{i}n\theta}}_{\text{Scattered field}}.$$

Remark: $\int_0^{2\pi} e^{\mathrm{i}(n-m)\theta} d\theta = 2\pi\delta_{nm}$, i.e., $e^{\mathrm{i}n\theta}$ is an orthogonal basis.

Expression of the incident field in $\mathcal{C}$



$$\mathbf{k}^i = \begin{bmatrix} k_1^i = -k\cos\left(\theta^i\right) \\ k_2^i = k\sin\left(\theta^i\right) \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 = r\cos\left(\theta\right) \\ x_2 = r\sin\left(\theta\right) \end{bmatrix}.$$

$$\begin{aligned} p^i(\mathbf{r}) &= e^{\mathrm{i}k_1^i x_1 - \mathrm{i}k_2^i x_2} \\ &= e^{-\mathrm{i}k\cos\left(\theta^i\right) r\cos(\theta) - \mathrm{i}k\sin\left(\theta^i\right) r\sin(\theta)} \\ &= e^{-\mathrm{i}kr\left[\cos\left(\theta^i\right)\cos(\theta) + \sin\left(\theta^i\right)\sin(\theta)\right]} \\ &= e^{-\mathrm{i}kr\cos\left(\theta - \theta^i\right)}. \end{aligned}$$

Refering to Abramowitz & Stegun, 1964:

$$e^{-\mathrm{i}kr\cos\left(\theta-\theta^i\right)} = \sum_{m\in\mathbb{Z}} (-\mathrm{i})^m \mathrm{J}_m\left(kr\right) e^{\mathrm{i}m(\theta-\theta^i)},$$

so the incident field may be written as

$$p^i(\mathbf{r}) = \sum_{m\in\mathbb{Z}} \underbrace{(-\mathrm{i})^m e^{-\mathrm{i}m\theta^i}}_{\mathcal{A}_m} \mathrm{J}_m\left(kr\right) e^{\mathrm{i}n\theta}.$$

Application of the BC and solution of the problem

$$p(\mathbf{r}) = \sum_{m\in\mathbb{Z}} \mathcal{A}_m \mathrm{J}_m(kr)e^{\mathrm{i}m\theta} + \sum_{n\in\mathbb{Z}} \mathcal{B}_n \mathrm{H}_n^{(1)}(kr)e^{\mathrm{i}n\theta}.$$

The normal derivative with respect of r reads as

$$\frac{\partial p(\mathbf{r})}{\partial r} = \sum_{m\in\mathbb{Z}} k\mathcal{A}_m \dot{\mathrm{J}}_m(kr)e^{\mathrm{i}m\theta} + \sum_{n\in\mathbb{Z}} k\mathcal{B}_n \dot{\mathrm{H}}_n^{(1)}(kr)e^{\mathrm{i}n\theta},$$

where $\dot{\chi}_n(x) = \partial\chi_n(x)/\partial x = \left(\chi_{n-1}(x) - \chi_{n+1}(x)\right)/2.$

Introducing $\alpha = kR$ and making use of $\displaystyle\int_0^{2\pi} e^{\mathrm{i}(n-m)\theta}d\theta = 2\pi\delta_{nm}$ after projection $\displaystyle\underbrace{R\times}_{optional} \int_0^{2\pi} \left.\frac{\partial p(\mathbf{r})}{\partial r}\right|_{r=R} e^{-\mathrm{i}l\theta}d\theta = 0$, we get:

$$\mathcal{B}_n = -\frac{\dot{\mathrm{J}}_n(\alpha)}{\dot{\mathrm{H}}_n^{(1)}(\alpha)}\mathcal{A}_n = \underbrace{\mathcal{SC}_n}_{\text{Scattering coefficient}} \mathcal{A}_n,$$

and finally $\displaystyle p(\mathbf{r}) = \sum_{n\in\mathbb{Z}} \mathcal{A}_n \left(\mathrm{J}_n(kr) + \mathcal{SC}_n \mathrm{H}_n^{(1)}(kr)\right) e^{\mathrm{i}n\theta}.$

Penetrable scactterers

- when the cylindrical scatterer is penetrable (fluid), the pressure field in $\Omega^{[1]}$ reads as (Rayleigh hyposthesis):

$$p^{[1]}(\mathbf{r}) = \sum_{m\in\mathbb{Z}} \mathcal{C}_m \mathrm{J}_n(kr) e^{\mathrm{i}n\theta} e^{\mathrm{i}n\theta}.$$

Application of the BC $\left(\text{after projection } \int_0^{2\pi} \cdot e^{-\mathrm{i}l\theta} d\theta\right)$ leads to

$$\mathcal{B}_n = \frac{\beta^{[1]} \dot{\mathrm{J}}_n(\alpha^{[1]}) \mathrm{J}_n(\alpha^{[0]}) - \beta^{[0]} \dot{\mathrm{J}}_n(\alpha^{[0]}) \mathrm{J}_n(\alpha^{[1]})}{\beta^{[0]} \dot{\mathrm{H}}_n^{(1)}(\alpha^{[0]}) \mathrm{J}_n(\alpha^{[1]}) - \beta^{[1]} \mathrm{H}_n^{(1)}(\alpha^{[0]}) \dot{\mathrm{J}}_n(\alpha^{[1]})} \mathcal{A}_n = \mathcal{SC}_n \mathcal{A}_n,$$

where $\alpha^{[j]} = k^{[j]} R$, $\beta^{[j]} = \alpha^{[j]} / \rho^{[j]}$, $j = 0, 1$.
Remark: the low frequency approximation reads as $\left(\mathcal{O}\left(\alpha^{[0]}\right)^2\right)$:

$$\mathcal{SC}_0 \approx \frac{\mathbf{i}\pi \left(\alpha^{[0]}\right)^2}{4} \left(1 - \frac{K^{[0]}}{K^{[1]}}\right), \quad \mathcal{SC}_{\pm 1} \approx \frac{\mathbf{i}\pi \left(\alpha^{[0]}\right)^2}{4} \frac{\rho^{[1]} - \rho^{[0]}}{\rho^{[0]} + \rho^{[1]}}.$$

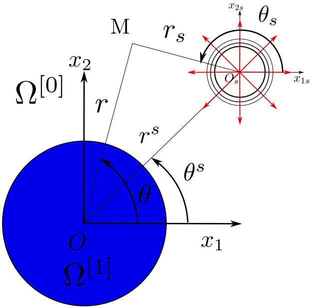
- at low frequency, split ring or Helmholtz resonators leads to full scattering matrices Krynkin *et al.*, J. Phys. D: Appl. Phys., 2011.

Look for $p^{[0]}(\mathbf{r}),\ \forall \mathbf{r} \in \Omega^{[0]}$,

$$\begin{cases} (\nabla + k^2)p^{[0]}(\mathbf{r}) = 0, \\ + \\ p^{[0]}(\mathbf{r}) - p^i(\mathbf{r}) \sim \text{ outgoing waves,} \end{cases}$$

wherein $p^i(\mathbf{r}_s) = \frac{\mathrm{i}}{4}\mathrm{H}_0^{(1)}(kr_s)$.

Boundary conditions: $\left.\frac{\partial p^{[0]}}{\partial r}\right|_{r=R} = 0.$



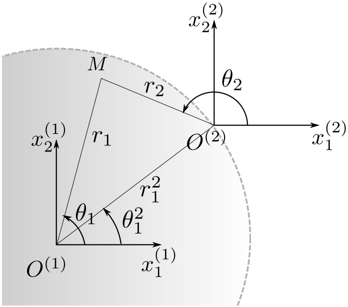
Notation:

- superscript indicates the object
- subscript indicates the object the coordinate system is attached to

The pressure field in $\Omega^{[0]}$ takes the following form

$$p(\mathbf{r}) = \underbrace{\frac{\mathrm{i}}{4}\mathrm{H}_0^{(1)}(kr_s)}_{\text{Coordinate system attached to the source}} + \overbrace{\sum_{n\in\mathbb{Z}} \mathcal{B}_n \mathrm{H}_n^{(1)}(kr)e^{\mathrm{i}n\theta}}^{\text{Coordinate system attached to the cylinder}}.$$

Graf's addition theorem Abramowitz & Stegun, 1964



$$\mathrm{H}_n^{(1)}(kr_2)e^{\mathrm{i}n\theta_2} = \begin{cases} \sum_{q\in\mathbb{Z}} e^{\mathrm{i}(n-q)\theta_1^2}\mathrm{H}_{q-n}^{(1)}(kr_1^2)\mathrm{J}_q(kr_1)e^{\mathrm{i}q\theta_1}, & \text{if } r_1 < r_1^2 \\ \sum_{q\in\mathbb{Z}} e^{\mathrm{i}(n-q)\theta_1^2}\mathrm{J}_{q-n}(kr_1^2)\mathrm{H}_q^{(1)}(kr_1)e^{\mathrm{i}q\theta_2}, & \text{if } r_1 > r_1^2 \end{cases}$$

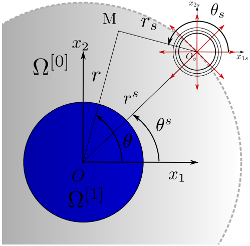
Remark: may also be found with $\theta_1^{2'} = \theta_1^2 + \pi$.

