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Department of Materials Science and Engineering

Topological acoustics

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NewFoS
NEW FRONTIERS OF SOUND

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Past 300 years

PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

AUCTORE
ISAACO NEWTONO, Eq. Aur.

Editio tertia aucta & emendata.

LONDINI:
Apud GUIL. & JOH. INNYS, Regiæ Societatis typographos.
MDCCXXVI.

PROPOSITIO XLIII. THEOREMA XXXIV.

Corpus omne tremulum in medio elastico propagabit motum pulsuum undique in directum; in medio vero non elastico motum circularem excitabit.

THE
THEORY OF SOUND

BY

JOHN WILLIAM STRUTT, BARON RAYLEIGH, Sc.D., F.R.S.
HONORARY FELLOW OF TRINITY COLLEGE, CAMBRIDGE.

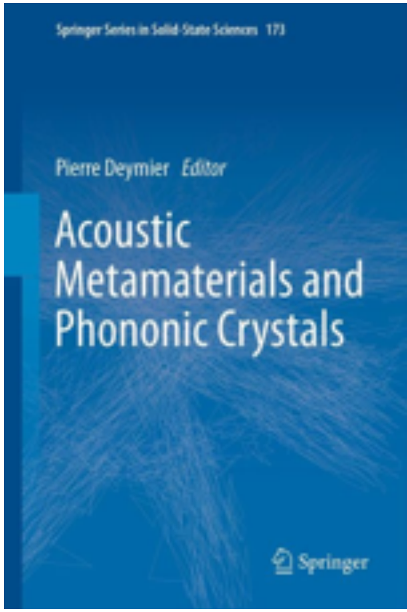
IN TWO VOLUMES

VOLUME II.

SECOND EDITION REVISED AND ENLARGED

London
MACMILLAN AND CO., LTD.
NEW YORK: MACMILLAN & CO.
1896

Phononics and acoustic metamaterials: The past 25 years



$$u(\vec{r},t)=u_0 e^{i\vec{k}\vec{r}}\, e^{i(\omega t+\varphi)}$$

Spectral Properties (*ω*-space)

Exploitation of partial/complete band gaps

Stop bands → insulators to acoustic/elastic waves

Narrow pass bands → frequency filtering

Defected structures → wave-guiding & mode-localization

Wave Vector Properties (*k*-space)

Negative Refraction

Flat lenses → Focusing & Sub-wavelength imaging

Zero-angle Refraction

Collimation

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VOLUME 86, NUMBER 14 PHYSICAL REVIEW LETTERS 2 APRIL 2001

Spectral gaps

Experimental and Theoretical Evidence for the Existence of Absolute Acoustic Band Gaps in Two-Dimensional Solid Phononic Crystals

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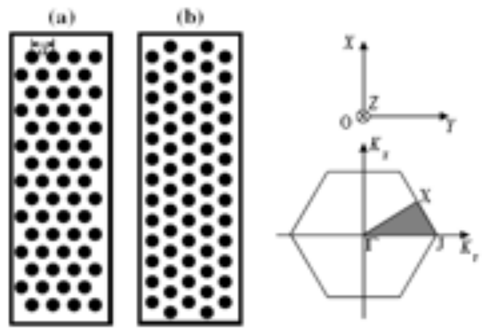

FIG. 1. Two-dimensional cross sections of the triangular array of steel cylinders embedded in an epoxy matrix: (a) the "ΓJ" sample and (b) the "ΓX" sample. The steel cylinders, of circular cross section, are parallel to the Z axis of the Cartesian coordinate system $(0, X, Y, Z)$. The lattice parameter a is defined as the distance between two nearest neighboring cylinders. The inset shows the irreducible Brillouin zone of the triangular array.

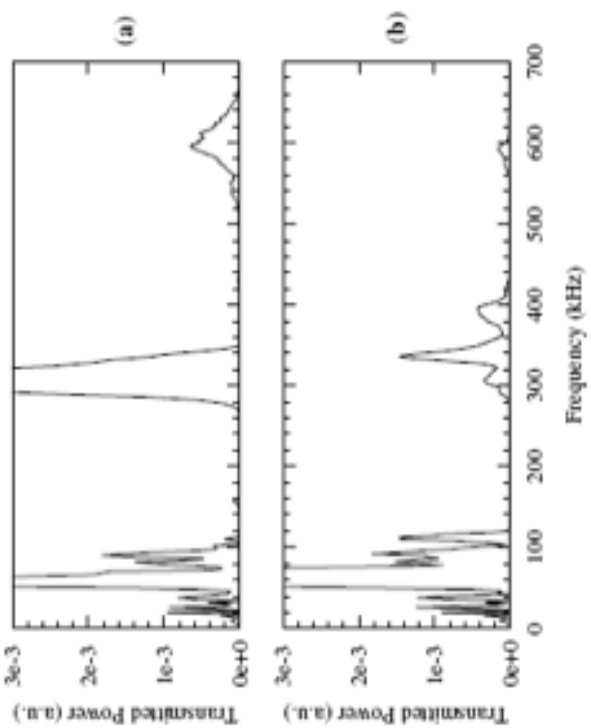

FIG. 2. Transmission power spectrum measured perpendicular to the vertical faces of the (a) "ΓJ" sample and (b) "ΓX" sample. The transmitted power is given in arbitrary units. The probing signal is a longitudinal wave.

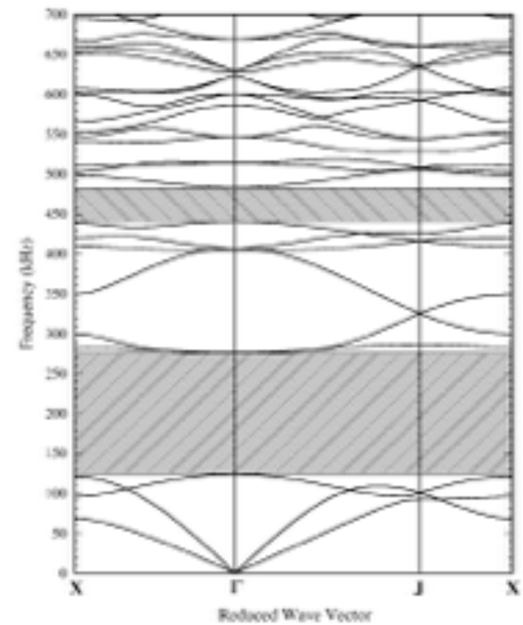

FIG. 3. PWE results for the band structure of the two-dimensional XY modes of vibration in the periodic triangular array of steel cylinders in an epoxy resin matrix for a filling fraction $f = 0.4$. The reduced wave vector $\vec{k}(k_X, k_Y)$ is defined as $\vec{K}a/2\pi$ where $\vec{K}$ is a two-dimensional wave vector. The points Γ, J, and X are defined in the inset of Fig. 1. Absolute band gaps are represented as hatched areas.

iemn Recherche Formation Transfert

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PRL 102, 154301 (2009) PHYSICAL REVIEW LETTERS week ending 17 APRIL 2009

Experimental and Theoretical Evidence for Subwavelength Imaging in Phononic Crystals

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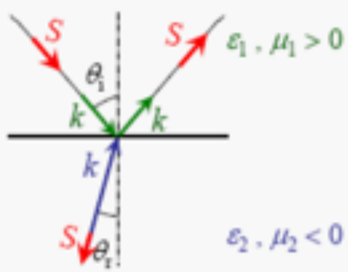
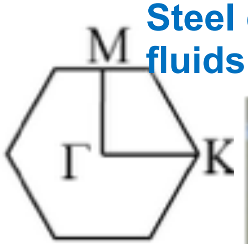
Wave vector domain

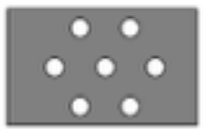
Veselago[1] also predicted the unusual phenomenon of negative refraction:

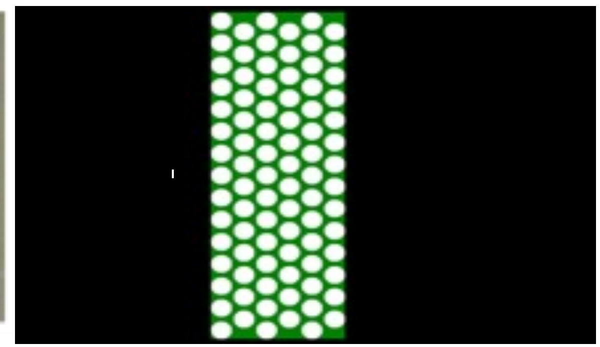


[1] V. G. Veselago, Sov. Phys. Usp. 92, 517 (1964)

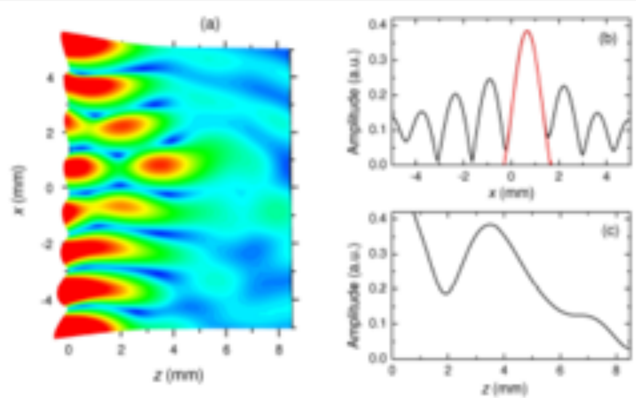
Steel cylindrical inclusions in fluids (methanol)





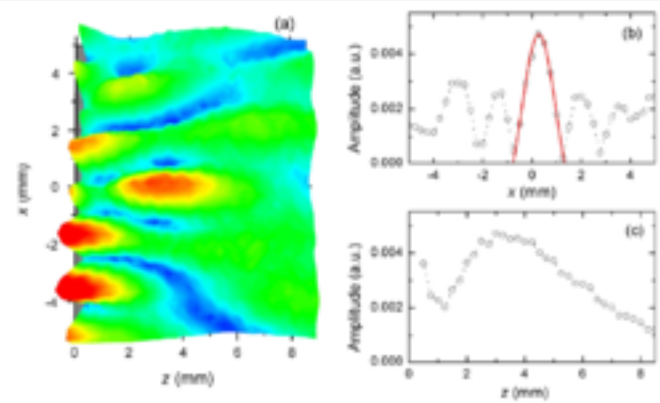


FDTD Model



(a) Calculated normalized absolute value of pressure, (b) Amplitude through focus along direction parallel to lens surface, (c) The data fit to a sinc function gives a half width of the primary peak of 0.35λ, (c) amplitude along the direction perpendicular to lens. Surface at Z=0.

Experiment



(a) Experimental field amplitude, (b) Amplitude through focus along direction parallel to lens surface, (c) The data fit to a sinc function gives a half width of the primary peak of 0.37λ, (c) amplitude along the direction perpendicular to lens. Surface at Z=0.

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Phase Domain symmetry breaking and Topology

$$u(\vec{r},t) = u_0 e^{i\vec{k}\vec{r}} e^{i(\omega t+\varphi)}$$

The Nobel Prize in Physics 2016

Photo: A. Mahmoud
David J. Thouless
Prize share: 1/2

Photo: A. Mahmoud
F. Duncan M. Haldane
Prize share: 1/4

Photo: A. Mahmoud
J. Michael Kosterlitz
Prize share: 1/4

The Nobel Prize in Physics 2016 was divided, one half awarded to David J. Thouless, the other half jointly to F. Duncan M. Haldane and J. Michael Kosterlitz *"for theoretical discoveries of topological phase transitions and topological phases of matter"*.

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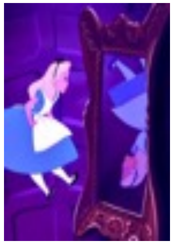
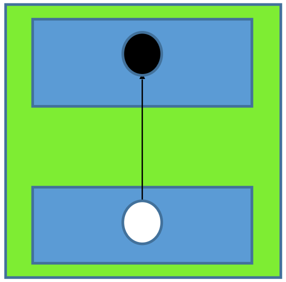
Symmetry breaking of elastic and acoustic waves equation

- **Time-reversal symmetry**

- **Parity symmetry**



- **Chiral symmetry**



- **Particle-hole symmetry**

Conventional 1D wave equation:

$$\frac{\partial^2 u}{\partial t^2} - \beta^2 \frac{\partial^2 u}{\partial x^2} = 0$$

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Intrinsic vs Extrinsic symmetry breaking

(a) intrinsic symmetry breaking occurs from the internal structural characteristics,

(b) extrinsic symmetry breaking occurs from an external stimulus such as spatio-temporal modulations of the physical properties of the medium.

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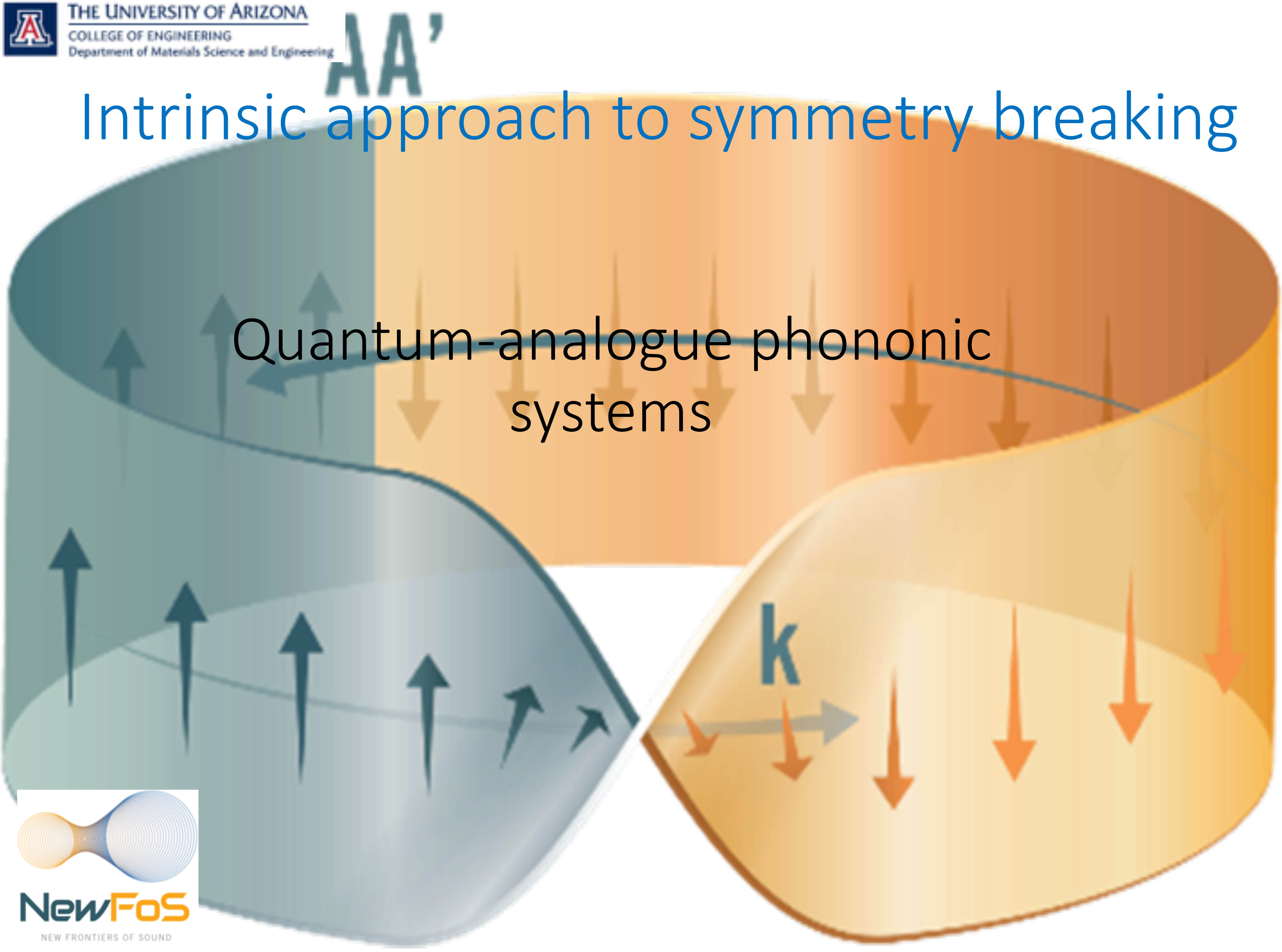
Intrinsic approach to symmetry breaking

