iemn Institut d'Electronique, de Microélectronique et de Nanotechnologie

Introduction to Phononic Crystals and Metamaterials

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ePoV

Université de Lille 1 Sciences et Technologies

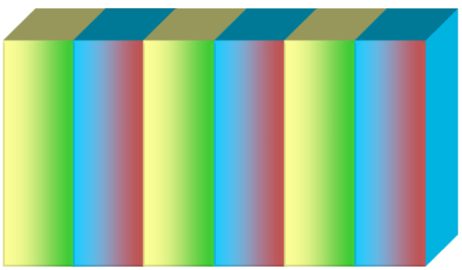
PHONONIC CRYSTALS

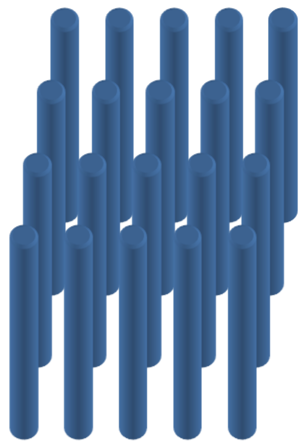
Heterogeneous materials whose elastic constants and density are periodic functions of the position (1D, 2D, 3D)

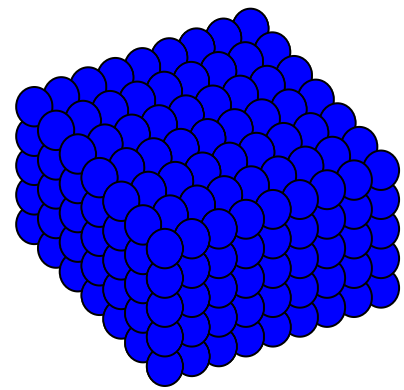
1D: Multilayers materials

2D: Array of cylinders of circular, square, cross section embedded in a matrix

3D: Array of spheres embedded in a matrix



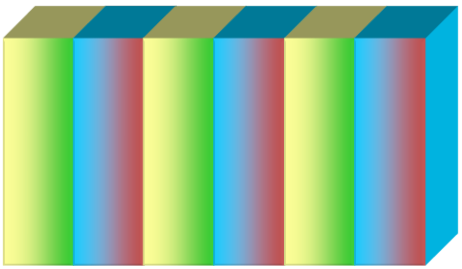
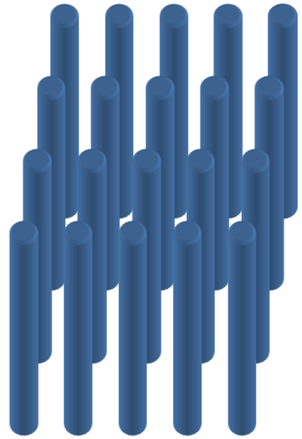
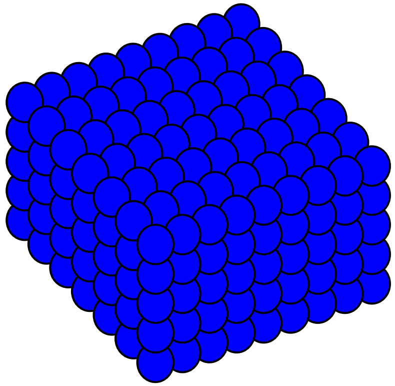




⇒ Engineering of the band structure, in particular Forbidden bands for acoustic waves

⇔ Elastic analogs of photonic crystals : Heterogeneous materials which refraction index ***n*** is a periodic function of the position

Periodicity ➡ determines the properties of materials

Material	Description	Waves	Gap
Crystalline Solid	Periodic arrangement of atoms ~ 5 Å	Electrons (Ψ) **Schrödinger** Eq.	Absence of **electron** states
Photonic Crystal	Periodic modulation of *ε μ* on a macroscopic scale	EM (**E B**) **Maxwell** Eqs.	Absence of states of the **EM** field
Phononic Crystal	Periodic modulation of *ρ λ μ* on a macroscopic scale	Elastic (**U**) **Elasticity** Eqs.	Absence of states of the **elastic** field

Classical waves in **artificial periodic** structures:
controlling the propagation of **light** and **sound**.