Applying the Graf's theorem, we end with

$$p^i(\mathbf{r}) = \begin{cases} \displaystyle\sum_{n\in\mathbb{Z}} \frac{\mathrm{i}}{4} e^{-\mathrm{i}n\theta^s} \mathrm{J}_n(kr^s)\mathrm{H}_n^{(1)}(kr)e^{\mathrm{i}n\theta} & \text{, for } r > r^s, \\ \displaystyle\sum_{n\in\mathbb{Z}} \underbrace{\frac{\mathrm{i}}{4} e^{-\mathrm{i}n\theta^s} \mathrm{H}_n^{(1)}(kr^s)}_{\mathcal{A}_n} \mathrm{J}_n(kr)e^{\mathrm{i}n\theta} & \text{, for } r < r^s, \end{cases}$$

so the problem reads as

$$p(\mathbf{r}) = \sum_{n\in\mathbb{Z}} \left(\mathcal{A}_n \mathrm{J}_n(kr) + \mathcal{B}_n \mathrm{H}_n^{(1)}(kr)\right) e^{\mathrm{i}n\theta}, \text{ for } r < r^s.$$

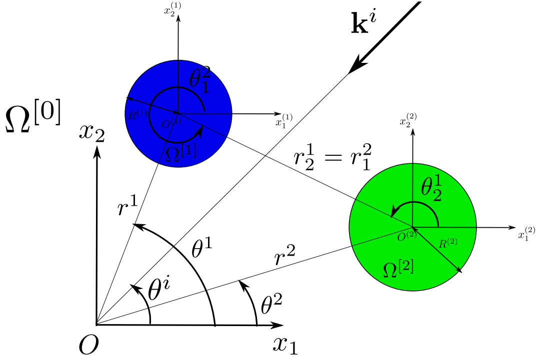


Or, we show previously that

$$\mathcal{B}_n = -\frac{\dot{\mathrm{J}}_n(\alpha)}{\dot{\mathrm{H}}_n^{(1)}(\alpha)} \mathcal{A}_n = \mathcal{SC}_n \mathcal{A}_n.$$

$$p(\mathbf{r}) = \frac{\mathrm{i}}{4} \mathrm{H}_0^{(1)}(kr_s) + \sum_{n\in\mathbb{Z}} \mathcal{SC}_n \mathcal{A}_n \mathrm{H}_n^{(1)}(kr) e^{\mathrm{i}n\theta}.$$

Scattering by a N cylinders



Boundary conditions:

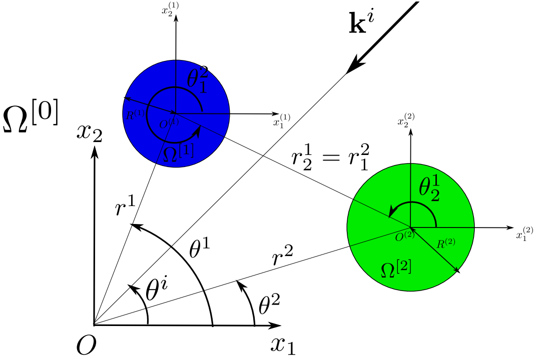
$$\left.\frac{\partial p^{[0]}}{\partial r}\right|_{r_1=R^{(1)}} = 0,$$

$$\left.\frac{\partial p^{[0]}}{\partial r}\right|_{r_2=R^{(2)}} = 0.$$

Look for $p^{[0]}(\mathbf{r}), \ \forall \mathbf{r} \in \Omega^{[0]}$,

$$\left\{\begin{array}{l} (\nabla + k^2)p^{[0]}(\mathbf{r}) = 0, \\ + \\ p^{[0]}(\mathbf{r}) - p^i(\mathbf{r}) \sim \text{ outgoing waves,} \end{array}\right.$$

wherein $p^i(\mathbf{r}) = e^{\mathrm{i}k_1^i x_1 - \mathrm{i}k_2^i x_2}$, with $k_1^i = -k\cos\left(\theta^i\right)$ and $k_2^i = \sqrt{k^2 - (k_1^i)^2}$, with $\mathrm{Re}(k_2^i) \geq 0$.



Boundary conditions:

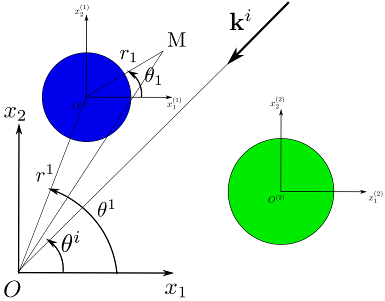
$$\left.\frac{\partial p^{[0]}}{\partial r}\right|_{r_1=R^{(1)}} = 0,$$

$$\left.\frac{\partial p^{[0]}}{\partial r}\right|_{r_2=R^{(2)}} = 0.$$

The pressure field in $\Omega^{[0]}$ takes the following form

$$p(\mathbf{r}) = \underbrace{\sum_{m\in\mathbb{Z}} (-\mathrm{i})^m \mathrm{J}_m(kr)\, e^{\mathrm{i}m(\theta-\theta^i)}}_{\text{Global coordinate system}} + \overbrace{\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(1)} \mathrm{H}_n^{(1)}(kr_1) e^{\mathrm{i}n\theta_1}}^{\text{Coordinate system } \mathcal{C}_1} + \underbrace{\sum_{q\in\mathbb{Z}} \mathcal{B}_q^{(2)} \mathrm{H}_q^{(1)}(kr_2) e^{\mathrm{i}q\theta_2}}_{\text{Coordinate system } \mathcal{C}_2}.$$

$$p(\mathbf{r}) = \overbrace{e^{\mathrm{i}k_1^i x_1 - \mathrm{i}k_2^i x_2}}^{p^i(\mathbf{r})} + \overbrace{\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(1)} \mathrm{H}_n^{(1)}(kr_2) e^{\mathrm{i}n\theta_2}}^{p_{scat}^{(1)}(\mathbf{r}_1)} + \overbrace{\sum_{q\in\mathbb{Z}} \mathcal{B}_q^{(2)} \mathrm{H}_q^{(1)}(kr_2) e^{\mathrm{i}q\theta_2}}^{p_{scat}^{(2)}(\mathbf{r}_2)}.$$



- Incident field

$$\begin{aligned} p^i(\mathbf{r}) &= e^{\mathrm{i}\mathbf{k}^i\cdot\mathbf{r}} \\ &= e^{\mathrm{i}\mathbf{k}^i\cdot\left(\mathbf{r}^1+\mathbf{r}_1\right)} \\ &= e^{\mathrm{i}\mathbf{k}^i\mathbf{r}^1} \times \underbrace{e^{\mathrm{i}\mathbf{k}^i\cdot\mathbf{r}_1}}_{\text{Cylinder 1}} \\ &= e^{-\mathrm{i}k^i r^1 \cos\left(\theta^1-\theta^i\right)} \times \sum_{n\in\mathbb{Z}} (-\mathrm{i})^n \mathrm{J}_n\left(kr_1\right) e^{\mathrm{i}n(\theta_1-\theta^i)} \\ &= \sum_{n\in\mathbb{Z}} \mathcal{A}_n^{1i} \mathrm{J}_n\left(kr_1\right) e^{\mathrm{i}n\theta_1}. \end{aligned}$$

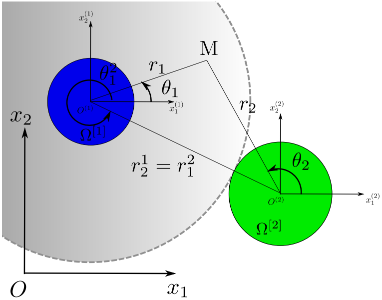
Expression of the field in $\mathcal{C}_1$

$$p(\mathbf{r}) = \overbrace{e^{\mathrm{i}k_1^i x_1 - \mathrm{i}k_2^i x_2}}^{p^i(\mathbf{r})} + \overbrace{\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(1)} \mathrm{H}_n^{(1)}(kr_2) e^{\mathrm{i}n\theta_2}}^{p_{scat}^{(1)}(\mathbf{r}_1)} + \overbrace{\sum_{q\in\mathbb{Z}} \mathcal{B}_q^{(2)} \mathrm{H}_q^{(1)}(kr_2) e^{\mathrm{i}q\theta_2}}^{p_{scat}^{(2)}(\mathbf{r}_2)}.$$

- Scattered field by the cylinder 2

Graf's theorem, for $r_1 < r_1^2 - R^{(2)}$:

$$\mathrm{H}_q^{(1)}(kr_2)e^{\mathrm{i}q\theta_2} = \sum_{n\in\mathbb{Z}} e^{\mathrm{i}(q-n)\theta_1^2} \mathrm{H}_{n-q}^{(1)}(kr_1^2) \mathrm{J}_n(kr_1) e^{\mathrm{i}n\theta_1},$$



so we got,

$$p_{scat}^{(2)}(\mathbf{r}_1) = \sum_{n\in\mathbb{Z}} \sum_{q\in\mathbb{Z}} \mathcal{B}_q^{(2)} e^{\mathrm{i}(q-n)\theta_1^2} \mathrm{H}_{n-q}^{(1)}(kr_1^2) \mathrm{J}_n(kr_1) e^{\mathrm{i}n\theta_1}, \text{ for } r_1 < r_1^2 - R^{(2)}.$$

$$
\left.p(\mathbf{r_1}) = \sum_{n\in\mathbb{Z}} \mathcal{A}_n^{1i} \mathrm{J}_n\left(kr_1\right) e^{\mathrm{i}n\theta_1}\right\} \text{ Incident field}
$$

$$
\left.+ \sum_{n\in\mathbb{Z}} \sum_{q\in\mathbb{Z}} \mathcal{B}_q^{(2)} e^{\mathrm{i}(q-n)\theta_1^2} \mathrm{H}_{n-q}^{(1)}(kr_1^2) \mathrm{J}_n(kr_1) e^{\mathrm{i}n\theta_1}\right\} \text{ Scattered field by 2}
$$

$$
\left.+ \sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(1)} \mathrm{H}_n^{(1)}(kr_1) e^{\mathrm{i}n\theta_1}\right\} \text{ Scattered field by 1}
$$

Keeping in mind that $\int_0^{2\pi} e^{\mathrm{i}(n-m)\theta} d\theta = 2\pi\delta_{nm}$, this field may be written as

$$
p(\mathbf{r_1}) = \sum_{n\in\mathbb{Z}} \left(\overbrace{\left[\mathcal{A}_n^{1i} + \sum_{q\in\mathbb{Z}} \mathcal{B}_q^{(2)} e^{\mathrm{i}(q-n)\theta_1^2} \mathrm{H}_{n-q}^{(1)}(kr_1^2) \right]}^{\mathcal{A}_n^{(1)}} \mathrm{J}_n(kr_1) + \mathcal{B}_n^{(1)} \mathrm{H}_n^{(1)}(kr_1) \right) e^{\mathrm{i}n\theta_1}.
$$

Once again, we have

$$\mathcal{B}_n^{(1)} = -\frac{\dot{\mathrm{J}}_n(\alpha^1)}{\dot{\mathrm{H}}_n^{(1)}(\alpha^1)}\mathcal{A}_n^1 = \mathcal{SC}_n^1\mathcal{A}_n^1$$

$$= \mathcal{SC}_n^1\left(\mathcal{A}_n^{1i} + \sum_{q\in\mathbb{Z}} \mathcal{B}_q^{(2)} e^{\mathrm{i}(q-n)\theta_1^2}\mathrm{H}_{n-q}^{(1)}(kr_1^2)\right),$$

which may be written in matrix form

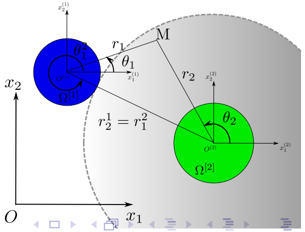
$$\mathbf{B}^1 = \mathbf{A}^1 + \mathbf{C}_1^2\mathbf{B}^2.$$