Quantum-analogue phononic systems



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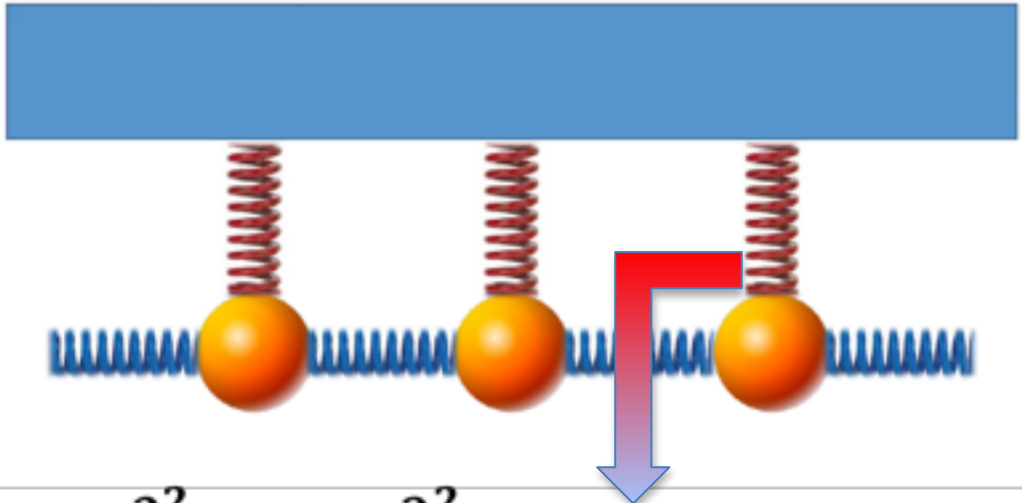
Intrinsic phononic structure



$$\frac{\partial^2 u}{\partial t^2} - \beta^2 \frac{\partial^2 u}{\partial x^2} + \alpha^2 u = 0$$

Relativistic quantum mechanics
Klein-Gordon Equation

Other examples of physical systems

Intrinsic acoustic and phononic structures

1-D Elastic wave equation with graded properties

$$\rho(x)\frac{\partial^2 \tilde{u}}{\partial t^2} - \frac{\partial}{\partial x}\left(E(x)\frac{\partial \tilde{u}}{\partial x}\right) = 0$$

$$\begin{cases} \rho(x) = \rho_0 S(x) \\ E(x) = E_0 S(x) \end{cases}$$

where $\beta^2 = \frac{E_0}{\rho_0}$

Graded elastic wave guide

Pressure wave in a duct of varying cross-sectional area $S(x)$

$$\frac{\partial^2 P}{\partial t^2} - c^2\left\{\frac{1}{S(x)}\frac{\partial}{\partial x}\left(S(x)\frac{\partial P}{\partial x}\right)\right\} = 0$$

where $\beta^2 = c^2$

Webster equation

$$\frac{\partial^2 \tilde{u}}{\partial t^2} - \beta^2\left\{\frac{1}{S(x)}\frac{\partial}{\partial x}\left(S(x)\frac{\partial \tilde{u}}{\partial x}\right)\right\} = 0$$

$$u = \tilde{u}S(x)^{1/2}$$

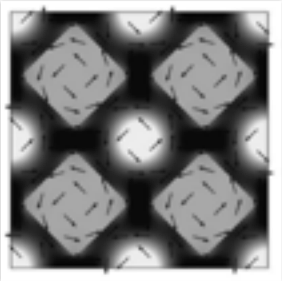
Generalized Klein-Gordon equation

$$\frac{\partial^2 u}{\partial t^2} - \beta^2\left\{\frac{\partial^2 u}{\partial x^2} - V(x)u\right\} = 0$$

with $V(x) = \left(d^2S(x)^{1/2}/dx^2\right)/S(x)^{1/2}$

$$S(x) = e^{ax}$$

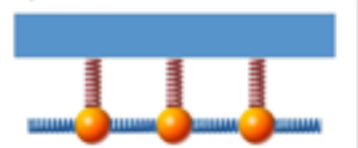
with $\alpha^2 = a\beta^2$



Rotational modes in phononic crystals

Relativistic Klein-Gordon equation

$$\frac{\partial^2 u}{\partial t^2} - \beta^2\frac{\partial^2 u}{\partial x^2} + \alpha^2 u = 0$$



1D mass-spring harmonic system
+
Plate modes

1-D Dirac equation

$$\left[\boldsymbol{\sigma}_x\frac{\partial}{\partial t} + i\beta\boldsymbol{\sigma}_y\frac{\partial}{\partial x} \pm i\alpha\boldsymbol{I}\right]\psi = 0$$

$\boldsymbol{\sigma}_x$ and $\boldsymbol{\sigma}_y$ are the 2x2 Pauli matrices and $\boldsymbol{I}$ is the 2x2 identity matrix

Dirac-like equation

$$\left[\boldsymbol{\sigma}_x \frac{\partial}{\partial t} + i\beta \boldsymbol{\sigma}_y \frac{\partial}{\partial x} - i\alpha I\right]\Psi = 0 \qquad \text{“particle”}$$

$$\left[\boldsymbol{\sigma}_x \frac{\partial}{\partial t} + i\beta \boldsymbol{\sigma}_y \frac{\partial}{\partial x} + i\alpha I\right]\bar{\Psi} = 0 \qquad \text{“anti-particle”}$$

σ_x and σ_y are the 2x2 Pauli matrices: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and I is the 2x2 identity matrix.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Some matrix algebra

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$$

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Dirac-like equation

$$\left[\boldsymbol{\sigma}_x \frac{\partial}{\partial t} + i\beta \boldsymbol{\sigma}_y \frac{\partial}{\partial x} - i\alpha I\right]\Psi = 0$$ "particle"

$$\left[\boldsymbol{\sigma}_x \frac{\partial}{\partial t} + i\beta \boldsymbol{\sigma}_y \frac{\partial}{\partial x} + i\alpha I\right]\bar{\Psi} = 0$$ "anti-particle"

$t \rightarrow -t$ alone changes the equations, *time-reversal symmetry is broken*

$x \rightarrow -x$ alone changes the equations, *parity symmetry is broken*

$t \rightarrow -t$ and $x \rightarrow -x$ does not change the equations, *time-reversal and parity symmetry are not broken simultaneously*

Spinor solutions

Eigen vectors:
Plane wave solutions

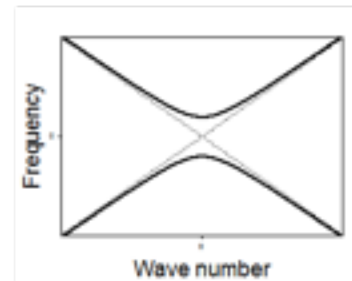
$$\begin{cases} \psi_k = \psi(k,\omega_k) = c_0 \xi_k(k,\omega_k) e^{(\pm)i\omega_k t} e^{(\pm)ikx} \\ \bar{\psi}_k = \bar{\psi}(k,\omega_k) = c_0 \bar{\xi}_k(k,\omega_k) e^{(\pm)i\omega_k t} e^{(\pm)ikx} \end{cases}$$

where ξ_k and $\bar{\xi}_k$ are two by one spinors with $\xi_k = \begin{pmatrix} \xi_{k,L} \\ \xi_{k,R} \end{pmatrix}$

$$\omega = \pm\sqrt{\alpha^2 + \beta^2 k^2}$$

Eigen values:
Dispersion relation

Band structure is symmetrical ($\pm$), *particle-hole (antiparticle) symmetry is **not** broken*



	$e^{+ikx}e^{+i\omega_k t}$	$e^{-ikx}e^{+i\omega_k t}$	$e^{+ikx}e^{-i\omega_k t}$	$e^{-ikx}e^{-i\omega_k t}$
ξ_k	$\begin{pmatrix} \sqrt{\omega+\beta k} \\ \sqrt{\omega-\beta k} \end{pmatrix}$	$\begin{pmatrix} \sqrt{\omega-\beta k} \\ \sqrt{\omega+\beta k} \end{pmatrix}$	$\begin{pmatrix} -\sqrt{\omega-\beta k} \\ \sqrt{\omega+\beta k} \end{pmatrix}$	$\begin{pmatrix} -\sqrt{\omega+\beta k} \\ \sqrt{\omega-\beta k} \end{pmatrix}$
$\bar{\xi}_k$	$\begin{pmatrix} \sqrt{\omega-\beta k} \\ -\sqrt{\omega+\beta k} \end{pmatrix}$	$\begin{pmatrix} \sqrt{\omega+\beta k} \\ -\sqrt{\omega-\beta k} \end{pmatrix}$	$\begin{pmatrix} \sqrt{\omega+\beta k} \\ \sqrt{\omega-\beta k} \end{pmatrix}$	$\begin{pmatrix} \sqrt{\omega-\beta k} \\ \sqrt{\omega+\beta k} \end{pmatrix}$

Left (L) propagation and right (R) propagations are note independent, *chiral symmetry is broken*

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Spinor solutions (2)

Symmetry

$$\underset{k\to -k}{T_{\omega\to\omega}}(\Psi(\omega,k)) = \Psi(\omega,k)$$

$$\underset{k\to k}{T_{\omega\to -\omega}}(\Psi(\omega,k)) = i\sigma_x\bar{\Psi}(-\omega,k)$$

$$\underset{k\to -k}{T_{\omega\to -\omega}}(\Psi(\omega,k)) = i\sigma_x\bar{\Psi}(-\omega,-k)$$

$\underset{k\to -k}{T_{\omega\to\omega}}$ and $\underset{k\to k}{T_{\omega\to -\omega}}$ are transformations that change the sign of the frequency and wave number.

Orthogonality condition $\bar{\Psi}\sigma_x\Psi = 0$

Topology

Parallel transport of a vector field on a manifold with ¼ turn twist ("i" leads to a phase of π/2)

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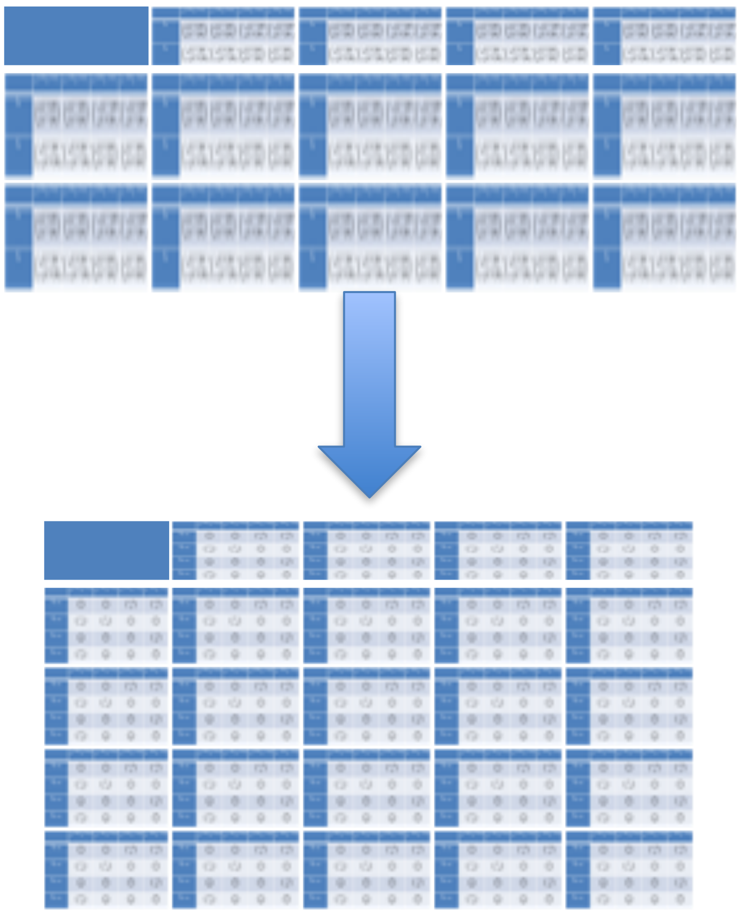
Pseudospin φ-bit



Spin-like states in the direction of propagation of elastic states (forward $|F\rangle = \binom{1}{0}$ or $|0\rangle$ and backward $|B\rangle = \binom{0}{1}$ or $|1\rangle$) and crucially superposition of states: $\left(s_1\sqrt{\omega \pm \beta k}\right)|0\rangle +$ $\left(s_2\sqrt{\omega \mp \beta k}\right)|1\rangle$. The superposition of states is tunable by frequency, ω and/or wavenumber k.

An analogue to qubits necessary for quantum computing

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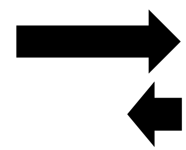
The Phi-Bit

Spin Polarization

20% 0 **80% 1**

Spin

Direction of Propagation



80% 1
20% 0

Pseudo-spin

Elastic Pseudospin

C. R. Mecanique 343 (2015) 700–711

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ELSEVIER

Acoustic metamaterials and phononic crystals

Torsional topology and fermion-like behavior of elastic waves in phononic structures

CrossMark

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ABSTRACT

A one-dimensional block-spring model that supports rotational waves is analyzed within Dirac formalism. We show that the wave functions possess a spinor and a spatio-temporal part. The spinor part leads to a non-conventional torsional topology of the wave function. In the long-wavelength limit, field theoretical methods are used to demonstrate that rotational phonons can exhibit fermion-like behavior. Subsequently, we illustrate how information can be encoded in the spinor-part of the wave function by controlling the phonon wave phase.

crystals

MDPI

Article

One-Dimensional Mass-Spring Chains Supporting Elastic Waves with Non-Conventional Topology

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Abstract: There are two classes of phononic structures that can support elastic waves with non-conventional topology, namely intrinsic and extrinsic systems. The non-conventional topology of elastic wave results from breaking time reversal symmetry (T-symmetry) of wave propagation. In extrinsic systems, energy is injected into the phononic structure to break T-symmetry. In intrinsic systems symmetry is broken through the medium microstructure that may lead to internal resonances. Mass-spring composite structures are introduced as metaphors for more complex phononic crystals with non-conventional topology. The elastic wave equation of motion of an intrinsic phononic structure composed of two coupled one-dimensional (1D) harmonic chains can be factored into a Dirac-like equation, leading to antisymmetric modes that have spinor character and therefore non-conventional topology in wave number space. The topology of the elastic waves can be further modified by subjecting phononic structures to externally-induced spatio-temporal modulation of their elastic properties. Such modulations can be actuated through photo-elastic effects, magneto-elastic effects, piezo-electric effects or external mechanical effects. We also uncover an analogy between a combined intrinsic-extrinsic systems composed of a simple one-dimensional harmonic chain coupled to a rigid substrate subjected to a spatio-temporal modulation of the side spring stiffness and the Dirac equation in the presence of an electromagnetic field. The modulation is shown to be able to tune the spinor part of the elastic wave function and therefore its topology. This analogy between classical mechanics and quantum phenomena offers new modalities for developing more complex functions of phononic crystals and acoustic metamaterials.