METAMATERIALS

-Artificial materials with properties or wave manipulation functionalities that cannot be found in nature or realized with conventional materials

- A common feature of metamaterials is their **subwavelength characteristics**: the wavelength in the background is much larger than the size of the constituting building blocks (“meta-molecules” or unit cells)

-The functionalities arise as the collective manifestations of the internal constituent units (possibly locally resonant).

-Characteristic of metamaterials

- Properties are based on effective parameters
- Material response does not depend on the size and shape of the sample
- Building blocks may display low frequency (subwavelength) resonating elements
- Periodicity not required (although maintained in most of the structures)

Background and Motivations

1.Existence of band gaps

- Evanescent waves inside the gaps (tunneling, superluminal transmission)
- Strong confinement of waveguide and cavity modes. Filtering applications. Slow waves
- Tunable structures. Sensor applications

2.Refractive properties

- Positive and negative refraction
- Imaging and Focusing applications

3. Local resonances and metamaterials

- Sound isolation
- Effective properties. Negative dynamic mass density and compressibility. Zero index
- GRIN devices. Cloaking. Focusing and imaging (superlens, hyperlens)
- Metasurfaces. Superabsorption. Phase manipulation, control of refractive properties (space coiling)
- Active and time dependent materials . Non reciprocal behaviors

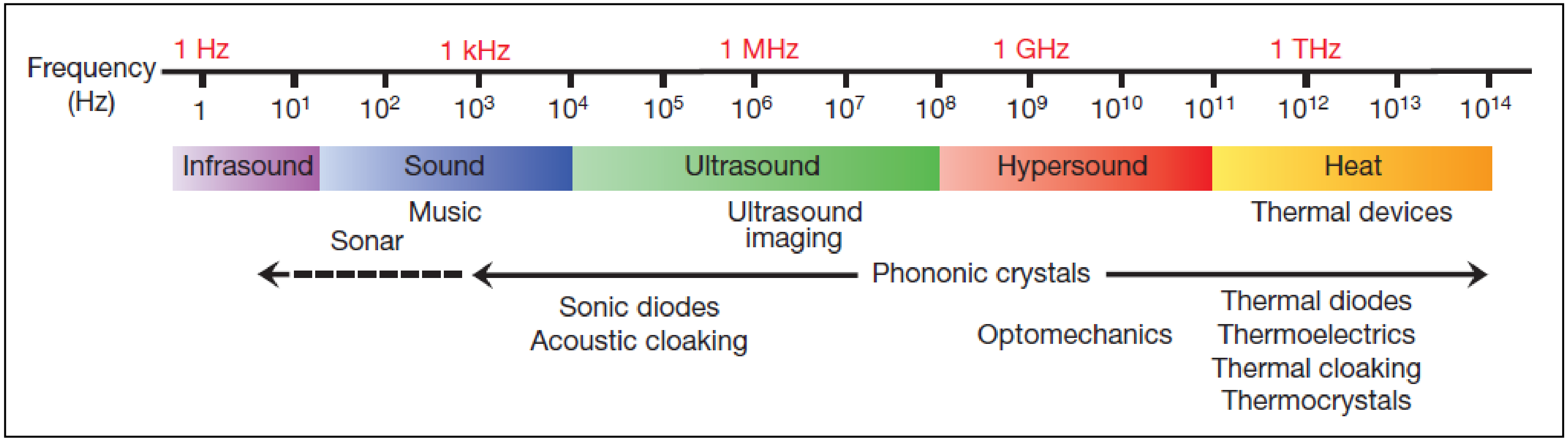
4. Dual phononic-photonic(phoXonic) crystals