Similarly, we can express the field in $\mathcal{C}_2$ for $r_2 < r_2^1 - R^{(1)}$ and we get:

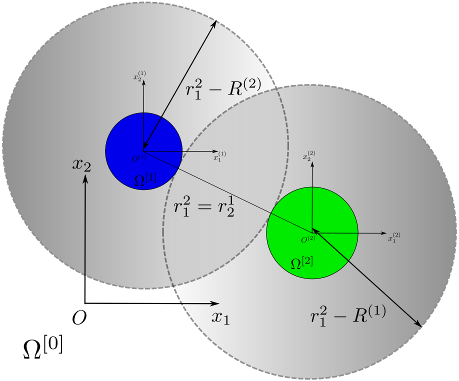
$$\mathbf{B}^2 = \mathbf{A}^2 + \mathbf{C}_2^1\mathbf{B}^1.$$

Finally, the final system reads as:

$$\begin{bmatrix} \mathbf{Id} & -\mathbf{C}_1^2 \\ -\mathbf{C}_2^1 & \mathbf{Id} \end{bmatrix}\begin{bmatrix} \mathbf{B}^1 \\ \mathbf{B}^2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}^1 \\ \mathbf{A}^2 \end{bmatrix}.$$

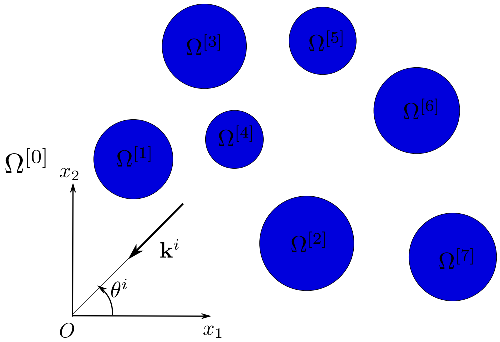


Warning! Take care with field representation domains



For the solution of the problem we experessed the fields in both $r_1 < r_1^2 - R^{(2)}$ and $r_2 < r_1^2 - R^{(1)}$, but we should keep in mind that

$$p(\mathbf{r}) = \sum_{n\in\mathbb{Z}} (-\mathrm{i})^n \mathrm{J}_n(kr)\, e^{\mathrm{i}n(\theta-\theta^i)} + \mathcal{B}_n^{(1)} \mathrm{H}_n^{(1)}(kr_1) e^{\mathrm{i}n\theta_1} + \mathcal{B}_n^{(2)} \mathrm{H}_n^{(1)}(kr_2) e^{\mathrm{i}n\theta_2}, \forall r \in \Omega^{[0]}.$$



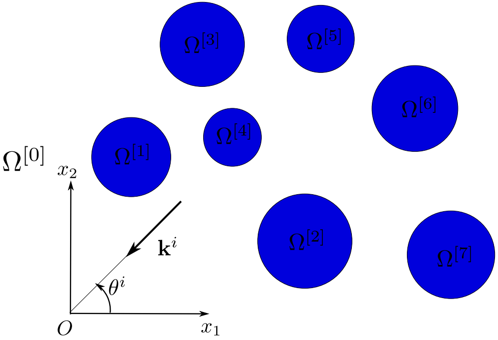
Boundary conditions:

$$\left.\frac{\partial p^{[0]}}{\partial r_j}\right|_{r_j=R^{(j)}} = 0,\ j \in \mathcal{J}$$

Look for $p^{[0]}(\mathbf{r}),\ \forall \mathbf{r} \in \Omega^{[0]}$,

$$\begin{cases} (\nabla + k^2)p^{[0]}(\mathbf{r}) = 0, \\ + \\ p^{[0]}(\mathbf{r}) - p^i(\mathbf{r}) \sim \text{ outgoing waves,} \end{cases}$$

wherein $p^i(\mathbf{r}) = e^{\mathrm{i}k_1^i x_1 - \mathrm{i}k_2^i x_2}$, with $k_1^i = -k\cos\left(\theta^i\right)$ and $k_2^i = \sqrt{k^2 - (k_1^i)^2}$, with $\mathrm{Re}(k_2^i) \geq 0$.



Boundary conditions:

$$\left.\frac{\partial p^{[0]}}{\partial r_j}\right|_{r_j=R^{(j)}} = 0,\ j \in \mathcal{J}$$

The pressure field in $\Omega^{[0]}$ takes the following form

$$p(\mathbf{r}) = \underbrace{\sum_{n\in\mathbb{Z}} (-\mathrm{i})^n \mathrm{J}_n\left(kr\right) e^{\mathrm{i}n(\theta-\theta^i)}}_{\text{Global coordinate system}} + \sum_{j\in\mathcal{J}} \overbrace{\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(j)} \mathrm{H}_n^{(1)}(kr_j) e^{\mathrm{i}n\theta_j}}^{p_{scat}^{(j)}}.$$

Solution of the problem

- Express the field around each j-th cylinder $\forall r_j < \min_{o\neq j}\left(r_j^o - R^{(o)}\right)$

$$p(\mathbf{r}_j) = \sum_{n\in\mathbb{Z}}\left(\overbrace{\left[\mathcal{A}_n^{ji} + \sum_{o\in\mathcal{J}\neq j}\sum_{q\in\mathbb{Z}}\mathcal{B}_q^{(o)} e^{\mathrm{i}(q-n)\theta_j^o}\mathrm{H}_{n-q}^{(1)}(kr_j^o)\right]}^{\mathcal{A}_n^{(j)}}\mathrm{J}_n(kr_j) + \mathcal{B}_n^{(1)}\mathrm{H}_n^{(1)}(kr_j)\right)e^{\mathrm{i}n\theta_j}.$$

- Apply the BC on the j-th cylinder

$$\mathcal{B}_n^{(j)} = \mathcal{SC}_n^j\left(\mathcal{A}_n^{ji} + \sum_{o\in\mathcal{J}\neq j}\sum_{q\in\mathbb{Z}}\mathcal{B}_q^{(o)} \mathrm{e}^{\mathrm{i}(q-n)\theta_j^o}\mathrm{H}_{n-q}^{(1)}(kr_j^o)\right).$$

- Final system for the solution of $\mathcal{B}_n^{(j)}$, $\forall n\in\mathbb{Z}$ and $\forall j\in\mathcal{J}$

$$\begin{bmatrix} \mathbf{Id} & -\mathbf{C}_1^2 & \cdot & \cdot & \cdot & -\mathbf{C}_1^{J-1} & -\mathbf{C}_1^J \\ -\mathbf{C}_2^1 & \mathbf{Id} & \cdot & \cdot & \cdot & -\mathbf{C}_2^{J-1} & -\mathbf{C}_2^J \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\mathbf{C}_{J-1}^1 & -\mathbf{C}_{J-1}^2 & \cdot & \cdot & \cdot & \mathbf{Id} & -\mathbf{C}_{J-1}^J \\ -\mathbf{C}_J^1 & -\mathbf{C}_J^2 & \cdot & \cdot & \cdot & -\mathbf{C}_J^{J-1} & \mathbf{Id} \end{bmatrix}\begin{bmatrix} \mathbf{B}^1 \\ \mathbf{B}^2 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{B}^{J-1} \\ \mathbf{B}^J \end{bmatrix} = \begin{bmatrix} \mathbf{A}^1 \\ \mathbf{A}^2 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{A}^{J-1} \\ \mathbf{A}^J \end{bmatrix}.$$

Comments

- Replacing the expression of $\mathcal{A}_n^{ji}$ by $\mathcal{A}_n^{ji} = \frac{\mathrm{i}}{4} e^{-\mathrm{i}n\theta_j^s} \mathrm{H}_n^{(1)}(kr_j^s)$, enable the calculation of $\mathbf{B}^j$, $j \in \mathcal{J}$ when the configuration is excited by a line source.
  In other words, you calculate the Green's function of the system!

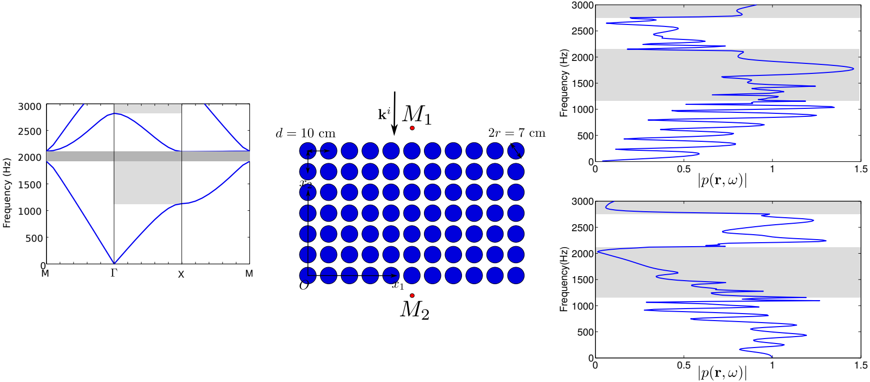
  $\Rightarrow$ usefull to calculate the density of state Asatryan *et al.*, Waves Random Media, 2003
  $\Rightarrow$ usefull to solve inverse problem Groby and Lesselier *et al.*, J. Opt. Soc. Am. A, 2008

- Sum are truncated in practice and reads $\sum_{m=-M}^{M}$, with

$$M = \mathrm{int}\left(4.05(kR)^{1/3} + kR\right) + \underbrace{\text{security coefficient}}_{=10},$$

  Barber and Hill, 1990

Example: 77 element finite dimension sonic crystal

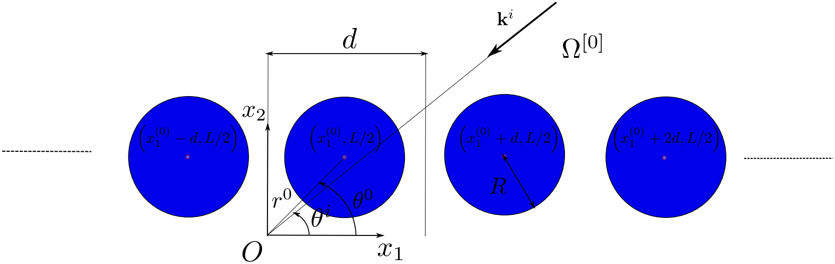


We should use the pointing vector instead of the pressure field.