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Pseudospin φ-bit

1-D Elastic wave equation with graded properties

$$\rho(x)\frac{\partial^2 \tilde{u}}{\partial t^2} - \frac{\partial}{\partial x}\left(E(x)\frac{\partial \tilde{u}}{\partial x}\right) = 0 \quad \begin{cases} \rho(x) = \rho_0 S(x) \\ E(x) = E_0 S(x) \end{cases}$$

Webster equation

$$\frac{\partial^2 \tilde{u}}{\partial t^2} - \beta^2\left\{\frac{1}{S(x)}\frac{\partial}{\partial x}\left(S(x)\frac{\partial \tilde{u}}{\partial x}\right)\right\} = 0 \text{ where } \beta^2 = \frac{E_0}{\rho_0}$$

$$u = \tilde{u}S(x)^{1/2}$$

Making the quantum analogue

Generalized Klein-Gordon equation

$$\frac{\partial^2 u}{\partial t^2} - \beta^2\left\{\frac{\partial^2 u}{\partial x^2} - V(x)u\right\} = 0$$

with $V(x) = \left(d^2 S(x)^{1/2}/dx^2\right)/S(x)^{1/2}$

$$S(x) = e^{ax}$$

Relativistic Klein-Gordon equation

$$\frac{\partial^2 u}{\partial t^2} - \beta^2\frac{\partial^2 u}{\partial x^2} + \alpha^2 u = 0 \quad \text{with} \quad \alpha^2 = a\beta^2$$

1-D Dirac equation

$$\left[\boldsymbol{\sigma}_x\frac{\partial}{\partial t} + i\beta\boldsymbol{\sigma}_y\frac{\partial}{\partial x} \pm i\alpha\boldsymbol{I}\right]\psi = 0$$

$\boldsymbol{\sigma}_x$ and $\boldsymbol{\sigma}_y$ are the 2x2 Pauli matrices and $\boldsymbol{I}$ is the 2x2 identity matrix

Spin-like states in the direction of propagation of elastic states (forward $|F\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ or $|0\rangle$ and backward $|B\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ or $|1\rangle$) and crucially superposition of states: $\left(s_1\sqrt{\omega \pm \beta k}\right)|0\rangle + \left(s_2\sqrt{\omega \mp \beta k}\right)|1\rangle$. The superposition of states is tunable by frequency, ω and/or wavenumber k.

$$\omega = \pm\sqrt{\alpha^2 + \beta^2 k^2}$$

1Plane wave solutions

$$\begin{cases} \psi_k = \psi(k,\omega_k) = c_0\xi_k(k,\omega_k)e^{(\pm)i\omega_k t}e^{(\pm)ikx} \\ \bar{\psi}_k = \bar{\psi}(k,\omega_k) = c_0\bar{\xi}_k(k,\omega_k)e^{(\pm)i\omega_k t}e^{(\pm)ikx} \end{cases}$$

where ξ_k and $\bar{\xi}_k$ are two by one spinors

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Observables

Number operator

$$N = \int dx\, \bar{\psi}\sigma_x \psi$$

Direction switching operators

$$S_+ = \frac{1}{2}(\sigma_x + i\sigma_y) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$S_- = \frac{1}{2}(\sigma_x - i\sigma_y) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Direction occupancy

$$n_k^+ = \xi_k \frac{1}{2\omega} S_+ S_- \sigma_x \xi_k$$

$$n_k^- = \xi_k \frac{1}{2\omega} S_- S_+ \sigma_x \xi_k$$

$$n_k^{\pm} = \frac{1}{2} \pm \frac{\beta k/\alpha}{2\sqrt{1 + \left(\frac{\beta k}{\alpha}\right)^2}}$$

"Amount" of traveling wave character of the wave function

$$(n_k^+ - n_k^-) = \frac{\sqrt{\omega^2 - \alpha^2}}{\omega}$$

$$(n_k^+ - n_k^-)^{0.5}$$

Transmission coefficient is measurable

In contrast with quantum superposition, an elastic superposition of states

$$\left(s_1\sqrt{\omega \pm \beta k}\right)|0\rangle + \left(s_2\sqrt{\omega \mp \beta k}\right)|1\rangle$$

is measurable directly through the transmission coefficient, without need for wave function collapse.

For frequencies 100kHz to 1MHz wavelength is cm to mm which is significantly larger than possible defect scattering length:
Signal to noise ratio ~ 10^{+3}.

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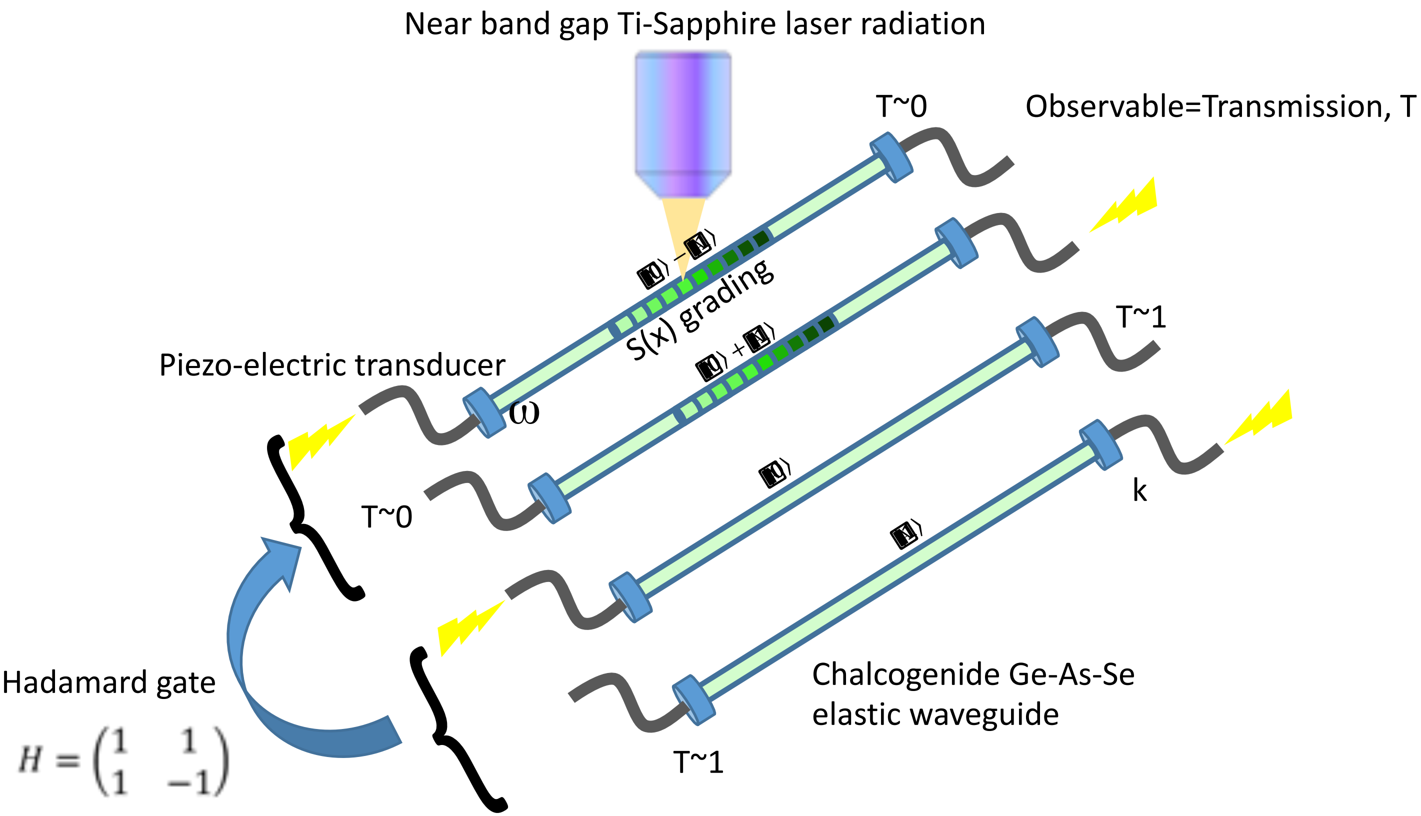
Physical realization and operation of a φ-bit



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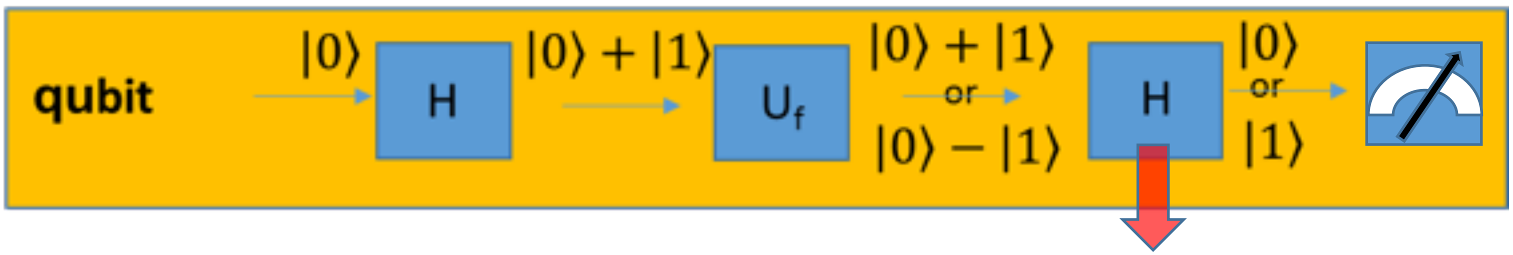
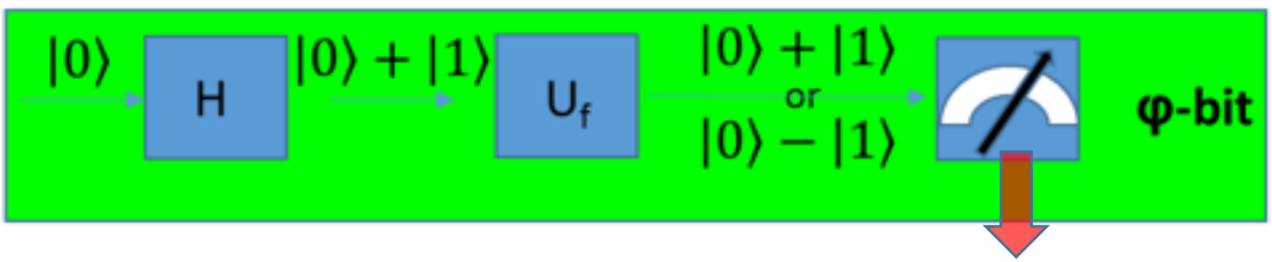
Qubit vs φ-bit Deutsch-Jozsa algorithm

Deutsch-Jozsa Objective: Determine if the function $f:\{0,1\} \rightarrow \{0,1\}$ is constant or balanced.



Hadamard gate applied a second time to ‘collapse’ the wavefunction.



No second Hadamard gate is needed, the superposition of states is read directly.

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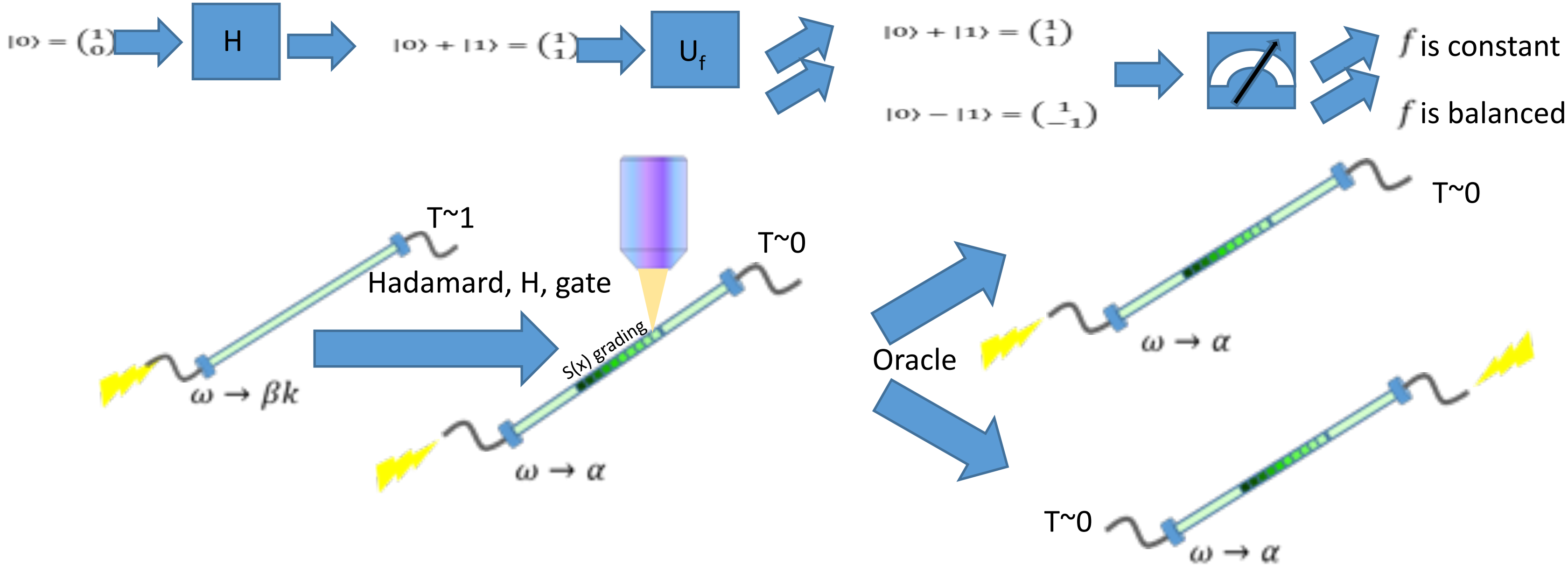
Single φ-bit Deutsch-Jozsa algorithm

Step 1: Apply Hadamard gate **Step 2**: Oracle unitary transformation **Step 3**: Measure superposition of states



Note: In a qubit-based algorithm, the superpositions of states $|0\rangle + |1\rangle$ or $|0\rangle - |1\rangle$ cannot be measured. One needs to apply the Hadamard gate after step 2 to collapse the superposition of state into measurable pure states $|0\rangle$ if the function is constant or $|1\rangle$ if it is balanced. Using a φ-bit, one can directly and advantageously measure the superposition of state by the voltage at a transducer.