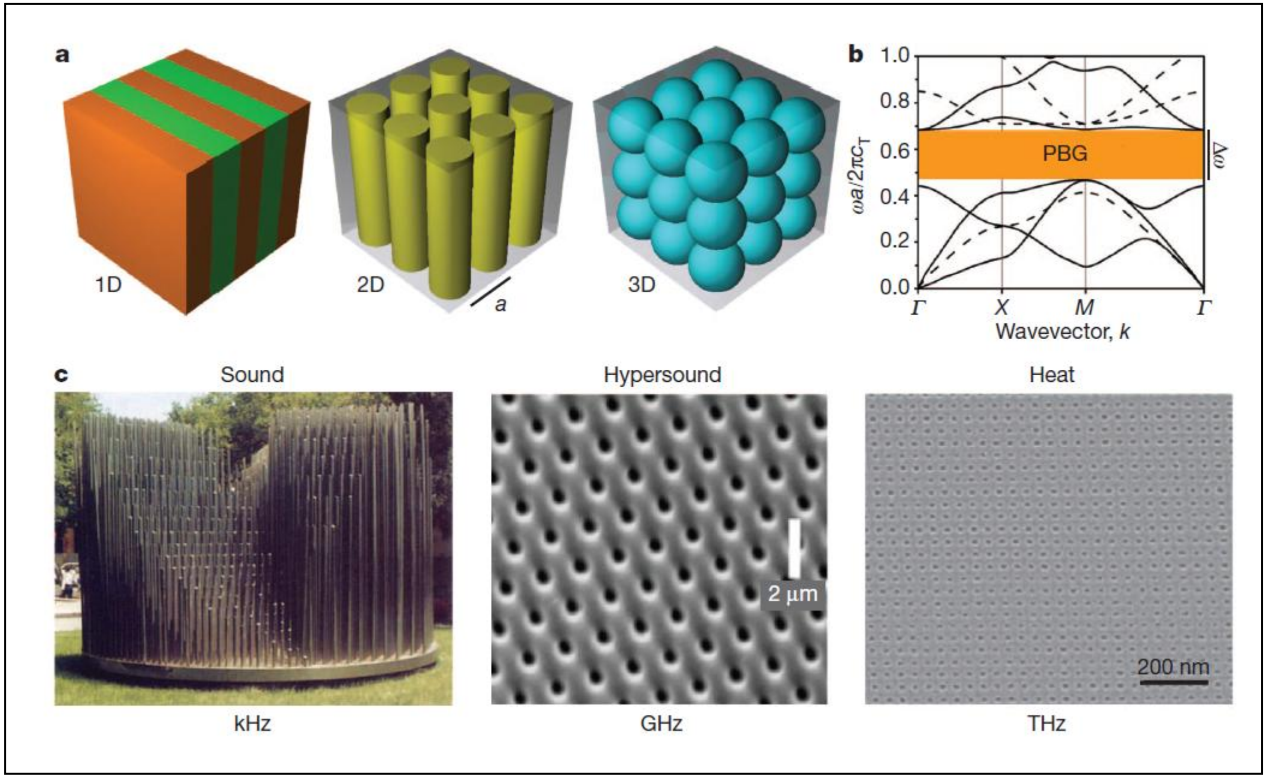
- Simultaneous photonic-photonic band gaps and phonon-photon confinements
- Enhanced phonon-photon interaction. Optomechanic crystals
- Dual sensors