Part III. Scattering by a periodic arrangement of circular cylinders

- Scattering of a plane incident by an array of rigid cylinders
- Reflection and transmission coefficients by an array of rigid cylinders
- Reflection and Transmission coefficients by a stack of gratings
- Band diagram calculation

Scattering of a plane incident by an array of rigid cylinders

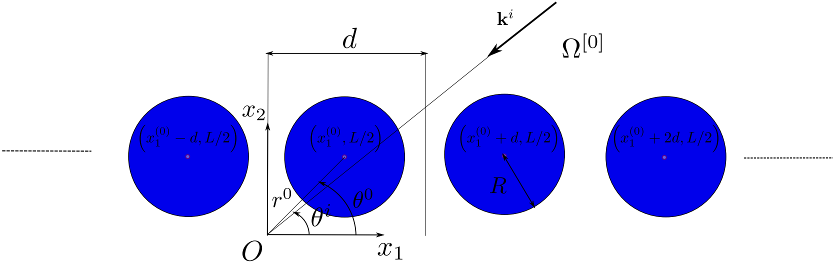


Look for $p^{[0]}(\mathbf{r}),\ \forall \mathbf{r} \in \Omega^{[0]}$,

$$\begin{cases} (\nabla + k^2) p^{[0]}(\mathbf{r}) = 0, \\ + \\ p^{[0]}(\mathbf{r}) - p^i(\mathbf{r}) \sim \text{ outgoing waves,} \end{cases}$$

wherein $p^i(\mathbf{r}) = e^{\mathrm{i}k_1^i x_1 - \mathrm{i}k_2^i x_2}$, with $k_1^i = -k\cos\left(\theta^i\right)$ and $k_2^i = \sqrt{k^2 - (k_1^i)^2}$, with $\mathrm{Re}(k_2^i) \geq 0$.

Scattering by an array of rigid cylinders



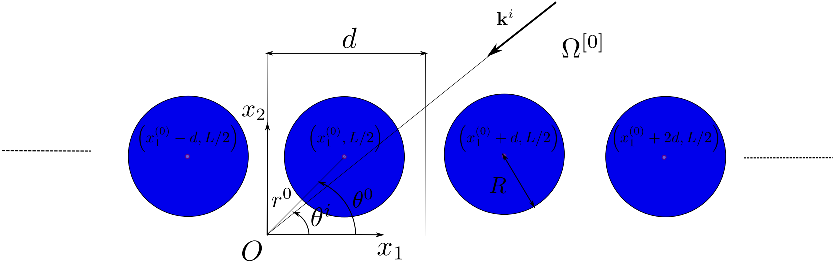
Boundary conditions: $\left.\dfrac{\partial p^{[0]}}{\partial r}\right|_{r=R} = 0.$

The field is quasi-periodic (*Floquet-Bloch condition*):

$$p^{[0]}(x_1 + nd, x_2) = p^{[0]}(x_1, x_2)e^{ik_1^i nd}, \ \forall \mathbf{x} \in \mathbb{R}^2 \text{ and } \forall n \in \mathbb{Z}.$$

$\Rightarrow$ It is sufficient to determine the field in the unit cell $\mathcal{C}$.

Scattering by an array of rigid cylinders



The pressure field in $\Omega^{[0]}$ takes the following form

$$p^{[0]}(\mathbf{r}) = \sum_{n\in\mathbb{Z}} (-\mathrm{i})^n \mathrm{J}_n\left(kr\right) e^{\mathrm{i}n(\theta-\theta^i)} + \sum_{j\in\mathbb{Z}} \overbrace{\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(j)} \mathrm{H}_n^{(1)}(kr_j) e^{\mathrm{i}n\theta_j}}^{p_{scat}^{(j)}} .$$

The periodicity implies

$$\mathcal{B}_n^{(j)} = \mathcal{B}_n^{(0)} e^{\mathrm{i}jk_1^i d}.$$

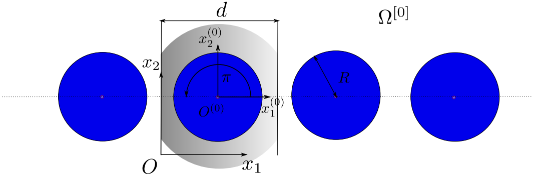
Scattered field in $\mathcal{C}_0$

The scattered field may be written in the form

$$
\begin{aligned}
p_{scat}^{[0]}(\mathbf{r}) &= \sum_{j\in\mathbb{Z}}\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)} e^{\mathrm{i}jk_1^i d}\mathrm{H}_n^{(1)}(kr_j)e^{\mathrm{i}n\theta_j} \\
&= \sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)}\mathrm{H}_n^{(1)}(kr_0)e^{\mathrm{i}n\theta_0} \\
&+ \sum_{j<0}\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)} e^{\mathrm{i}jk_1^i d}\mathrm{H}_n^{(1)}(kr_j)e^{\mathrm{i}n\theta_j} + \sum_{j>0}\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)} e^{\mathrm{i}jk_1^i d}\mathrm{H}_n^{(1)}(kr_j)e^{\mathrm{i}n\theta_j}.
\end{aligned}
$$

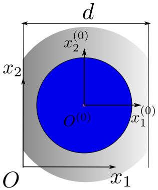
Applying the Graf's theorem ($r_0 < d - R$) leads to

$$
\mathrm{H}_n^{(1)}(kr_j)e^{\mathrm{i}n\theta_j} = \begin{cases} \sum_{q\in\mathbb{Z}}(-1)^{n-q}\mathrm{H}_{q-n}^{(1)}(k|j|d)\mathrm{J}_q(kr_0)e^{\mathrm{i}q\theta_0}, \text{ for } j<0, \\ \sum_{q\in\mathbb{Z}}\mathrm{H}_{q-n}^{(1)}(kjd)\mathrm{J}_q(kr_0)e^{\mathrm{i}q\theta_0}, \text{ for } j>0. \end{cases}
$$



The scattered field may be written in the form

$$
\begin{aligned}
p_{scat}^{[0]}(\mathbf{r}) &= \sum_{j\in\mathbb{Z}}\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)} e^{\mathrm{i}jk_1^i d}\mathrm{H}_n^{(1)}(kr_j)e^{\mathrm{i}n\theta_j} \\
&= \sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)}\mathrm{H}_n^{(1)}(kr_0)e^{\mathrm{i}n\theta_0} \\
&+ \sum_{j<0}\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)} e^{\mathrm{i}jk_1^i d}\mathrm{H}_n^{(1)}(kr_j)e^{\mathrm{i}n\theta_j} + \sum_{j>0}\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)} e^{\mathrm{i}jk_1^i d}\mathrm{H}_n^{(1)}(kr_j)e^{\mathrm{i}n\theta_j}.
\end{aligned}
$$



For $(r_0 < d - R)\bigcup$ unit cell

$$
p_{scat}^{[0]}(\mathbf{r}) = \sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)}\mathrm{H}_n^{(1)}(kr_0)e^{\mathrm{i}n\theta_0}
$$

$$
+\sum_{n\in\mathbb{Z}}\sum_{q\in\mathbb{Z}} \mathcal{B}_q^{(0)} \underbrace{\sum_{j>0} \mathrm{H}_{n-q}^{(1)}(kjd)\left(e^{\mathrm{i}jk_1^i d} + (-1)^{n-q}e^{-\mathrm{i}jk_1^i d}\right)}_{\text{Schlömilch serie}} \mathrm{J}_n(kr_0)e^{\mathrm{i}n\theta_0}
$$

The serie:

$$\mathcal{S}_n = \sum_{j>0} \mathrm{H}_n^{(1)}(kjd) \left(e^{\mathrm{i}jk_1^i d} + (-1)^n e^{-\mathrm{i}jk_1^i d} \right)$$

is known to be slowly converging in absence of losses.
A large litterature exists on the numerical evaluation of this serie

- V. Twersky, *Elementary function representations of Schlömilch series*, Arch. Ration. Mech. An., 8(1):323-332, 1961
- C.M. Linton, *Schlömilch series that arise in diffraction theory and their efficient computation*, J. Phys. A. : Math. Gen., 39:3325-3339, 2006
- R.C. McPhedran, N.A. Nicorovici, and L.C. Botten, *Schlömilch series and grating sums*, J. Phys. A. : Math. Gen., 38 :8353-8366, 2005.
- ...

Solution of the scattering problem

The incident field reads as

$$p^i(\mathbf{r}_0) = e^{-\mathrm{i}k^i r^0 \cos\left(\theta^0-\theta^i\right)} \times \sum_{n\in\mathbb{Z}} (-\mathrm{i})^n \mathrm{J}_n\left(kr_0\right) e^{\mathrm{i}n(\theta_0-\theta^i)} = \sum_{n\in\mathbb{Z}} \mathcal{A}_n^{0i} \mathrm{J}_n\left(kr_0\right) e^{\mathrm{i}n\theta_0},$$

so we end with

$$p^{[0]}(\mathbf{r}) = \sum_{n\in\mathbb{Z}} \left(\mathcal{B}_n^{(0)} \mathrm{H}_n^{(1)}(kr_0) + \underbrace{\left(\sum_{q\in\mathbb{Z}} \mathcal{B}_q^{(0)} \mathcal{S}_{n-q} + \mathcal{A}_n^{0i} \right)}_{\mathcal{A}_n} \mathrm{J}_n(kr_0) \right) e^{\mathrm{i}n\theta_0}.$$

Once again, we have

$$\mathcal{B}_n^{(1)} = \mathcal{S}\mathcal{C}_n^1 \left(\mathcal{A}_n^{0i} + \sum_{q\in\mathbb{Z}} \mathcal{B}_q^{(0)} \mathcal{S}_{n-q} \right),$$

which may be written in matrix form

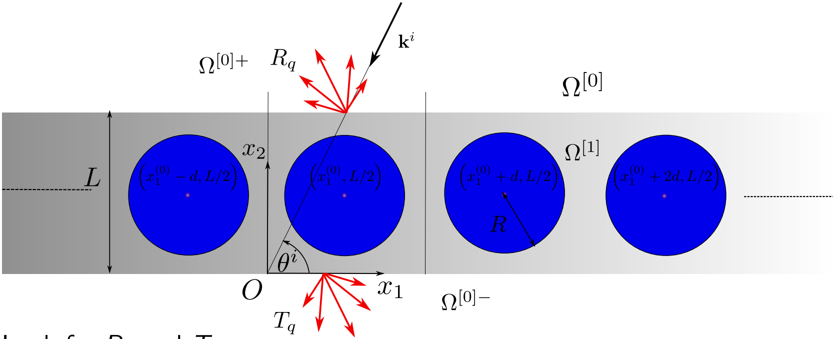
$$\left[\mathbf{Id} - \mathbf{S}\right] \mathbf{B}^0 = \mathbf{A}^0.$$

$$p^{[0]}(\mathbf{r}) = \sum_{n\in\mathbb{Z}} (-\mathrm{i})^n \mathrm{J}_n\left(kr\right) e^{\mathrm{i}n(\theta-\theta^i)} + \sum_{j\in\mathbb{Z}} \sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(j)} \mathrm{H}_n^{(1)}(kr_j) e^{\mathrm{i}n\theta_j}.$$

We do not really use the reciprocal space and Bloch waves!