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Non-separable superposition of states in parallel φ-bit arrays

- The power of quantum computing lies in the concept of *entanglement*.
- The state of two quantum subsystems in *separable* superposition is the tensor product of the states of the two individual subsystems:

$$\psi_{12} = |\psi_1\rangle \otimes |\psi_2\rangle$$

- The state of two quantum subsystems in *non-separable* superposition cannot be written as a tensor product of the states of the subsystems.

$$\psi_{12} \neq |\psi_1\rangle \otimes |\psi_2\rangle$$

The power of exponential complexity

- Two non-separable φ-bits 1 and 2 support 2^2 "bits."
- Affecting the state of subsystem 1 in a non-separable superposition of states impacts the state of subsystem 2, thus operating on the 2^2 "bits."

- N non-separable φ-bits support 2^N "bits."
- Operating on any subsystem in a non-separable superposition operates on the 2^N "bits."

- Hence, arrays of φ-bits in non-separable states offer massively parallel processing of phonons. For example, an array of N=50 φ-bits, which is easily technologically realizable, has a parallel computing capacity of 2^{50} or ~1×10^{15} bits (Petascale).

Elastic waves in non-separable states

Example: two φ-bits, "*a*" and "*b*," coupled in parallel through an elastic mediu
Elastic displacements are "*u*" and "*v*".

Coupled 1-D Elastic Klein-Gordon wave equations

$$\left(\frac{\partial^2}{\partial t^2} - \beta^2 \frac{\partial^2}{\partial x^2}\right) u + \gamma^2 (u - v) = 0$$

$$\left(\frac{\partial^2}{\partial t^2} - \beta^2 \frac{\partial^2}{\partial x^2}\right) v - \gamma^2 (u - v) = 0$$

: coupling terms

1-D Elastic Dirac equation

$$\left[\boldsymbol{\sigma}_x \otimes \boldsymbol{\sigma}_x \frac{\partial}{\partial t} + i\beta \boldsymbol{\sigma}_x \otimes \boldsymbol{\sigma}_y \frac{\partial}{\partial x} \pm i\delta \boldsymbol{C}\right] \Psi = 0 \quad \text{with } \boldsymbol{C} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \delta = \frac{1}{\sqrt{2}}\gamma$$

4x1 spinorial plane wave solutions

$$\Psi = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} e^{+i\omega t} e^{+ikx}$$

A two φ-bit non-separable superposition of states relates to the amplitude of forward propagating waves in bits "a" and "b"(a_1 and a_3) as well as backward propagating waves in bits "a" and "b"(a_2 and a_4).

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Elastic waves in non-separable states (continues)

Elastically coupled φ-bits

4x1 spinorial plane wave solutions

$$\Psi = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} e^{+i\omega t} e^{+ikx}$$

Antisymmetric solutions

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = a_0 \begin{pmatrix} \sqrt{+}\sqrt{+} \\ -\sqrt{+}\sqrt{-} \\ -\sqrt{+}\sqrt{+} \\ \sqrt{+}\sqrt{-} \end{pmatrix}$$

$$\psi_{ab} \neq |\psi_a\rangle \otimes |\psi_b\rangle$$

Individual φ-bit solutions

Uncoupled φ-bits

$$\begin{pmatrix} s_1^a\sqrt{\pm} \\ s_2^a\sqrt{\mp} \end{pmatrix} \otimes \begin{pmatrix} s_1^b\sqrt{\pm} \\ s_2^b\sqrt{\mp} \end{pmatrix} = \begin{pmatrix} s_1^a s_1^b\sqrt{\pm}\sqrt{\pm} \\ s_1^a s_2^b\sqrt{\pm}\sqrt{\mp} \\ s_2^a s_1^b\sqrt{\mp}\sqrt{\pm} \\ s_2^a s_2^b\sqrt{\mp}\sqrt{\mp} \end{pmatrix}$$

But for special cases

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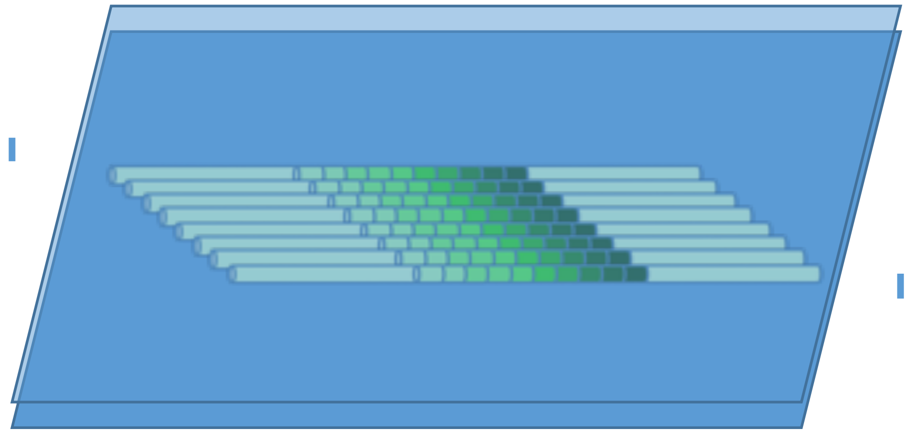
N coupled φ-bits



Nx1 non-separable spinorial plane wave solutions

$$\Psi_{123\ldots N} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_{N-1} \\ a_N \end{pmatrix} e^{+i\omega t} e^{+ikx}$$

Multi-pseudospin superpositions of ϕ-bit states are experimentally measurable from the transmission coefficients of individual fibers constituting the array.

$$\Psi_{123\ldots N} \neq \Psi_1 \otimes \Psi_2 \otimes \Psi_3 \otimes \ldots \otimes \Psi_N$$

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Deutsch-Jozsa algorithm with entanglement

Consider a Boolean function defined from a two-bit domain space to a one-bit range space: $f(x) : \{0,1\}^2 \to \{0,1\}$. There are four possible input values $(00), (01), (10)$ and (11) and the output for each of these could be either 0 or 1. There are thus 16 functions in all. For a given function, the output can have either: all ones, three ones and a zero, two ones and two zeros, three zeros and one one or all zeros. We can divide the function into classes $[0,4]$, $[1,3]$, $[2,2]$, $[3,1]$, and $[4,0]$, the first entry indicating the number of ones and the second indicating the number of zeros in the output. The functions with an even number $(0,2,4)$ of ones (i.e. the functions $[0,4]$, $[2,2]$ and $[4,0]$) are defined as "Even" functions while the functions with an odd number $(1,3)$ of ones in the output (i.e. the $[1,3]$ and $[3,1]$ functions) are defined to be "Odd" functions. Using this evaluation criterion, of the 16 possible functions for the two-bit case, eight are even and eight are odd.

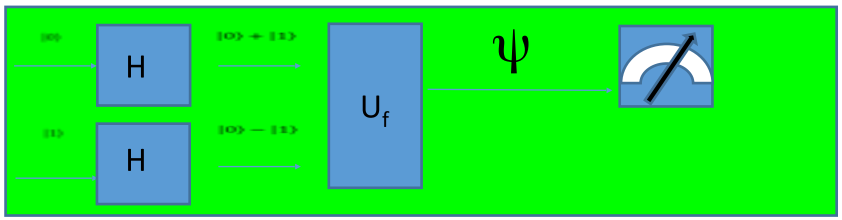
Given a function *f*, how does one decide whether it is even or odd, without computing the function at all input points?

A two-qubit algorithm involving quantum entanglement

Arvind[1*] and N. Mukunda[2†‡]

arXiv: quant-ph/0006069v1

Deutsch-Jozsa algorithm with entanglement (2)



$$U_f = \begin{pmatrix} (-1)^{f(00)} & 0 & 0 & 0 \\ 0 & (-1)^{f(01)} & 0 & 0 \\ 0 & 0 & (-1)^{f(10)} & 0 \\ 0 & 0 & 0 & (-1)^{f(11)} \end{pmatrix}$$

Class	Number	Nature	U_f	DJ Class
[0,4]	1	Even	Separable	Constant
[1,3]	4	Odd	Entangling	——
[2,2]	6	Even	Separable	Balanced
[3,1]	4	Odd	Entangling	——
[4,0]	1	Even	Separable	Constant

TABLE I. Characteristics of different classes of functions. In each class we give number of functions, their even or odd nature, the entangling or separable nature of U_f and their status in DJ problem.

For an "even" function, the final state is separable
For an "odd" function, the final state is non-separable (entangled)

Note: No unambiguous single measurement of entangled states of quantum systems (needs multiple measurement and statistics)

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Deutsch-Jozsa algorithm with entanglement (3)

For an “even” function, the final state is separable

For an “odd” function, the final state is non-separable (entangled)

Two elastically coupled φ-bits

$k = 0,\ \omega = 2\delta$ and $\sqrt{+} = \sqrt{-} = \sqrt{2\delta}$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = a_0 2\delta \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = a_0 2\delta \begin{pmatrix} 1 \\ -1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

separable

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = a_0 \begin{pmatrix} \sqrt{+}\sqrt{+} \\ -\sqrt{+}\sqrt{-} \\ -\sqrt{+}\sqrt{+} \\ \sqrt{+}\sqrt{-} \end{pmatrix}$$ otherwise

entangled

$\delta \to 0$ then $\omega \to \beta k$ and $\sqrt{+} \to \sqrt{2\beta k}$ and $\sqrt{-} \to 0$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = a_0\sqrt{2\beta k} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = a_0\sqrt{2\beta k} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

separable

Single measurement of transmission

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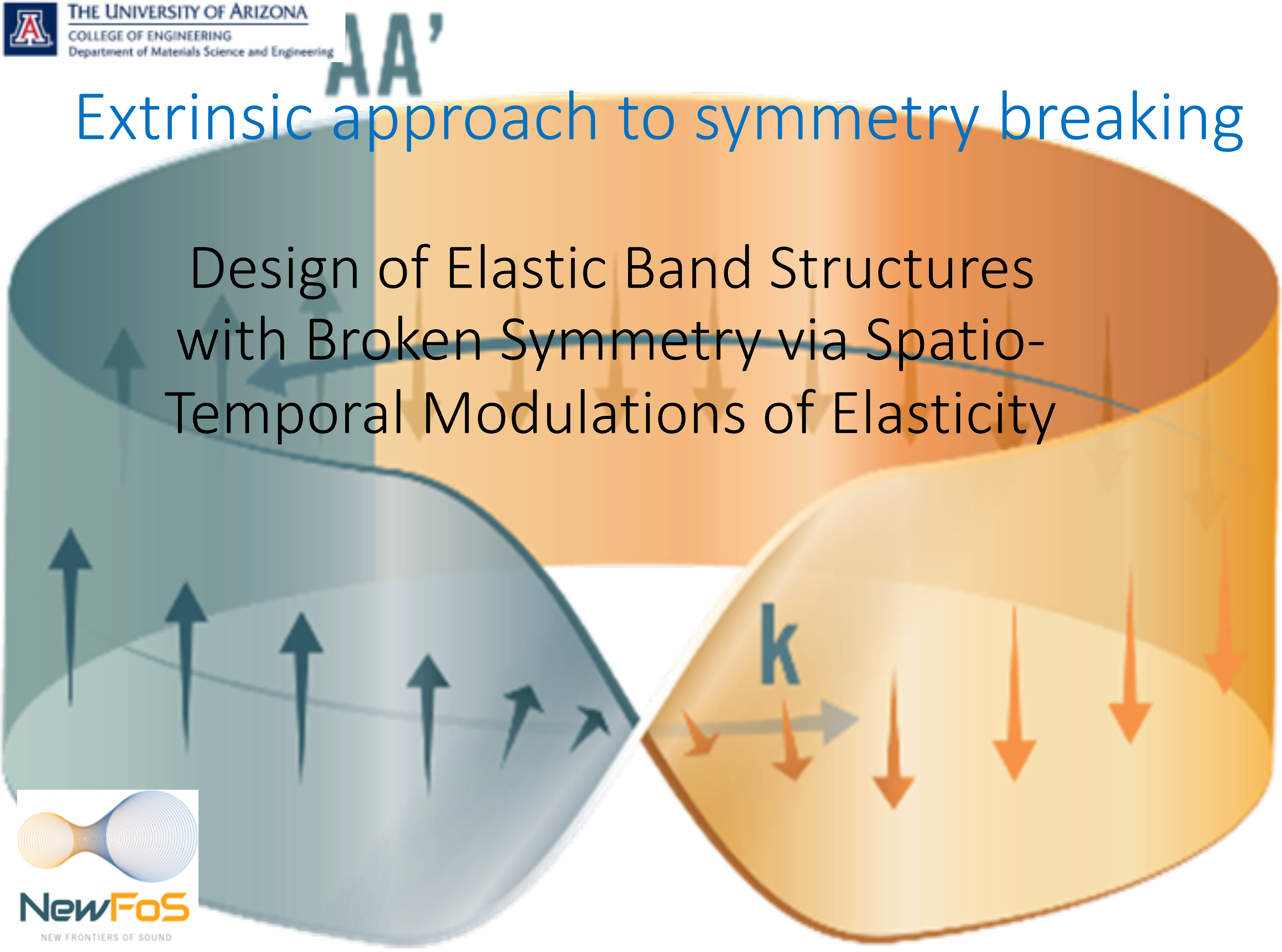
Extrinsic approach to symmetry breaking

Design of Elastic Band Structures with Broken Symmetry via Spatio-Temporal Modulations of Elasticity



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Non-reciprocal elastic wave propagation

JOURNAL OF APPLIED PHYSICS **118**, 063103 (2015)

Bulk elastic waves with unidirectional backscattering-immune topological states in a time-dependent superlattice

N. Swinteck,[1,a)] S. Matsuo,[1] K. Runge,[1] J. O. Vasseur,[2] P. Lucas,[1] and P. A. Deymier[1]
[1]*Department of Materials Science and Engineering, University of Arizona, Tucson, Arizona 85721, USA*
[2]*Institut d'Electronique, de Micro-électronique et de Nanotechnologie, UMR CNRS 8520, Cité Scientifique, 59652 Villeneuve d'Ascq Cedex, France*

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Recent progress in electronic and electromagnetic topological insulators has led to the demonstration of one way propagation of electron and photon edge states and the possibility of immunity to backscattering by edge defects. Unfortunately, such topologically protected propagation of waves in the bulk of a material has not been observed. We show, in the case of sound/elastic waves, that bulk waves with unidirectional backscattering-immune topological states can be observed in a time-dependent elastic superlattice. The superlattice is realized via spatial and temporal modulation of the stiffness of an elastic material. Bulk elastic waves in this superlattice are supported by a manifold in momentum space with the topology of a single twist Möbius strip. Our results demonstrate the possibility of attaining one way transport and immunity to scattering of bulk elastic waves. [http://dx.doi.org/10.1063/1.4928619]