Reflection and Transmission coefficients by an array of rigid cylinders

Scattering by an array of rigid cylinders



Look for R_q and T_q

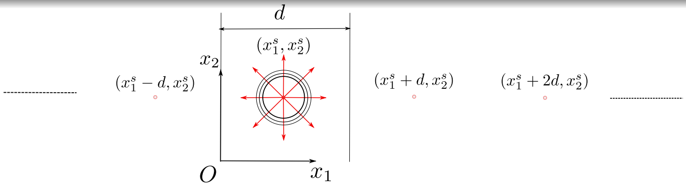
$$p^{[0]+} = \sum_{q \in \mathbb{Z}} \delta_{q0} e^{\mathbf{i}k_{1q}x_1 - \mathbf{i}k_{2q}(x_2 - L)} + R_q e^{\mathbf{i}k_{1q}x_1 + \mathbf{i}k_{2q}(x_2 - L)},$$

$$p^{[0]-} = \sum_{q \in \mathbb{Z}} T_q e^{\mathbf{i}k_{1q}x_1 - \mathbf{i}k_{2q}x_2},$$

where $k_{1q} = k_1^i + \frac{2\pi q}{d}$, $k_{2q} = \sqrt{k^2 - k_{1q}^2}$, with $\mathrm{Re}\left(k_{2q}\right) \geq 0$.

Warning: $\mathcal{A}_n^{0i} \leftarrow \mathcal{A}_n^{0i} e^{\mathbf{i}k_2 L}$.

Periodic Green's function



$$\left\{ \begin{array}{l} (\nabla + k^2)\mathcal{G}(\mathbf{x}, \mathbf{x^s}) = -\delta_{x_1^s + jd, x_2^s}, \; j \in \mathbb{Z} \\ + \\ \mathcal{G}(\mathbf{x}, \mathbf{x^s}) \sim \text{ outgoing waves when } x_2 \to \infty, \end{array} \right.$$

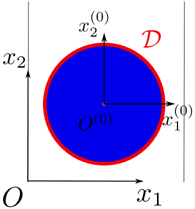
$$\mathcal{G}(\mathbf{x}, \mathbf{x^s}) = \sum_{j \in \mathbb{Z}} \frac{\mathbf{i}}{4\pi} \int_{-\infty}^{\infty} e^{\mathbf{i}k_1(x_1 - x_1^s - jd) + \mathbf{i}k_2|x_2 - x_2^s|} \frac{dk_1}{k_2},$$

with $k_2 = \sqrt{k^2 - k_1^2}$ and $\mathrm{Re}\,(k_2) \geq 0$, with $k_{1j} = \frac{2\pi j}{d}$.

Using the Poisson formula $\displaystyle\sum_{j=-\infty}^{\infty} e^{-\mathbf{i}k_1 jd} = \frac{2\pi}{d} \sum_{j=-\infty}^{\infty} \delta_{k_{1q}}$, we get

$$\mathcal{G}(\mathbf{x}, \mathbf{x^s}) = \sum_{j \in \mathbb{Z}} \frac{\mathbf{i}}{2d} \frac{e^{\mathbf{i}k_{1q}(x_1 - x_1^s) + \mathbf{i}k_{2q}|x_2 - x_2^s|}}{k_{2q}}.$$

Scattered field by the grating



From the Green's theorem we have

$$p^{[0]}(\mathbf{r}) = \int_{\mathcal{D}} \left(p^{[0]}(\mathbf{r}^s) \frac{\partial \mathcal{G}(\mathbf{r}, \mathbf{r}^s)}{\partial r^s} - \mathcal{G}(\mathbf{r}, \mathbf{r}^s) \frac{\partial p^{[0]}(\mathbf{r}^s)}{\partial r^s} \right) d\mathcal{D}.$$

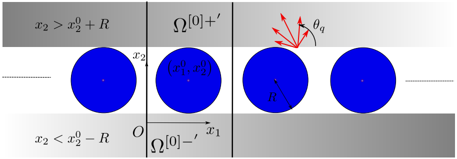
Using $p^{[0]}(\mathbf{r}^s) = \sum_{n \in \mathbb{Z}} \left(\mathcal{B}_q^{(0)} \mathrm{H}_n^{(1)}(kR) + \sum_{q \in \mathbb{Z}} \mathcal{B}_q^{(0)} \mathcal{S}_{n-q} \mathrm{J}_n(kR) \right) e^{\mathrm{i}n\theta_0}$,

orthogonality of $e^{\mathrm{i}n\theta}$, and Wronskian identity, we end with

$$p^{[0]\pm'}(\mathbf{x}) = \sum_{q \in \mathbb{Z}} \sum_{n \in \mathbb{Z}} \mathcal{B}_n^{(0)} K_{qn}^{\pm} e^{-\mathbf{i}k_{1q}x_1^0 \mp \mathbf{i}k_{2q}x_2^0} e^{\mathbf{i}k_{1q}x_1 \pm \mathbf{i}k_{2q}x_2},$$

with $K_{qn}^{\pm} = \frac{2(-\mathbf{i})^n}{dk_{2q}} e^{\pm \mathbf{i}n\theta_q}$,

where $ke^{\mathbf{i}\theta_q} = k_{1q} + \mathrm{i}k_{2q}$.



Identification of R_q and T_q

We have

$$\begin{cases} p^{[0]+'}(\mathbf{x}) = \displaystyle\sum_{q\in\mathbb{Z}}\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)} K_{qn}^{+} e^{-\mathrm{i}k_{1q}x_1^0-\mathrm{i}k_{2q}x_2^0} e^{\mathrm{i}k_{1q}x_1+\mathrm{i}k_{2q}x_2}, \text{ for } x_2 > x_2^0 + R, \\ p_{refl}^{[0]+}(\mathbf{x}) = \displaystyle\sum_{q\in\mathbb{Z}} R_q e^{\mathrm{i}k_{1q}x_1+\mathrm{i}k_{2q}(x_2-L)}, \text{ for } x_2 \geq L > x_2^0 + R. \end{cases}$$

Making use of $\displaystyle\int_0^d e^{\mathrm{i}(k_{1q}-k_{1m})x_1} dx_1 = 2\pi d \delta_{qm}$, we end with

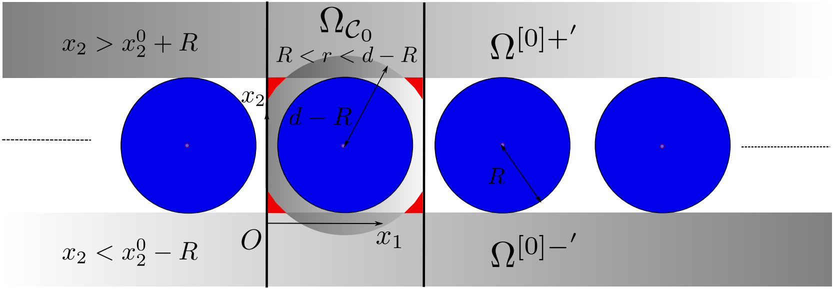
$$R_q = \sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)} K_{qn}^{+} e^{-\mathrm{i}k_{1q}x_1^0-\mathrm{i}k_{2q}(x_2^0-L)}.$$

On the other hand we have

$$\begin{cases} p^{[0]-'}(\mathbf{x}) - p^i(\mathbf{x}) = \displaystyle\sum_{q\in\mathbb{Z}}\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)} K_{qn}^{-} e^{-\mathrm{i}k_{1q}x_1^0+\mathrm{i}k_{2q}x_2^0} e^{\mathrm{i}k_{1q}x_1-\mathrm{i}k_{2q}x_2}, \text{ for } x_2 < x_2^0 - R, \\ p^{[0]-}(\mathbf{x}) = \displaystyle\sum_{q\in\mathbb{Z}} T_q e^{\mathrm{i}k_{1q}x_1-\mathrm{i}k_{2q}x_2}, \text{ for } x_2 \leq 0 < x_2^0 - R. \end{cases}$$

Making use of the orthogonality of the Bloch waves, we end with

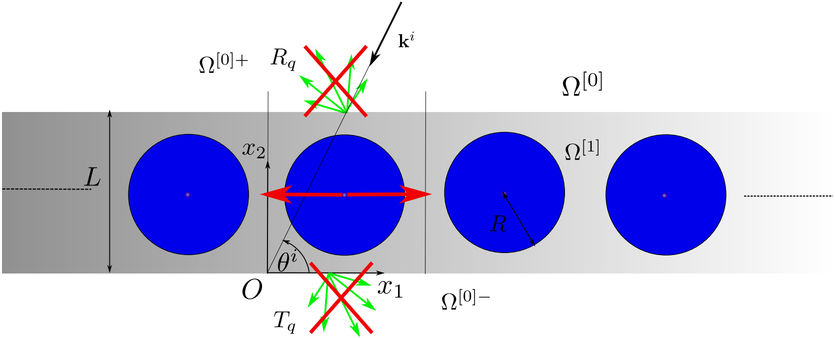
$$T_q = \sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(0)} K_{qn}^{-} e^{-\mathrm{i}k_{1q}x_1^0+\mathrm{i}k_{2q}x_2^0} + \delta_{q0} e^{\mathrm{i}k_{2q}L}.$$



- Field representation in cartesian coordinates in $\Omega^{[0]\pm}$
- Field representation in cylindrical coordinates in $\Omega_{\mathcal{C}_0}$
- For large radius cylinders, we should run the sum in the direct spatial domain in the red regions...

Evidence of the Wood anomaly (Wood, Phil. Mag. J. Sci., 1902)

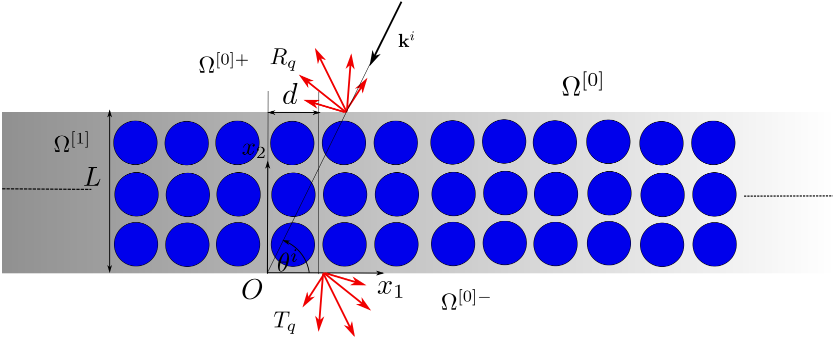
Far below the possible resonance of the scatterers, both R_q and T_q present a pole when $k_{2q} = 0$.
In particular, when $k_{21} = 0$, i.e., $k_1^i \pm \frac{2\pi}{d} = k$, all the energy is spread along the grating (at normal incidence $\lambda = d$) and $\alpha = 1$.



This has led several authors to study propagation of this type of guided/surface waves Porter and Evans, J. Fluid Mech., 1999

Reflection and Transmission by a stack of gratings

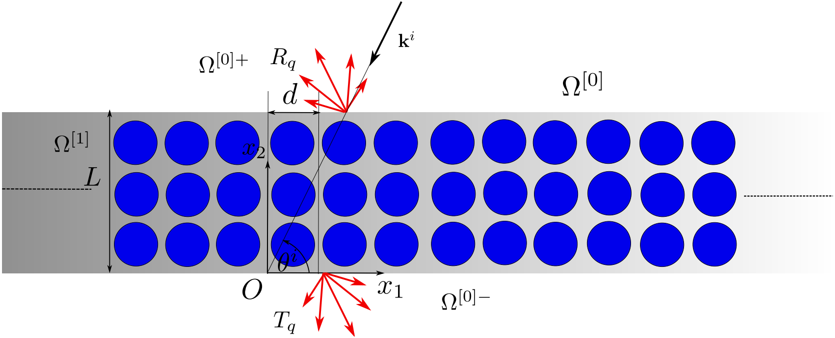
Scattering by a stack of array of rigid cylinders



Look for $p^{[0]}(\mathbf{r})$, $\forall \mathbf{r} \in \Omega^{[0]}$,

$$\begin{cases} (\nabla + k^2)p^{[0]}(\mathbf{r}) = 0, \\ + \\ p^{[0]}(\mathbf{r}) - p^i(\mathbf{r}) \sim \text{ outgoing waves,} \end{cases}$$

wherein $p^i(\mathbf{r}) = e^{\mathrm{i}k_1^i x_1 - \mathrm{i}k_2^i x_2}$, with $k_1^i = -k\cos\left(\theta^i\right)$ and $k_2^i = \sqrt{k^2 - (k_1^i)^2}$, with $\mathrm{Re}(k_2^i) \geq 0$.



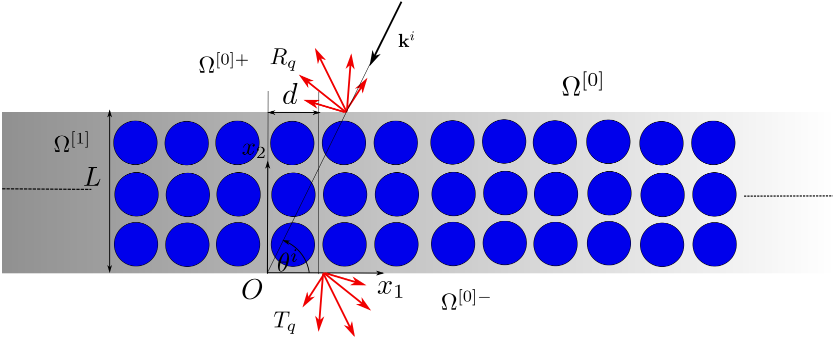
Boundary conditions: $\left.\frac{\partial p^{[0]}}{\partial r}\right|_{r=R} = 0.$

The field is quasi-periodic (*Floquet-Bloch condition*):

$$p^{[0]}(x_1 + nd, x_2) = p^{[0]}(x_1, x_2)e^{ik_1^i nd}, \ \forall \mathbf{x} \in \mathbb{R}^2 \text{ and } \forall n \in \mathbb{Z}.$$

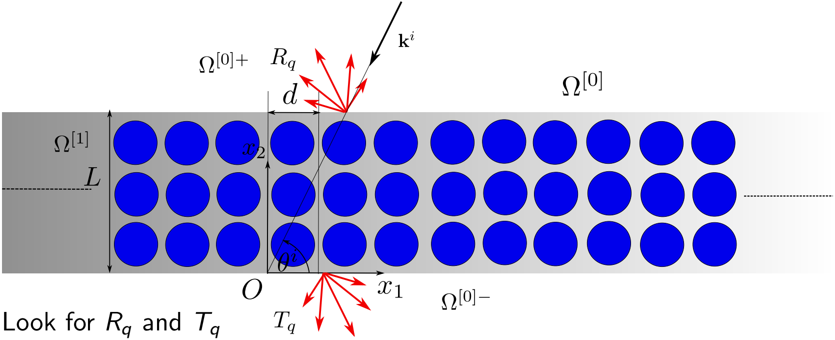
$\Rightarrow$ It is sufficient to determine the field in the unit cell $\mathcal{C}$.

Scattering by a stack of array of rigid cylinders



It exist several ways to solve this problem

- Scattering Matrix: large litterature notably by the group Mc Phedran and L. Botten
- Transfert Matrix
- Considering the unit cell as a kind of supercell



Look for R_q and T_q

$$p^{[0]+} = \sum_{q\in\mathbb{Z}} \delta_{q0} e^{\mathrm{i}k_{1q}x_1 - \mathrm{i}k_{2q}(x_2-L)} + R_q e^{\mathrm{i}k_{1q}x_1 + \mathrm{i}k_{2q}(x_2-L)},\ \forall \mathbf{x} \in \Omega^{[0]+},$$

$$p^{[0]-} = \sum_{q\in\mathbb{Z}} T_q e^{\mathrm{i}k_{1q}x_1 - \mathrm{i}k_{2q}x_2},\ \forall \mathbf{x} \in \Omega^{[0]-},$$

$$p^{[1]} = \sum_{q\in\mathbb{Z}} \left(f_q^+ e^{\mathrm{i}k_{2q}x_2} + f_q^- e^{\mathrm{i}k_{2q}x_2}\right) e^{\mathrm{i}k_{1q}x_1} + \sum_{j\in\mathbb{Z}}\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(j)} \mathrm{H}_n^{(1)}(kr_j) e^{\mathrm{i}n\theta_j},$$

where $k_{1q} = k_1^i + \frac{2\pi q}{d}$, $k_{2q} = \sqrt{k^2 - k_{1q}^2}$, with $\mathrm{Re}\,(k_{2q}) \geq 0$.

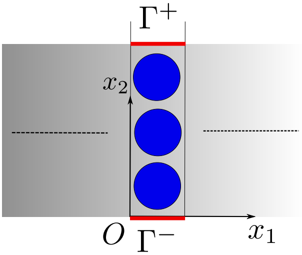
Solution of the scattering problem

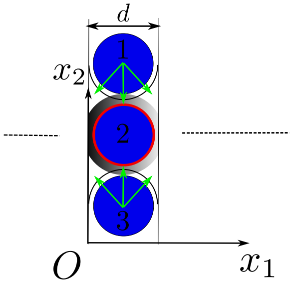
- Application of the BC on $\Gamma^{\pm}$

$$R_q = \sum_{j \in \mathcal{J}} \mathcal{B}_n^{(j)} K_{qn}^{+} e^{-\mathrm{i}k_{1q}x_1^0 - \mathrm{i}k_{2q}(x_2^0 - L)}$$

$$T_q = \sum_{j \in \mathcal{J}} \mathcal{B}_n^{(j)} K_{qn}^{-} e^{-\mathrm{i}k_{1q}x_1^0 + \mathrm{i}k_{2q}(x_2^0 - L)} + e^{\mathrm{i}k_{2q}L}\delta_q$$

$$f_q^{+} = 0, \text{ and } f_q^{-} = e^{\mathrm{i}k_{2q}L}\delta_q$$





- Application of the BC on the cylinders

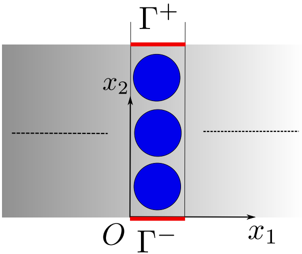
$$p_{inc}^{[1]}(\mathbf{r_j}) = e^{-\mathrm{i}k_{2q}(x_2+L)}\delta_q +$$

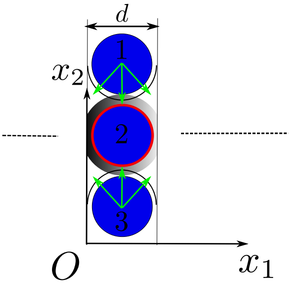
$$\sum_{o>j}\sum_{q\in\mathbb{Z}}\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(o)} K_{qn}^{+} e^{\mathrm{i}k_{1q}(x_1 - x_1^o) + \mathrm{i}k_{2q}(x_1 - x_2^o)} +$$

$$\sum_{o<j}\sum_{q\in\mathbb{Z}}\sum_{n\in\mathbb{Z}} \mathcal{B}_n^{(o)} K_{qn}^{-} e^{\mathrm{i}k_{1q}(x_1 - x_1^0) - \mathrm{i}k_{2q}(x_2 - x_2^0)}.$$

- Application of the BC on $\Gamma^{\pm}$

$$R_q = \sum_{j\in\mathcal{J}} \mathcal{B}_n^{(j)} K_{qn}^{+} e^{-\mathrm{i}k_{1q}x_1^0 - \mathrm{i}k_{2q}(x_2^0 - L)}$$
$$T_q = \sum_{j\in\mathcal{J}} \mathcal{B}_n^{(j)} K_{qn}^{-} e^{-\mathrm{i}k_{1q}x_1^0 + \mathrm{i}k_{2q}(x_2^0 - L)} + e^{\mathrm{i}k_{2q}L}\delta_q$$
$$f_q^+ = 0, \text{ and } f_q^- = e^{\mathrm{i}k_{2q}L}\delta_q$$





- Application of the BC on the cylinders
  - change of coordinate system $\tilde{x}_h = x_h - x_h^j$, $h = 1, 2$
  - coordinate type: cartesian $\rightarrow$ cylindrical

$$e^{\mathrm{i}k_{1q}\tilde{x}_1 \pm \mathrm{i}k_{1q}\tilde{x}_2} = \sum_{n\in\mathbb{Z}} J_{qn}^{\pm} \mathrm{J}_n(k_j)\, e^{\mathrm{i}n\theta_j},$$

where $J_{qn}^{\pm} = (\mathrm{i})^m e^{\mp \mathrm{i}\theta_q}$.