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Photo-elastic effect

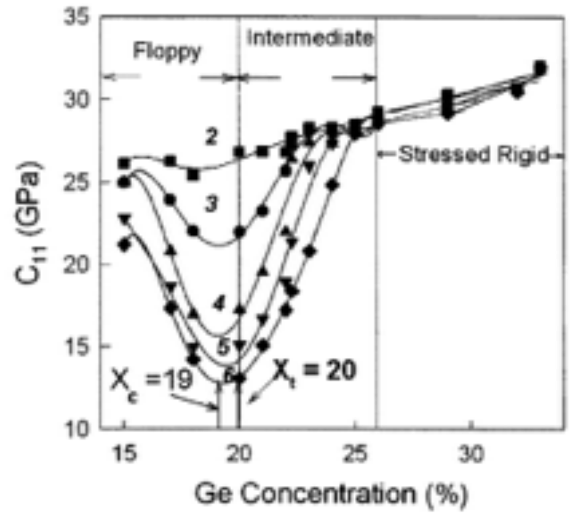
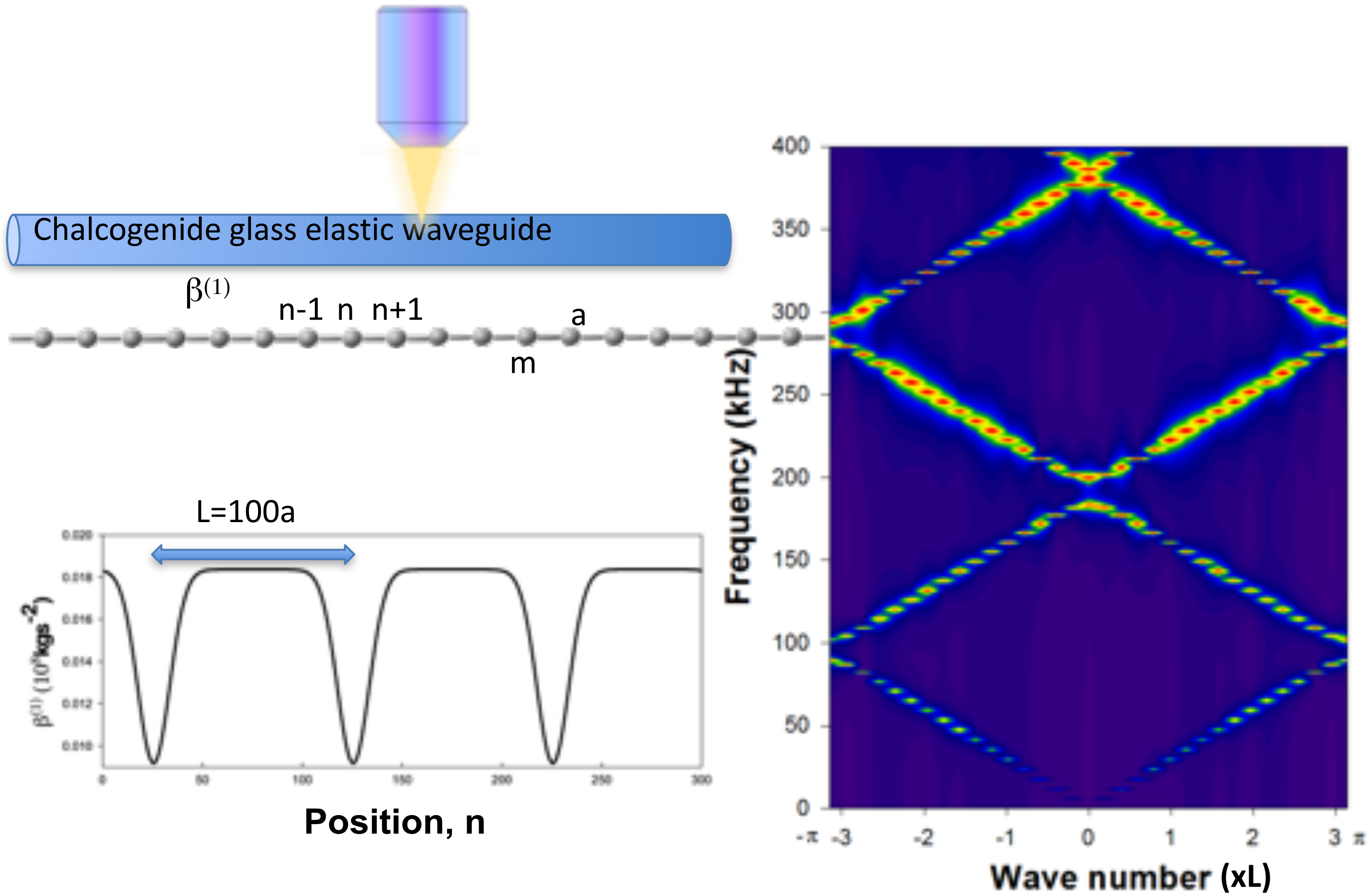


FIG. 4. Variations in longitudinal elastic constant $C_{11}(x)$ in Ge_xSe_{1-x} glasses as a function of power P_r (indicated for each curve). Here x_c and x_t designate, respectively, the observed threshold in light-induced softening of C_{11} and the mean-field rigidity transition. The lines at $x = 0.20$ and 0.26 designate, respectively, the rigidity and stress transition in the present glasses (see Ref. [21]).

J. Gump, I. Finckler, H. Xia, R. Sooryakumar, W. J. Bresser, and P. Boolchand, “Light-induced giant softening of network glasses observed near the mean-field rigidity transition,” Phys. Rev. Lett. 92, 245501 (2004).

Elastic writable superlattice



Spectral Energy Density (SED) method

$$\Phi\left(\vec{k},\omega\right) = \frac{1}{4\pi\tau_0 N}\sum_{\alpha}\sum_{b}^{B} m_b \left|\int_0^{\tau_0}\sum_{n_{x,y,z}}^{N} v_\alpha\begin{pmatrix} n_{x,y,z} \\ b \end{pmatrix}; t\right) \times e^{\left(i\vec{k}\cdot\vec{r_0} - i\omega t\right)} dt\right|^2$$

$v_\alpha\left(\begin{matrix} n_{x,y,z} \\ b \end{matrix}; t\right)$ represents the velocity of atom b (of mass m_b in unit cell $n_{x,y,z}$) in the α-direction.

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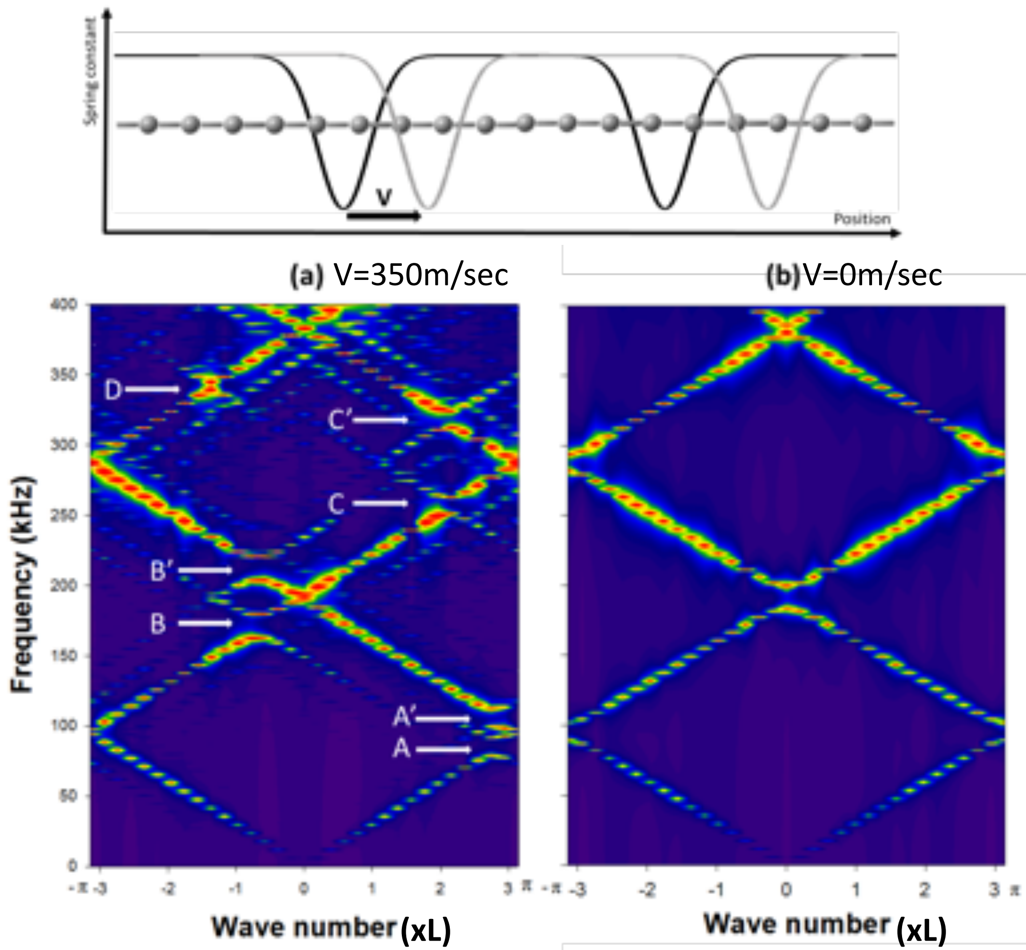
Elastic dynamically rewritable superlattice



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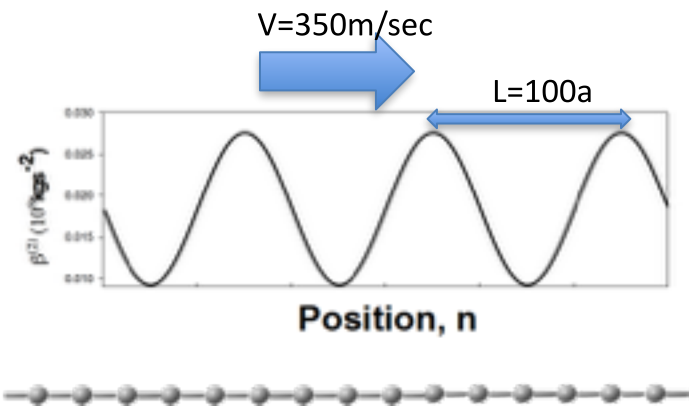
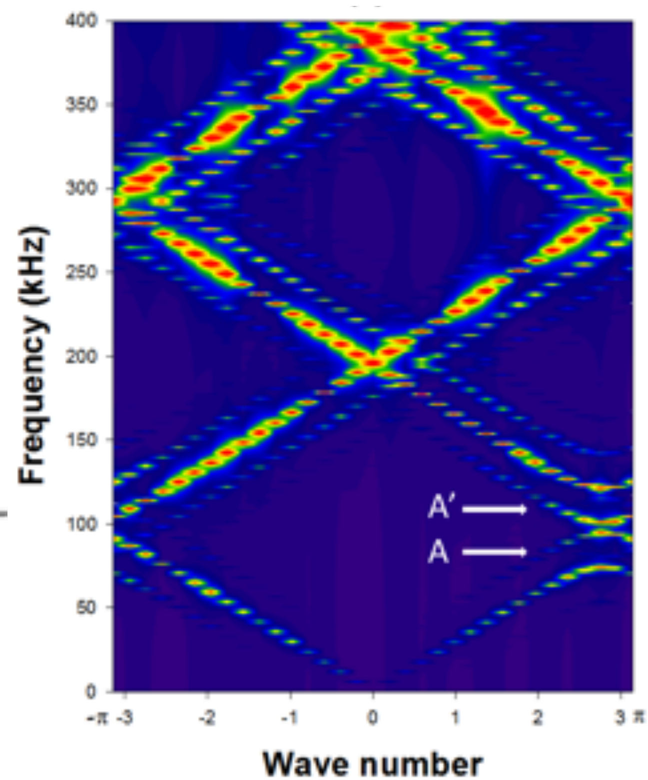
Understanding dynamically rewritable superlattices





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Topology

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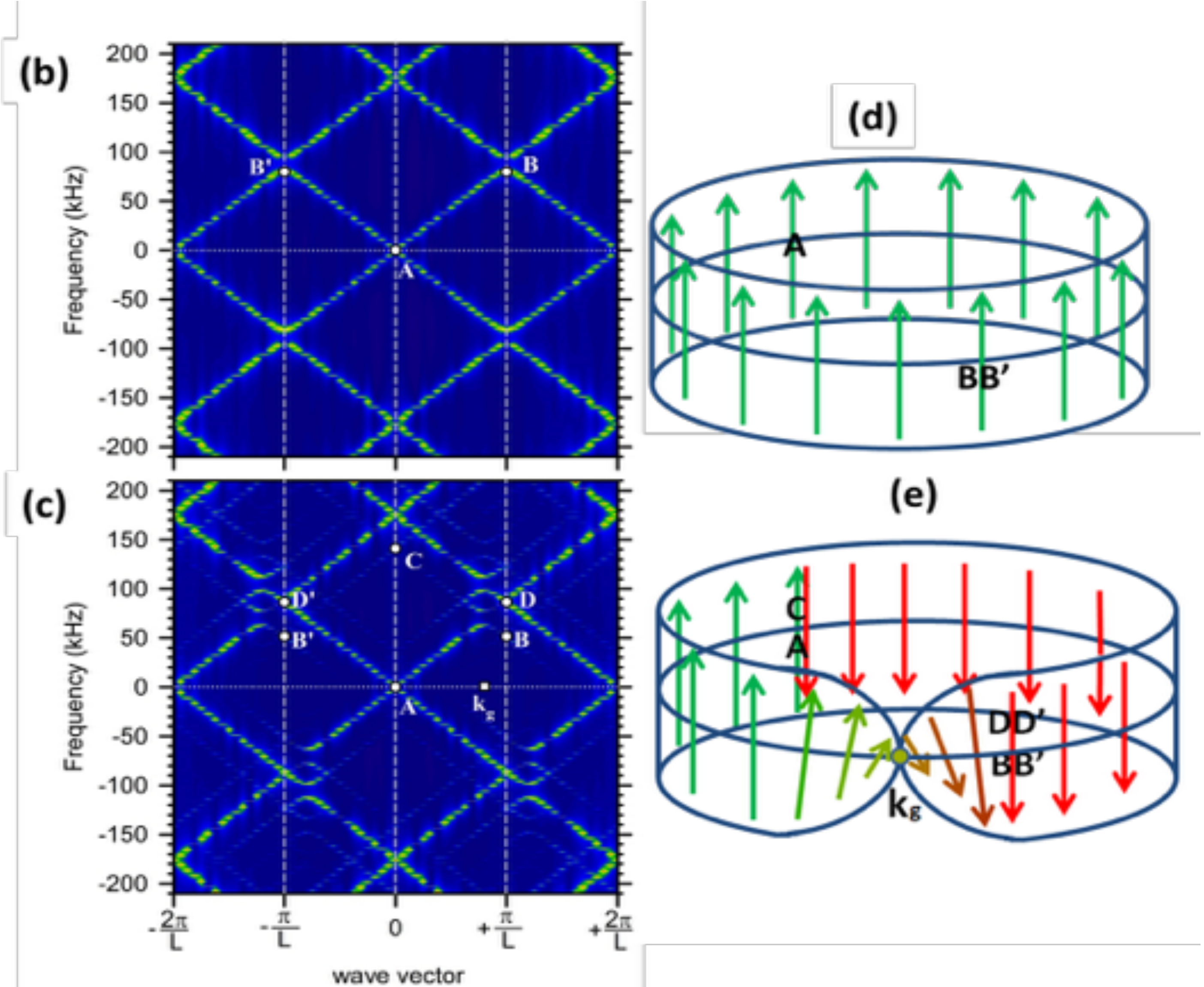