Final system

Formaly the pressure field for $\min_{o\neq j}\left(r_j^o - R^{(o)}\right)$ reads as

$$p^{[0]}(\mathbf{r}) = \sum_{n\in\mathbb{Z}}\left(\mathcal{B}_n^{(0)}\mathrm{H}_n^{(1)}(kr_0) + \underbrace{\left(\sum_{l\in\mathbb{Z}}\mathcal{B}_l^{(j)}\mathcal{S}_{n-q}^{j} + \sum_{o\neq j}\sum_{l\in\mathbb{Z}}\mathcal{B}_l^{(o)}\mathcal{S}_{n,l}^{(o,j)} + \mathcal{A}_n^{0i}\right)}_{\mathcal{A}_n}\mathrm{J}_n(kr_0)\right)e^{\mathrm{i}n\theta_0},$$

and we may apply again the realtion $\mathcal{B}_n^j = \mathcal{S}\mathcal{C}_n^j\mathcal{A}_n$.

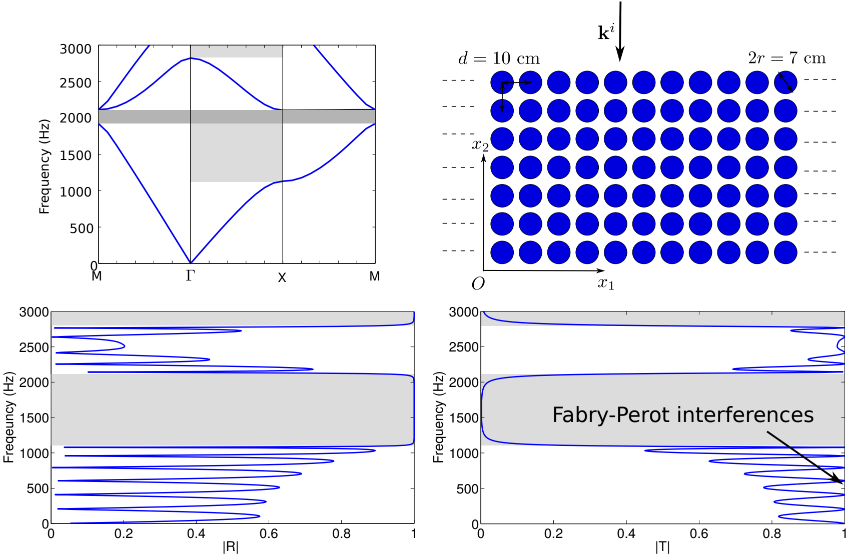
In this case $\mathcal{S}_{n,l}^{(o,j)} = \sum_{q\in\mathbb{Z}}\dfrac{2(-\mathbf{i})^{n-l}e^{\pm\mathbf{i}(n-l)\theta_q}}{dk_{2q}}e^{\mathbf{i}k_{1q}(x_1^j-x_1^o)\pm\mathbf{i}k_{2q}(x_2^j-x_2^o)}$, $(+ : x_2^j > x_2^o)$.

More complete expression may be found in Groby *et al.*, J.Acoust.Soc.Am., 11

Final system for the solution of $\mathcal{B}_n^{(j)}$, $\forall n\in\mathbb{Z}$ and $\forall j\in\mathcal{J}$

$$\begin{bmatrix} \mathbf{Id}-\mathbf{S} & -\mathbf{S}_1^2 & \cdot & \cdot & \cdot & -\mathbf{S}_1^{J-1} & -\mathbf{S}_1^J \\ -\mathbf{S}_2^1 & \mathbf{Id}-\mathbf{S} & \cdot & \cdot & \cdot & -\mathbf{S}_2^{J-1} & -\mathbf{S}_2^J \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\mathbf{S}_{J-1}^1 & -\mathbf{S}_{J-1}^2 & \cdot & \cdot & \cdot & \mathbf{Id}-\mathbf{S} & -\mathbf{S}_{J-1}^J \\ -\mathbf{S}_J^1 & -\mathbf{S}_J^2 & \cdot & \cdot & \cdot & -\mathbf{S}_J^{J-1} & \mathbf{Id}-\mathbf{S} \end{bmatrix}\begin{bmatrix}\mathbf{B}^1\\ \mathbf{B}^2\\ \cdot\\ \cdot\\ \cdot\\ \mathbf{B}^{J-1}\\ \mathbf{B}^J\end{bmatrix} = \begin{bmatrix}\mathbf{A}^1\\ \mathbf{A}^2\\ \cdot\\ \cdot\\ \cdot\\ \mathbf{A}^{J-1}\\ \mathbf{A}^J\end{bmatrix}.$$

Example: 7 rows sonic crystal of inifinte latteral extend

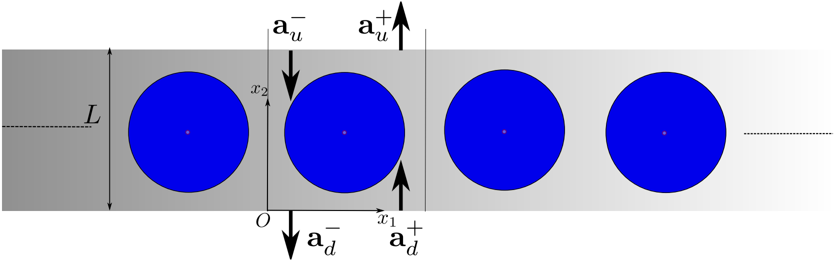


Band diagram calculation

Various methods

- Plane Wave Expansion (as presented previously by J. Vasseur)
  $\Rightarrow$ easy to use (eigenvalue problem)
  $\Rightarrow$ limited to lossless cases and a single type of material per unit cell
- Extended Plane Wave Expension
  $\Rightarrow$ easy to use (eigenvalue problem)
  $\Rightarrow$ single type of material per unit cell
- Method based on the Multiple Scattering Theory
  - Scattering Matrix: Botten *et al.*, Phys. Rev. E, 64:046603, 2001
    $\Rightarrow$ implicit in term of Bloch wave
  - 2D periodic Green's function: Poulton *et al.*, Proc. R. Soc. Lond. A, 456:2543-2559, 2000
    $\Rightarrow$ implicit in term of Bloch wave

Obtention of the eigenvalue problem



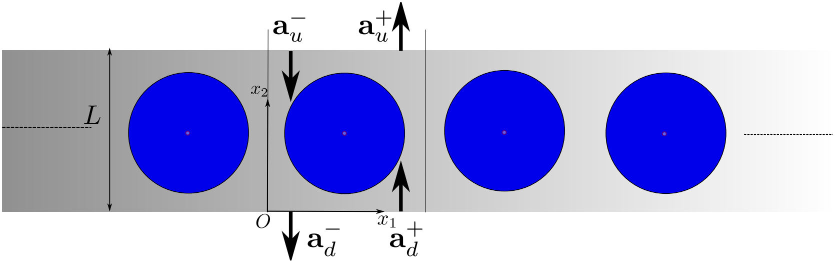
We have

$$p_u = \sum_{q\in\mathbf{Z}} \left(a_{uq}^- e^{-\mathbf{i}k_{2q}(x_2-L)} + a_{uq}^+ e^{\mathbf{i}k_{2q}(x_2-L)} \right) e^{\mathbf{i}k_{1q}x_1}$$

$$p_d = \sum_{q\in\mathbf{Z}} \left(a_{dq}^+ e^{\mathbf{i}k_{2q}x_2} + a_{dq}^- e^{-\mathbf{i}k_{2q}x_2} \right) e^{\mathbf{i}k_{1q}x_1}$$

The terms $a_{uq}^{\pm}$ and $a_{dq}^{\pm}$ may be arranged in $\mathbf{a}_u^{\pm}$ and $\mathbf{a}_d^{\pm}$.

Obtention of the eigenvalue problem

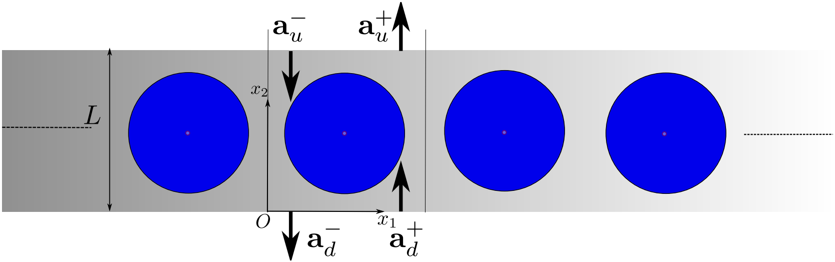


Because of the orthogonality of the Bloch waves, we write:

$$\begin{bmatrix} \mathbf{a}_u^+ \\ \mathbf{a}_u^- \end{bmatrix} = e^{\mathrm{i}k_{2B}L} \begin{bmatrix} \mathbf{a}_d^+ \\ \mathbf{a}_d^- \end{bmatrix} ,$$

where k_{2B} is the projection of the Bloch wave number k_B along x_2, such that $k_B = \sqrt{(k_1^i)^2 + k_{2B}^2}$.

Obtention of the eigenvalue problem



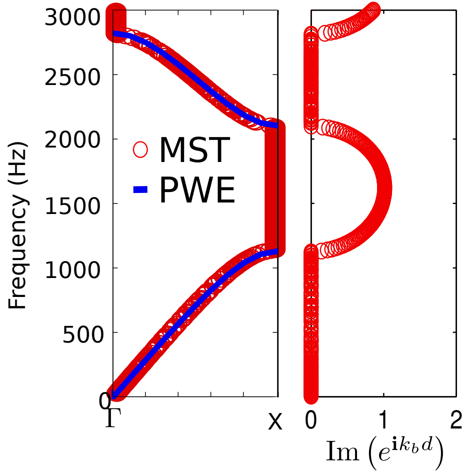
R_q^Q and T_q^Q might be calculated when the array is solicited by the Q-th Bloch wave. Therefore, we may construct a matrices **R** and **T**, and we have, again thanks to the orthogonality of the Bloch waves:

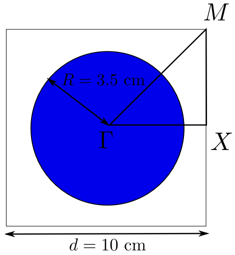
$$\begin{bmatrix} \mathbf{a}_u^+ \\ \mathbf{a}_u^- \end{bmatrix} = \begin{bmatrix} \mathbf{T} & \mathbf{R} \\ \mathbf{0} & \mathbf{Id} \end{bmatrix} \begin{bmatrix} \mathbf{a}_d^+ \\ \mathbf{a}_u^- \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \mathbf{a}_d^+ \\ \mathbf{a}_d^- \end{bmatrix} = \begin{bmatrix} \mathbf{Id} & \mathbf{0} \\ \mathbf{R} & \mathbf{T} \end{bmatrix} \begin{bmatrix} \mathbf{a}_d^+ \\ \mathbf{a}_u^- \end{bmatrix}.$$

So, we end with the following eigenvalue problem

$$\begin{bmatrix} \mathbf{Id} & \mathbf{0} \\ \mathbf{R} & \mathbf{T} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{T} & \mathbf{R} \\ \mathbf{0} & \mathbf{Id} \end{bmatrix} \begin{bmatrix} \mathbf{a}_d^+ \\ \mathbf{a}_u^- \end{bmatrix} = e^{\mathrm{i}k_{2B}L} \begin{bmatrix} \mathbf{a}_d^+ \\ \mathbf{a}_u^- \end{bmatrix}.$$

Example of band diagram





Along ΓX, $k_1^i = 0$.

⇒ Multiple Scattering might also be used to calculate band diagram

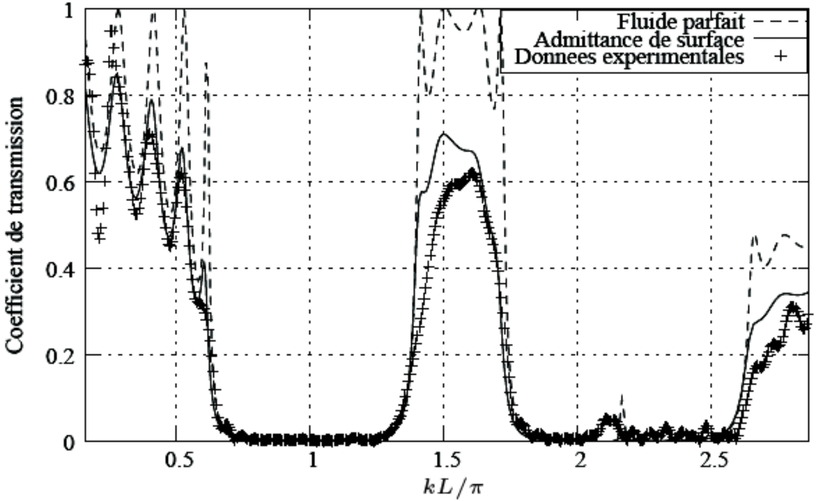
⇒ EquiFrequency Surface: might be complicated along the *XM* direction

Conclusion

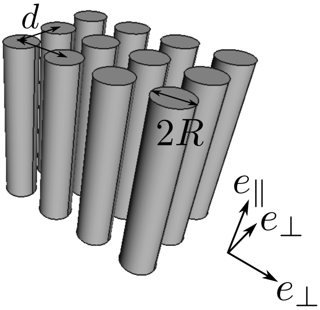
- Multiple Scattering Theory is an efficient tool for
  - calculating the response of finite dimension sonic crystals,
  - calculating the response of finite depth sonic crystals.
- Multiple Scattering Theory might also be used for band diagram calculation
- Multiple Scattering Theory might also be used in
  - phononic crystals
  - metaporous materials
  - vibroacoustics
  - ...
- Multiple Scattering Theory is efficient for cylindrical (ovaidal) shape scatterers

You have to take care of the losses!

$R = 1$ mm and $\phi = 0.5$, Duclos *et al.*, *EPJAP, 2009*



Comments on the longwavelength limit



In the longwavelength limit $\lambda \gg d$, the viscothermal problem reduces to

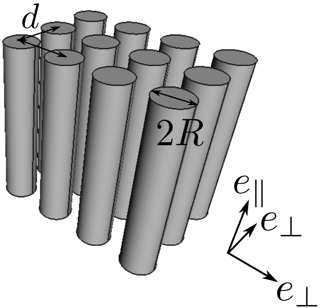
$$\begin{cases} \mathrm{i}\omega \begin{bmatrix} \rho_\parallel & 0 & 0 \\ 0 & \rho_\perp & 0 \\ 0 & 0 & \rho_\perp \end{bmatrix} \cdot \mathbf{V} = \nabla p, \\ \mathrm{i}\omega p = K \nabla \cdot \mathbf{V}, \end{cases}$$

where (Johnson-Lafarge model)

$$\begin{cases} \rho_j = \dfrac{\alpha_{\infty j}}{\phi}\left(1 + \dfrac{1}{\mathrm{i}\tilde{\omega}_j}\sqrt{1 + \mathrm{i}\tilde{\omega}_j \dfrac{M_j}{2}}\right), \text{ with } j = \perp, \parallel, \\ K = \dfrac{\gamma P_0}{\phi\left(\gamma - (\gamma - 1)\left[1 + \dfrac{1}{\mathrm{i}\tilde{\omega}'}\sqrt{1 + \mathrm{i}\tilde{\omega}'\dfrac{M'}{2}}\right]\right)}, \end{cases}$$

with $M' = \dfrac{8k_0'}{\phi \Lambda'^2}$, $\omega' = \dfrac{\omega k_0'}{\nu' \phi}$, $M_j = \dfrac{8\alpha_{\infty j} k_{0j}}{\phi \Lambda_j^2}$, $\omega' = \dfrac{\omega k_{0j} \alpha_{\infty j}}{\nu \phi}$, $j = \perp, \parallel$.

Comments on the longwavelength limit



In the longwavelength limit $\lambda \gg d$, the viscothermal problem reduces to

$$\begin{cases} i\omega \begin{bmatrix} \rho_{\|} & 0 & 0 \\ 0 & \rho_{\perp} & 0 \\ 0 & 0 & \rho_{\perp} \end{bmatrix} \cdot \mathbf{V} = \nabla p, \\ i\omega p = K \nabla \cdot \mathbf{V}, \end{cases}$$

where (Johnson-Lafarge model)

$$\phi = \frac{\pi R^2}{d^2}, \qquad \Lambda' = \frac{R\phi}{1-\phi}, \qquad k_0' = R^2 \frac{-2\log(1-\phi) - 2\phi - \phi^2}{8(1-\phi)},$$

$$\alpha_{\infty\perp} = 2 - \phi, \qquad \Lambda_{\perp} = R\frac{\phi(2-\phi)}{2(1-\phi)}, \qquad k_{0\perp} = R^2 \frac{-2\log(1-\phi) - 2\phi - \phi^2}{16(1-\phi)},$$

$$\alpha_{\infty\|} = 1, \qquad \Lambda_{\|} = \Lambda', \qquad k_{0\|} = k_0',$$

for dilute arrangement ($\phi > 0.5$) Tarnow, J. Acoust. Soc. Am., 1997.

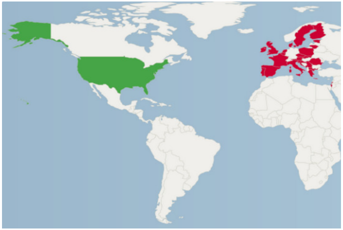
DENORMS Action (CA 15125)

DENORMS (Designs for Noise Reducing Materials and Structures) aims at **designing multifunctional, light and compact noise reducing treatments**, which will be used in **realistic environments**.

DENORMS **brings together** skills and knowledge of the complementary communities of scientists working on **acoustic metamaterials, sonic crystals and conventional acoustic materials** across Europe and overseas.

⇒ 3 interacting Working Groups (WG)

- WG1. Modelling of sound interaction with noise reducing materials and structures
- WG2. Experimental techniques
- WG3. Industrial applications



DENORMS currently gathers Laboratories and Industrial partners from 27 Participating countries and 1 (4) International Parner Country.

⇒ DENORMS Activities for 2017-2018

- Workshop *Brainstorming*, Novi-Sad, 14-15th Sept 2017.
- Training School *Experiments on porous materials and acoustic metamaterials*, Le Mans, 4th-6th Dec. 2017
- Workshop *Experimental techniques in porous materials and acoustic metamaterials*, Leuven, 7th-9th Feb 2018

Further information on
https://denorms.eu/
Contact denorms@univ-lemans.fr

Funded by the Horizon 2020 Framework Programme of the European Union

Symposium on the Acoustics of
Poro-Elastic Materials
December 6-7-8, 2017
Le Mans (France)

Welcome to SAPEM 2017

University of Le Mans and the Laboratory of Acoustics are happy to welcome you for the fifth edition of the Symposium on the Acoustics of Poro-Elastic Materials to be held on the 6th, 7th and 8th of December 2017.

The symposium aims:

- presenting the most up-to-date researches in modelling, characterisation
- promoting industrial applications of porous materials,
- bringing together researchers and engineers working in adjacent disciplines concerned with porous media,
- discussing future challenges for this area of research.

During this edition, a special focus will be put on metamaterials involving porous media and viscothermal dissipation and on the interaction of porous materials with flow.

Program

The non-parallel sessions of this fifth edition will include:

- Physical models for porous materials
- Experimental methods for porous materials
- Numerical models for porous materials
- Industrial applications
- Metamaterials with porous materials
- Interaction of Porous materials and flows

Special attention will be paid to reserving time for discussion.
A fruitful poster session will be organised alongside with exhibition by our industrial partners.

International Scientific Committee

Keith Attenborough (The Open-University London, UK)
Yves Aurégan (LAUM - CNRS, France)
Susan Boij (KTH, Sweden)
Stuart Bolton (Purdue University, USA)
Laurent De Ryck (LMS Siemens, Belgium)
Elke Deckers (KU Leuven Belgium)
Ludovic Desvard (Dyson, UK)
Arnaud Duval (Treves, France)
Peter Göransson (KTH, Sweden)
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Claude Boutin (ENTPE - CNRS)

Abstract submission

There is no requirement to submit a full paper, but presenters should submit an abstract of up to 1,500 words (about two pages), including details of their results and references. The Conference Committee reserves the right to decline submissions that are not in line with the objectives of the Symposium.

Presentations will be collected and compiled in a digital form. Selected papers will be recommended for publication in a special section of a peer-review journal.

Registration

The full registration fee is € 460, € 410 for EAA members and € 310 for students. It covers attendance, instructional materials, proceedings, coffee breaks, lunches and social events. There will be a € 60 discount off the full registration fee for registration made prior to October 20th 2017.
Registration must be carried out following the link available on the Conference website :

http://sapem2017.matelys.com

Important dates

Deadline for abstract submissions: Sep. 8th 2017
Deadline for early bird registrations: Oct. 20th 2017
Deadline for registrations: Nov. 17th 2017

Contacts

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Thank you for your attention.

Any questions?