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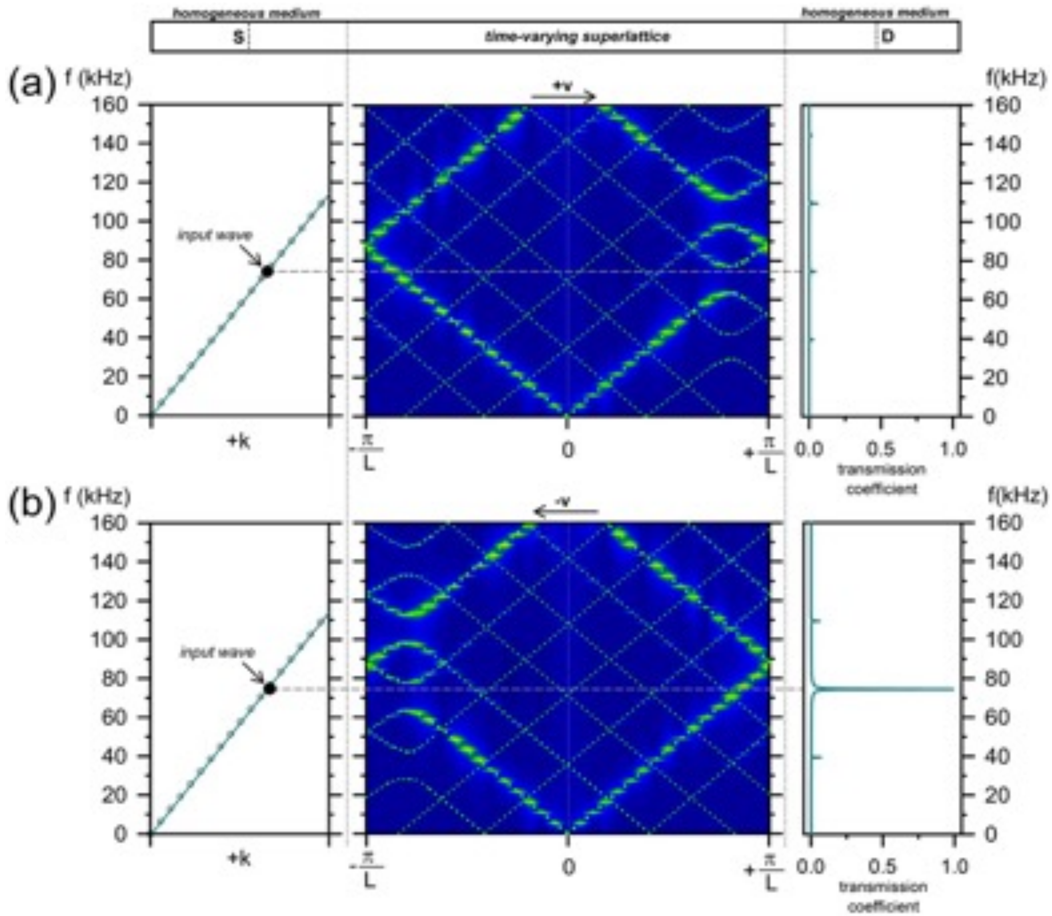
Non-reciprocity of elastic wave propagation



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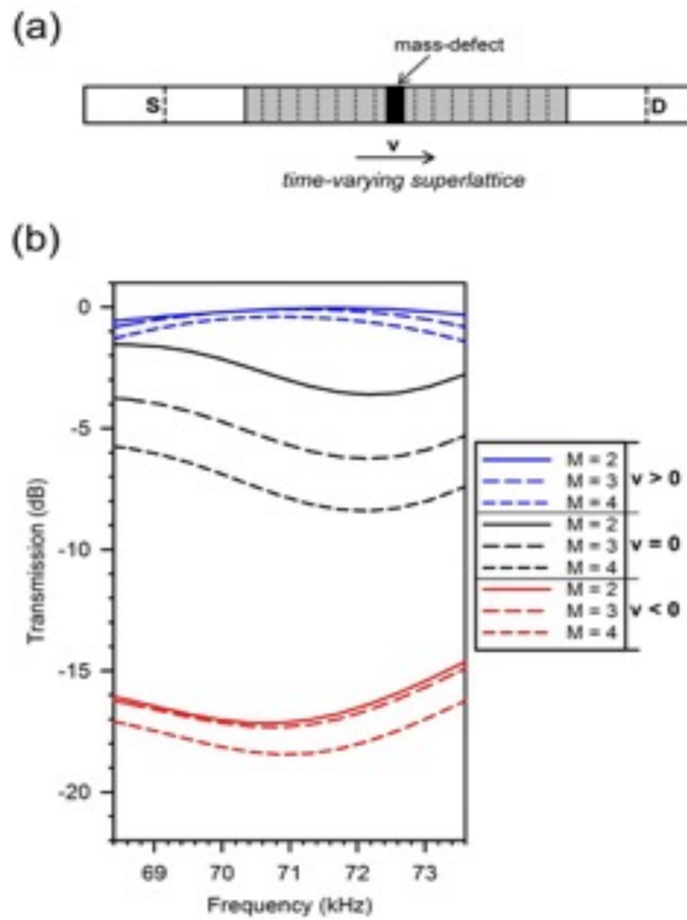
Immunity to backscattering by defects



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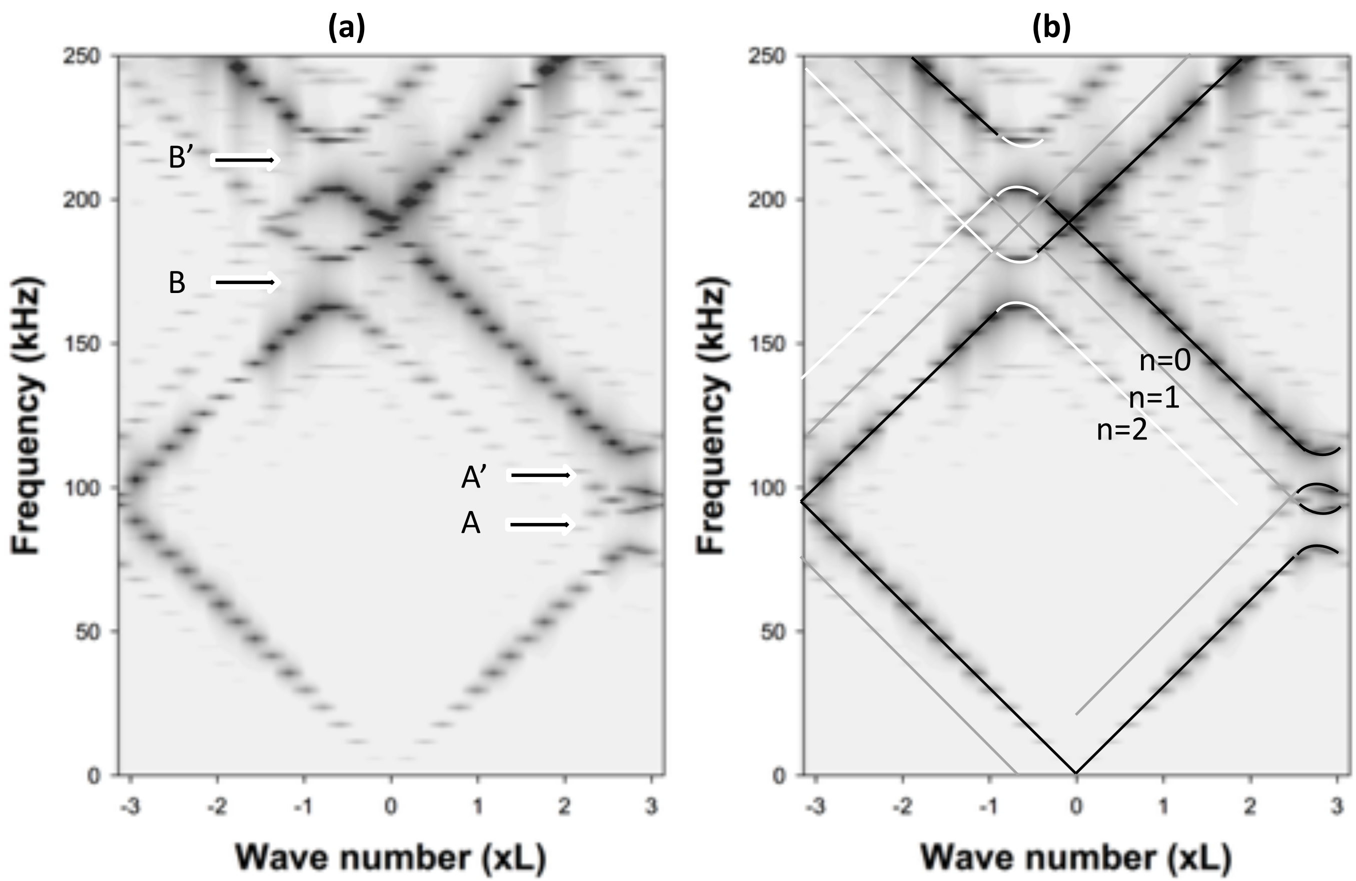
Hybridization gaps



Brillouin scattering Stokes and anti-Stokes modes

$$\upsilon_n = \upsilon_0 \pm nF,$$

where $F = \frac{\Omega}{2\pi} = \frac{V}{L}$

and $n = 1,2,3, \ldots$

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Introduction to multiple time scale perturbation theory

$$\frac{\partial^2 u}{\partial t^2} - \beta^2 \frac{\partial^2 u}{\partial x^2} + \varepsilon f(u) = 0$$

$$u(k+g,\tau_0,\tau_1,\tau_2) = u_0(k+g,\tau_0,\tau_1,\tau_2) + \varepsilon u_1(k+g,\tau_0,\tau_1,\tau_2) + \varepsilon^2 u_2(k+g,\tau_0,\tau_1,\tau_2) + \cdots$$

$\tau_0 = t$, $\tau_1 = \varepsilon t$, and $\tau_2 = \varepsilon^2 t = \varepsilon^2 \tau_0$.

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial \tau_0}\frac{\partial \tau_0}{\partial t} + \frac{\partial u}{\partial \tau_1}\frac{\partial \tau_1}{\partial t} + \frac{\partial u}{\partial \tau_2}\frac{\partial \tau_2}{\partial t} + \cdots = \frac{\partial u}{\partial \tau_0} + \varepsilon \frac{\partial u}{\partial \tau_1} + \varepsilon^2 \frac{\partial u}{\partial \tau_2} + \cdots$$

$$\frac{\partial u}{\partial t} = \frac{\partial u_0}{\partial \tau_0} + \varepsilon\left(\frac{\partial u_1}{\partial \tau_0} + \frac{\partial u_0}{\partial \tau_1}\right) + \varepsilon^2\left(\frac{\partial u_2}{\partial \tau_0} + \frac{\partial u_1}{\partial \tau_1} + \frac{\partial u_0}{\partial \tau_2}\right) + \cdots$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial \tau_0}\left(\frac{\partial u}{\partial t}\right) + \varepsilon \frac{\partial}{\partial \tau_1}\left(\frac{\partial u}{\partial t}\right) + \varepsilon^2 \frac{\partial}{\partial \tau_2}\left(\frac{\partial u}{\partial t}\right) + \cdots$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u_0}{\partial \tau_0^2} + \varepsilon\left(\frac{\partial^2 u_1}{\partial \tau_0^2} + 2\frac{\partial^2 u_0}{\partial \tau_1 \partial \tau_0}\right) + \varepsilon^2\left(\frac{\partial^2 u_2}{\partial \tau_0^2} + 2\frac{\partial^2 u_1}{\partial \tau_1 \partial \tau_0} + 2\frac{\partial^2 u_0}{\partial \tau_2 \partial \tau_0} + \frac{\partial^2 u_0}{\partial \tau_1^2}\right)$$

Introduction to multiple time scale perturbation theory (2)

$$\frac{\partial^2 u}{\partial t^2} - \beta^2 \frac{\partial^2 u}{\partial x^2} + \varepsilon f(u) = 0$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u_0}{\partial x^2} + \varepsilon \frac{\partial^2 u_1}{\partial x^2} + \varepsilon^2 \frac{\partial^2 u_2}{\partial x^2} + \cdots$$

$$f(u) = u^2 = \left(u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \cdots\right)^2 = u_0^2 + \varepsilon(u_0 u_1 + u_1 u_0) + \cdots$$

$$\left\{\frac{\partial^2 u_0}{\partial \tau_0^2} - \beta^2 \frac{\partial^2 u_0}{\partial x^2}\right\} + \varepsilon \left\{\frac{\partial^2 u_1}{\partial \tau_0^2} + 2\frac{\partial^2 u_0}{\partial \tau_1 \partial \tau_0} - \beta^2 \frac{\partial^2 u_1}{\partial x^2} + u_0^2\right\} + \varepsilon^2 \left\{\frac{\partial^2 u_2}{\partial \tau_0^2} + 2\frac{\partial^2 u_1}{\partial \tau_1 \partial \tau_0} + 2\frac{\partial^2 u_0}{\partial \tau_2 \partial \tau_0} + \frac{\partial^2 u_0}{\partial \tau_1^2} - \beta^2 \frac{\partial^2 u_2}{\partial x^2} + (u_0 u_1 + u_1 u_0)\right\} = 0$$

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Introduction to multiple time scale perturbation theory (3)

To zeroth order: $$\frac{\partial^2 u_0}{\partial \tau_0^2} - \beta^2 \frac{\partial^2 u_0}{\partial x^2} = 0$$

To first order: $$\frac{\partial^2 u_1}{\partial \tau_0^2} + 2\frac{\partial^2 u_0}{\partial \tau_1 \partial \tau_0} - \beta^2 \frac{\partial^2 u_1}{\partial x^2} + u_0^2 = 0$$

To second order: $$\frac{\partial^2 u_2}{\partial \tau_0^2} + 2\frac{\partial^2 u_1}{\partial \tau_1 \partial \tau_0} + 2\frac{\partial^2 u_0}{\partial \tau_2 \partial \tau_0} + \frac{\partial^2 u_0}{\partial \tau_1^2} - \beta^2 \frac{\partial^2 u_2}{\partial x^2} + (u_0 u_1 + u_1 u_0) = 0$$

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Introduction to multiple time scale perturbation theory (4)

To zeroth order: $\frac{\partial^2 u_0}{\partial \tau_0^2}$

To first order: $\frac{\partial^2 u_1}{\partial \tau_0^2} + 2\frac{\partial^2 u_0}{\partial \tau_1 \partial \tau_0}$

To second order: $\frac{\partial^2 u_2}{\partial \tau_0^2} + 2\frac{\partial^2 u_1}{\partial \tau_1 \partial \tau_0} + 2\frac{\partial^2 u_0}{\partial \tau_2 \partial \tau_0} + \frac{\partial^2 u_0}{\partial \tau_1^2}$

To third order: $\frac{\partial^2 u_3}{\partial \tau_0^2} + 2\frac{\partial^2 u_0}{\partial \tau_1 \partial \tau_2} + 2\frac{\partial^2 u_0}{\partial \tau_3 \partial \tau_0} + 2\frac{\partial^2 u_1}{\partial \tau_0 \partial \tau_2} + 2\frac{\partial^2 u_2}{\partial \tau_1 \partial \tau_0} + \frac{\partial^2 u_1}{\partial \tau_1^2}$

To fourth order: $\frac{\partial^2 u_4}{\partial \tau_0^2} + 2\frac{\partial^2 u_3}{\partial \tau_0 \partial \tau_1} + 2\frac{\partial^2 u_2}{\partial \tau_0 \partial \tau_2} + 2\frac{\partial^2 u_1}{\partial \tau_0 \partial \tau_3} + 2\frac{\partial^2 u_1}{\partial \tau_2 \partial \tau_1} + 2\frac{\partial^2 u_0}{\partial \tau_4 \partial \tau_0} + 2\frac{\partial^2 u_0}{\partial \tau_1 \partial \tau_3} +$
$\frac{\partial^2 u_2}{\partial \tau_1^2} + \frac{\partial^2 u_0}{\partial \tau_2^2}$

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Elastic wave equation

1D elastic wave equation with spatio-temporal variation in stiffness

$$\rho \frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial}{\partial x}\left(C(x,t)\frac{\partial u(x,t)}{\partial x}\right)$$ with $$C(x,t) = C_0 + 2C_1 \sin(Kx + \Omega t)$$

Seek solution in the form of Bloch waves

$$u(x,t) = \sum_k \sum_g u(k,g,t) e^{i(k+g)x}$$

The wave number k is limited to the first Brillouin zone: $\left[\frac{-\pi}{L}, \frac{\pi}{L}\right]$ and $g = \frac{2\pi}{L} m$ with m being a positive or negative integer

1D elastic wave equation in wave number space becomes

$$\frac{\partial^2 u(k+g,t)}{\partial t^2} + v_a^2 (k+g)^2 u(k+g,t) = i\varepsilon\left\{f(k')u(k',t)e^{i\Omega t} + h(k'')u(k'',t)e^{-i\Omega t}\right\}$$

where $f(k) = Kk + k^2, h(k) = Kk - k^2, k' = k + g - K$ and $k'' = k + g + K$.

and $v_a^2 = \frac{C_0}{\rho}$ and $\varepsilon = \frac{C_1}{\rho}$

Multiple time scale perturbation theory

Expand the displacement to second order in perturbation ε

$$u(k+g,\tau_0,\tau_1,\tau_2) = u_0(k+g,\tau_0,\tau_1,\tau_2) + \varepsilon u_1(k+g,\tau_0,\tau_1,\tau_2) + \varepsilon^2 u_2(k+g,\tau_0,\tau_1,\tau_2)$$

Define three time scales

$$\tau_0 = t,\ \ \tau_1 = \varepsilon t, \text{ and } \tau_2 = \varepsilon^2 t = \varepsilon^2 \tau_0$$

To 0th order $$\frac{\partial^2 u_0(k+g,\tau_0,\tau_1,\tau_2)}{\partial {\tau_0}^2} + \omega_0^2(k+g)u_0(k+g,\tau_0,\tau_1,\tau_2) = 0$$

To 1st order $$\frac{\partial^2 u_1(k+g,\tau_0,\tau_1,\tau_2)}{\partial {\tau_0}^2} + \omega_0^2(k+g)u_1(k+g,\tau_0,\tau_1,\tau_2) + 2\frac{\partial^2 u_0(k+g,\tau_0,\tau_1,\tau_2)}{\partial \tau_1 \partial \tau_0}$$
$$= i\{f(k')u_0(k',\tau_0,\tau_1,\tau_2)e^{i\Omega\tau_0} + h(k'')u_0(k'',\tau_0,\tau_1,\tau_2)e^{-i\Omega\tau_0}\}$$

To 2nd order $$\frac{\partial^2 u_2(k+g,\tau_0,\tau_2)}{\partial {\tau_0}^2} + \omega_0^2(k+g)u_2(k+g,\tau_0,\tau_2) + 2\frac{\partial^2 u_0(k+g,\tau_0,\tau_2)}{\partial \tau_2 \partial \tau_0}$$
$$= i\{f(k')u_1(k',\tau_0,\tau_2)e^{i\Omega\tau_0} + h(k'')u_1(k'',\tau_0,\tau_2)e^{-i\Omega\tau_0}\}$$

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Perturbative solutions (0th order)

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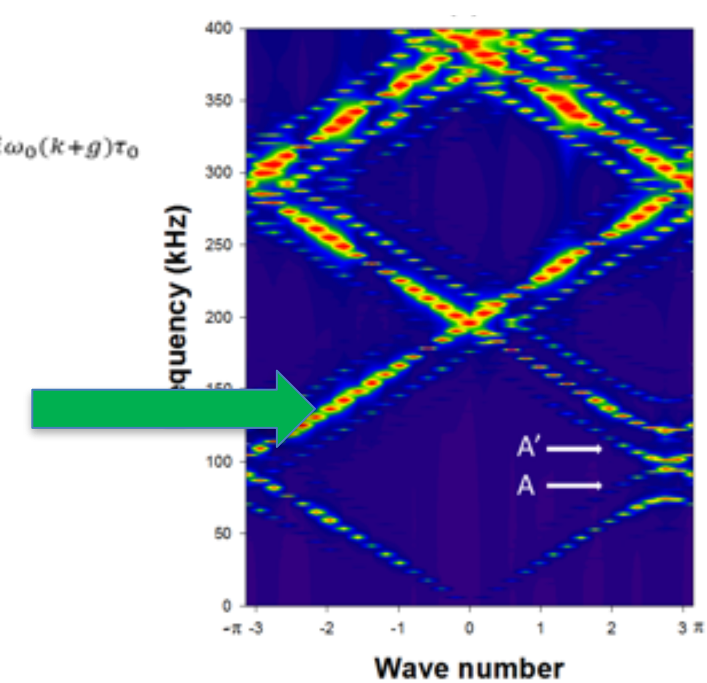
$$\frac{\partial^2 u_0(k+g,\tau_0,\tau_1,\tau_2)}{\partial {\tau_0}^2} + \omega_0^2(k+g)u_0(k+g,\tau_0,\tau_1,\tau_2) = 0$$

$$u_0(k+g,\tau_0,\tau_1,\tau_2) = a_0(k+g,\tau_1,\tau_2)e^{i\omega_0(k+g)\tau_0}$$



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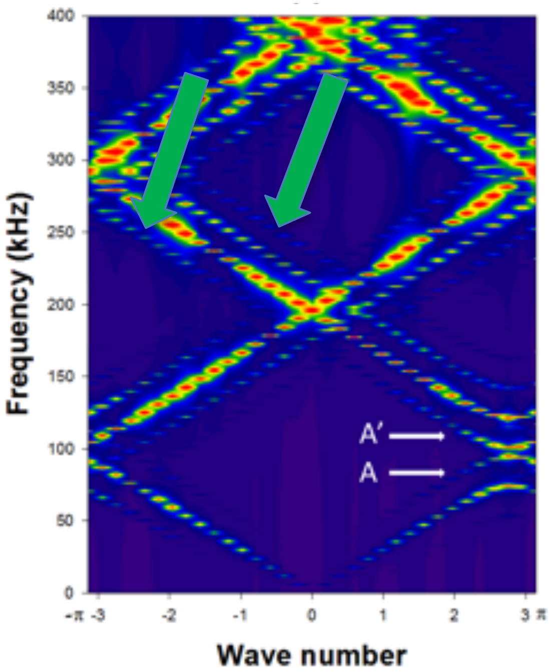
Perturbative solutions (1st order)

$$\frac{\partial^2 u_1(k+g,\tau_0,\tau_1,\tau_2)}{\partial {\tau_0}^2} + \omega_0^2(k+g)u_1(k+g,\tau_0,\tau_1,\tau_2) + 2\frac{\partial^2 u_0(k+g,\tau_0,\tau_1,\tau_2)}{\partial\tau_1\partial\tau_0}$$
$$= i\{f(k')u_0(k',\tau_0,\tau_1,\tau_2)e^{i\Omega\tau_0} + h(k'')u_0(k'',\tau_0,\tau_1,\tau_2)e^{-i\Omega\tau_0}\}$$

$$u_1(k+g,\tau_0,\tau_2)$$
$$= a_1(k+g,\tau_2)e^{i\omega_0(k+g)\tau_0}$$
$$+ i\frac{f(k')a_0(k',\tau_2)}{\omega_0^2(k+g) - (\omega_0(k')+\Omega)^2 + i\varphi}e^{i(\omega_0(k')+\Omega)\tau_0}$$
$$+ i\frac{h(k'')a_0(k'',\tau_2)}{\omega_0^2(k+g) - (\omega_0(k'')-\Omega)^2 + i\varphi}e^{i(\omega_0(k'')-\Omega)\tau_0}$$



Brillouin scattering like phenomenon

Perturbative solutions (2nd order)

$$\frac{\partial^2 u_2(k+g,\tau_0,\tau_2)}{\partial \tau_0{}^2} + \omega_0^2(k+g)u_2(k+g,\tau_0,\tau_2) + 2\frac{\partial^2 u_0(k+g,\tau_0,\tau_2)}{\partial\tau_2\partial\tau_0} = i\{f(k')u_1(k',\tau_0,\tau_2)e^{i\Omega\tau_0} + h(k'')u_1(k'',\tau_0,\tau_2)e^{-i\Omega\tau_0}\}$$

$$2\frac{\partial^2 u_0(k+g,\tau_0,\tau_2)}{\partial\tau_2\partial\tau_0} = -\left\{f(k')h(k+g)\left(\frac{1}{\omega_0^2(k')-(\omega_0(k+g)-\Omega)^2}\right) + h(k'')f(k+g)\left(\frac{1}{\omega_0^2(k'')-(\omega_0(k+g)+\Omega)^2}\right)\right\}\alpha_0(k+g,\tau_2)e^{i\omega_0(k+g)\tau_0}$$

$$u_0(k+g,\tau_0,\tau_2) = \alpha_0(k+g)e^{i\gamma\tau_2}e^{i\omega_0(k+g)\tau_0} = \alpha_0(k+g)e^{i[\omega_0(k+g)+\gamma\varepsilon^2]\tau_0} = \alpha_0(k+g)e^{i\omega_0^*(k+g)\tau_0}$$

$$\delta\omega_0(k+g) = \omega_0^*(k+g) - \omega_0(k+g) = \varepsilon^2(\gamma)_{pp} = \frac{\varepsilon^2}{2\omega_0(k+g)}\left\{\begin{matrix} f(k')h(k+g)\left(\frac{1}{\omega_0^2(k')-(\omega_0(k+g)-\Omega)^2}\right)_{pp} \\ +h(k'')f(k+g)\left(\frac{1}{\omega_0^2(k'')-(\omega_0(k+g)+\Omega)^2}\right)_{pp} \end{matrix}\right\}$$

A

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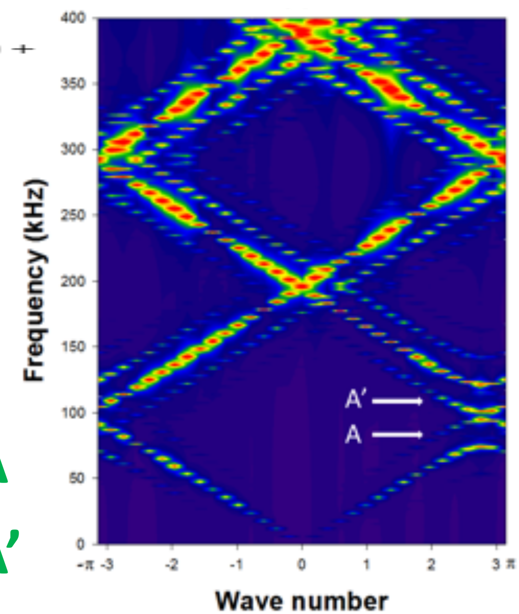

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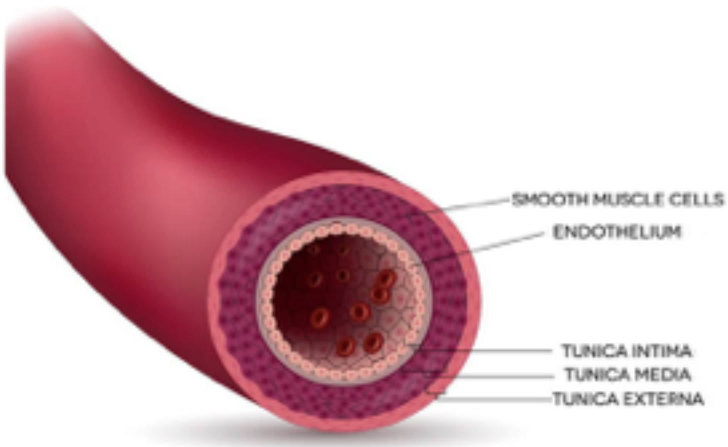
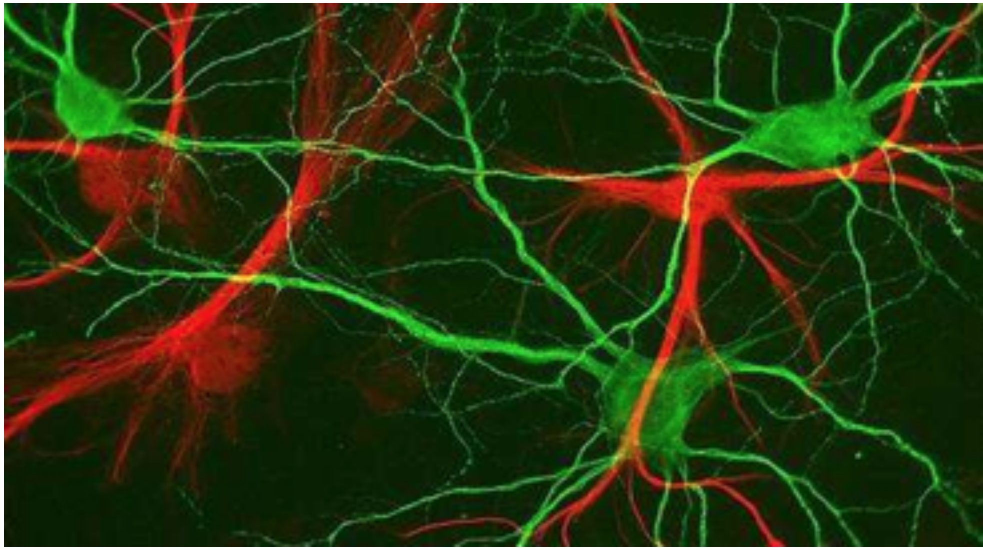
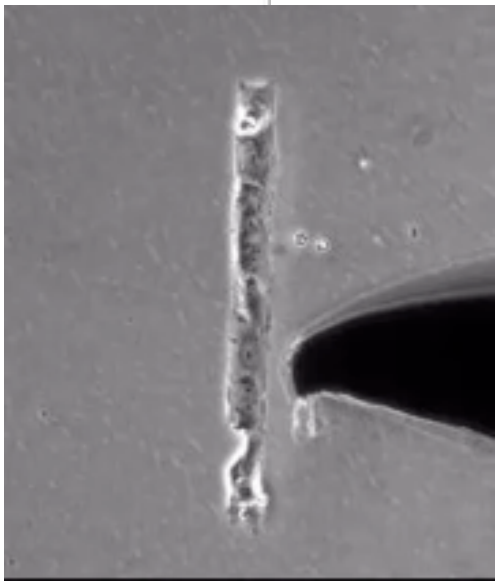
Other application domains

1. Extension to electromagnetic waves (Dynamical "microstructures" of index of refraction)
2. Extension to spin waves (Dynamical "microstructures" of exchange coupling)
3. Extension to electronic waves (Dynamical "microstructure" of potential)
4. Extension to chemical or biological waves (Dynamical "microstructure" of diffusion coefficient)

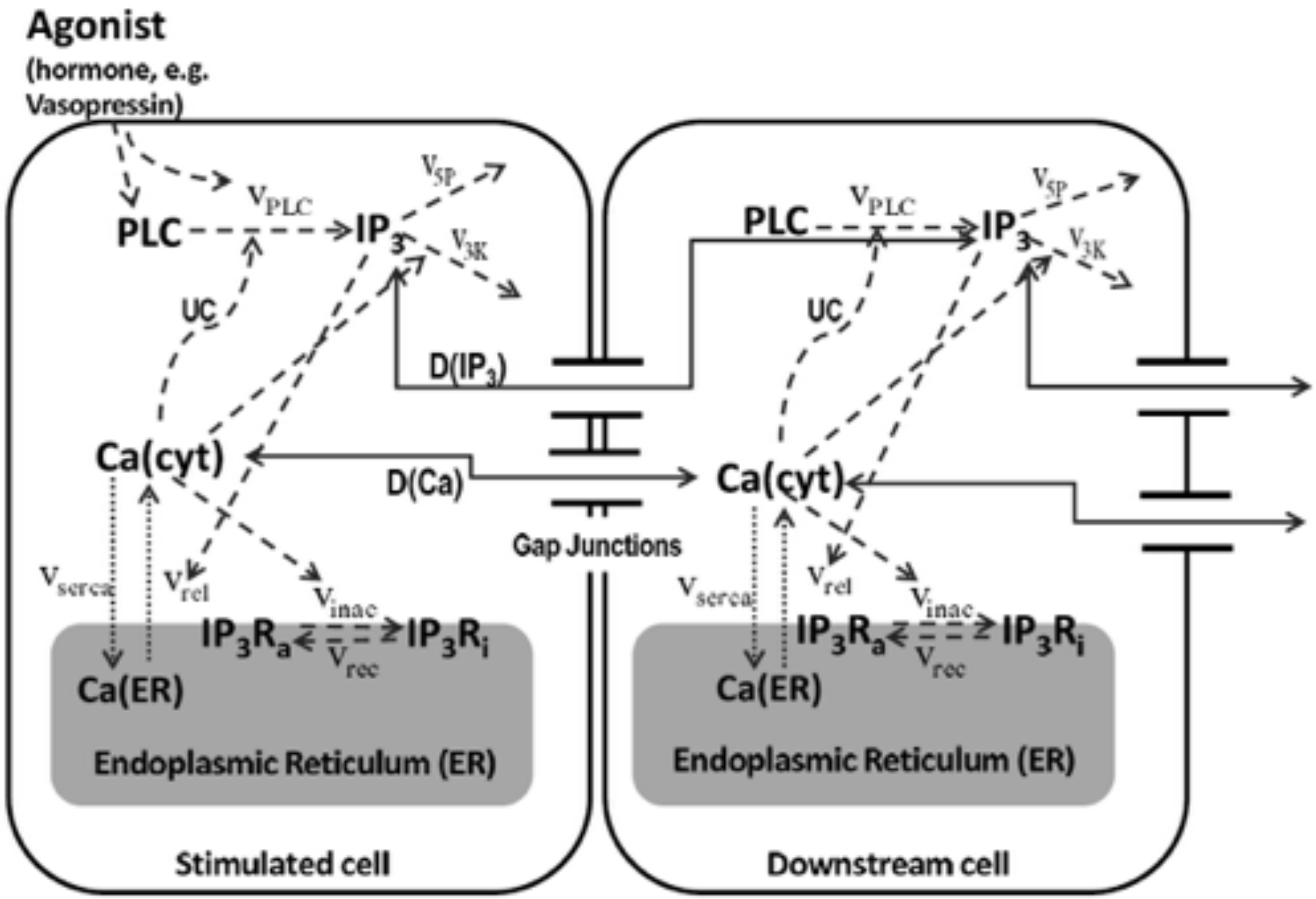
Calcium waves







Breaking symmetry of calcium waves



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Linear model

$$\frac{\partial C(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_c(x,t)\frac{\partial C(x,t)}{\partial x}\right] - rP(x,t)$$

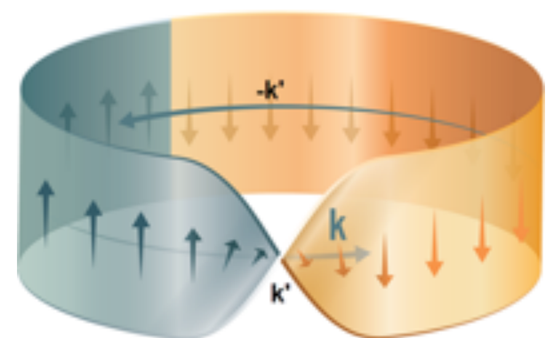
$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_p(x,t)\frac{\partial P(x,t)}{\partial x}\right] + rC(x,t)$$

with

$$D_c(x,t) = D_{0c} + 2D_{1c}\cos(Kx + \Omega t)$$

$$D_p(x,t) = D_{0p} + 2D_{1p}\cos(Kx + \Omega t)$$

the Ca^{2+} and IP_3 waves perturbed by the acoustic wave is not symmetrical about the origin k=0.



Need for nonlinear model

Turing noted the importance of studying the behavior of biological processes by considering the complementarity of both linear and nonlinear dynamical systems:

"Such systems (*with linear dynamics*) certainly have a special interest as giving the appearance of a pattern, but they are the exception rather than the rule. Most of an organism, most of the time, is developing from one pattern into another, rather than from homogeneity into a pattern. One would like to be able to follow this more general process (*nonlinear*) mathematically also. The difficulties are, however, such that one cannot hope to have any very embracing *theory* of such processes, beyond the statement of the equations. It might be possible, however, to treat a few particular cases in detail with the aid of a digital computer." (parenthetical comments added)

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Calcium wave dynamics

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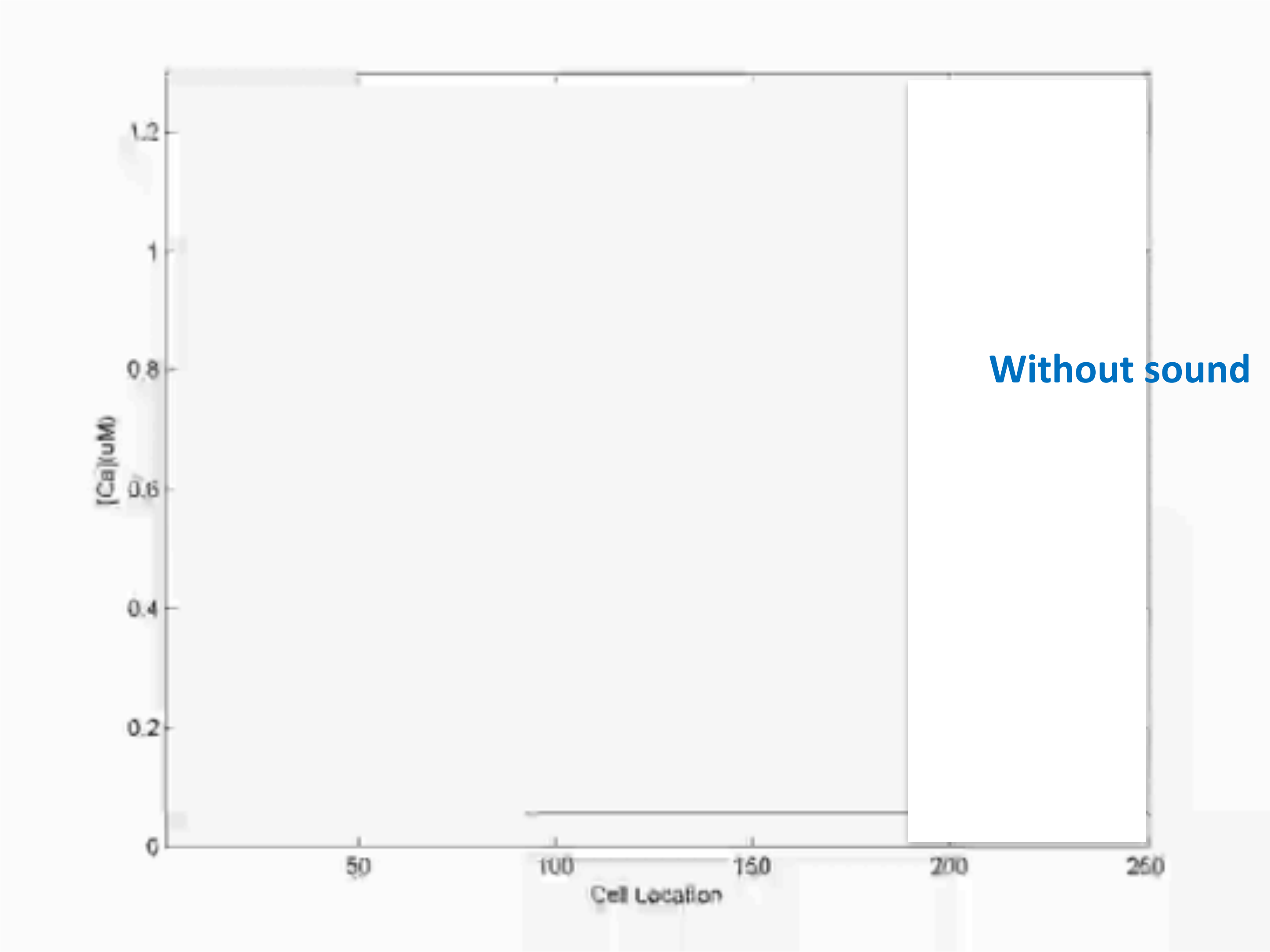

Without sound

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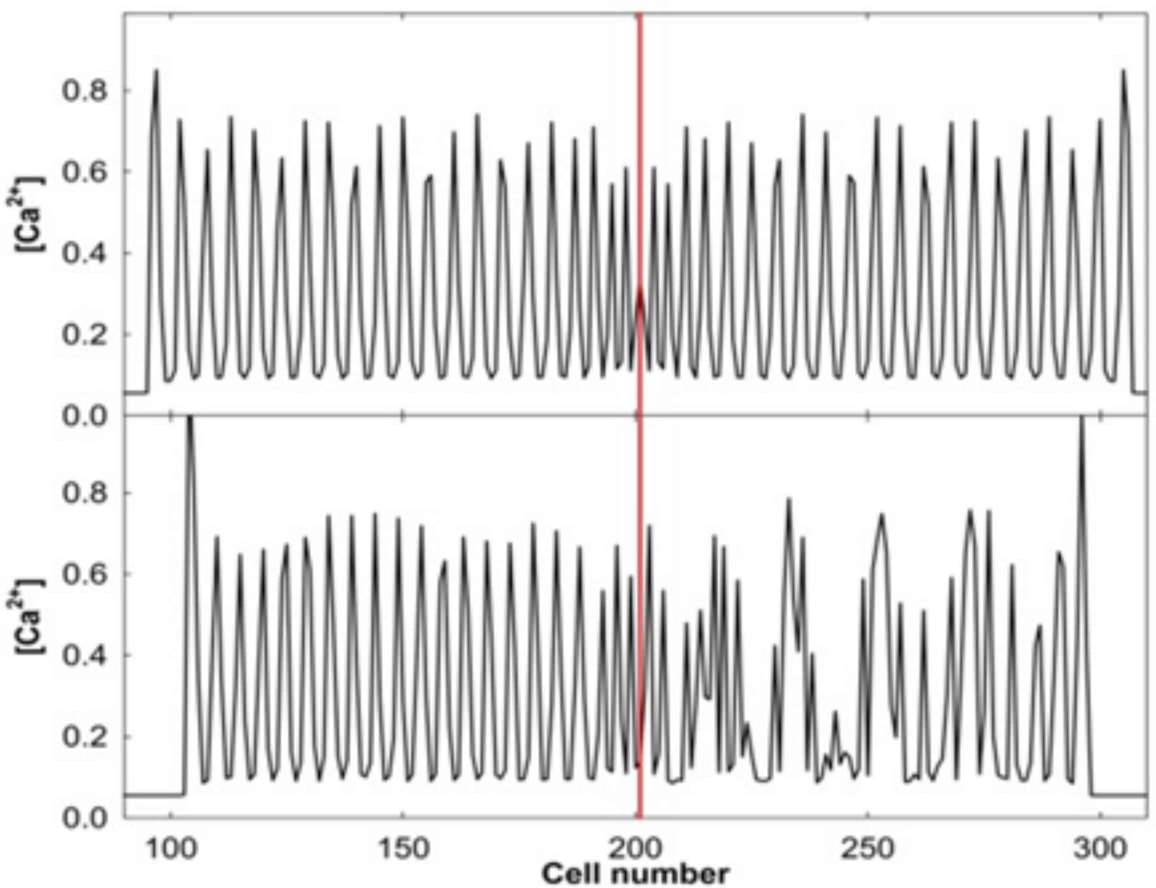
Symmetry breaking of Calcium wave propagation



Without sound

With sound

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PHYSICAL REVIEW E **92**, 052711 (2015)

Effect of sound on gap-junction-based intercellular signaling: Calcium waves under acoustic irradiation

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We present a previously unrecognized effect of sound waves on gap-junction-based intercellular signaling such as in biological tissues composed of endothelial cells. We suggest that sound irradiation may, through temporal and spatial modulation of cell-to-cell conductance, create intercellular calcium waves with unidirectional signal propagation associated with nonconventional topologies. Nonreciprocity in calcium wave propagation induced by sound wave irradiation is demonstrated in the case of a linear and a nonlinear reaction-diffusion model. This demonstration should be applicable to other types of gap-junction-based intercellular signals, and it is thought that it should be of help in interpreting a broad range of biological phenomena associated with the beneficial therapeutic effects of sound irradiation and possibly the harmful effects of sound waves on health.

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