5. Thermal management at the nanoscale

6. Emerging topics: PT symmetry, Time-space periodicity, Topological phononics

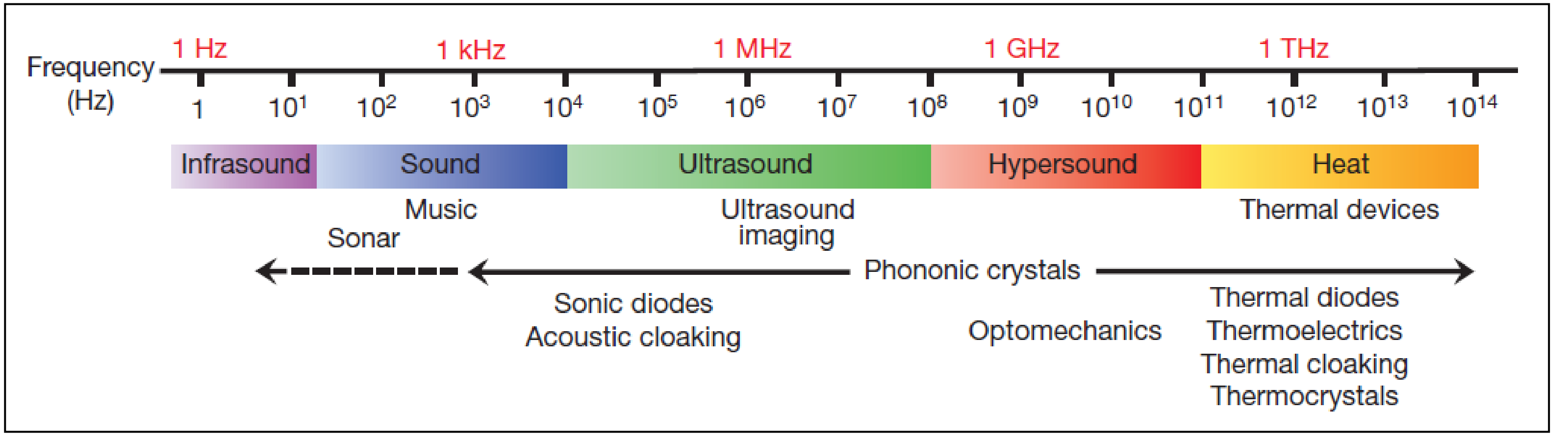
Phononic spectrum

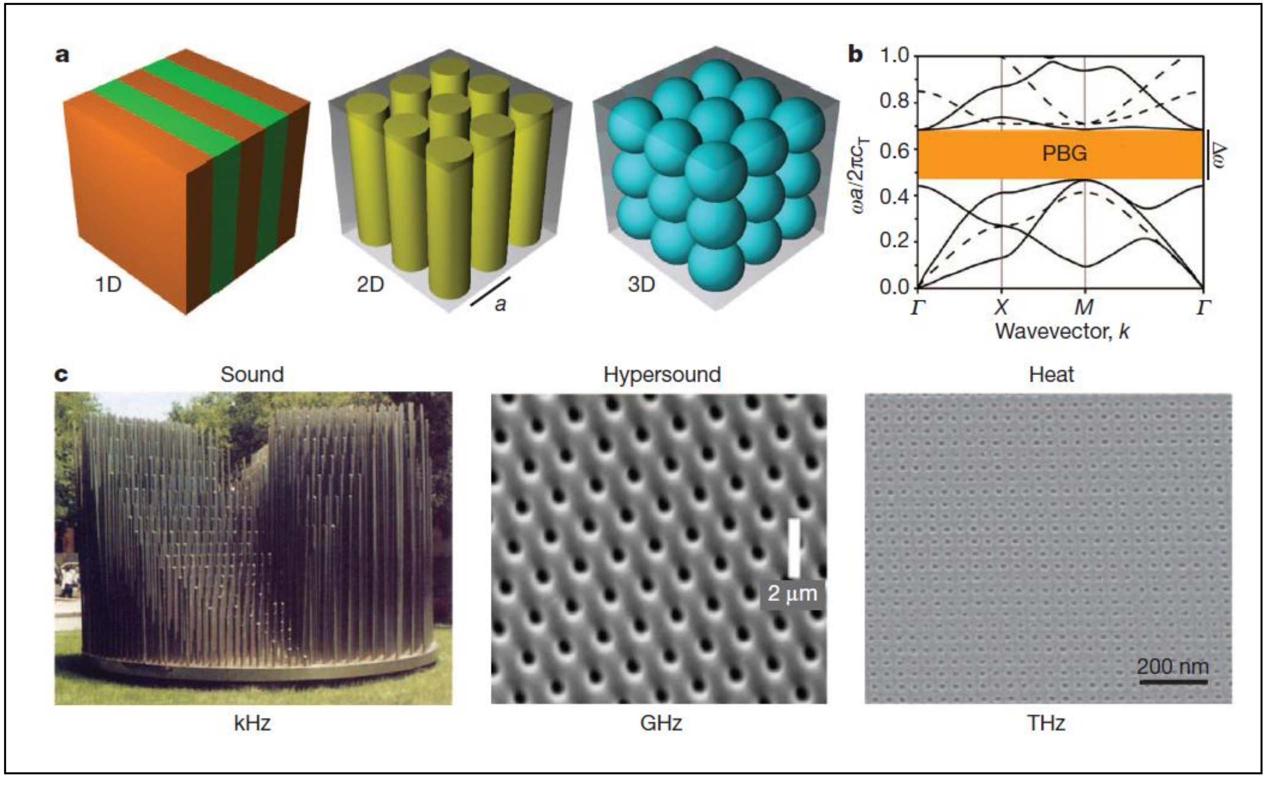




From M. Maldovan in Nature 503, 209 (2013)

Phononic spectrum





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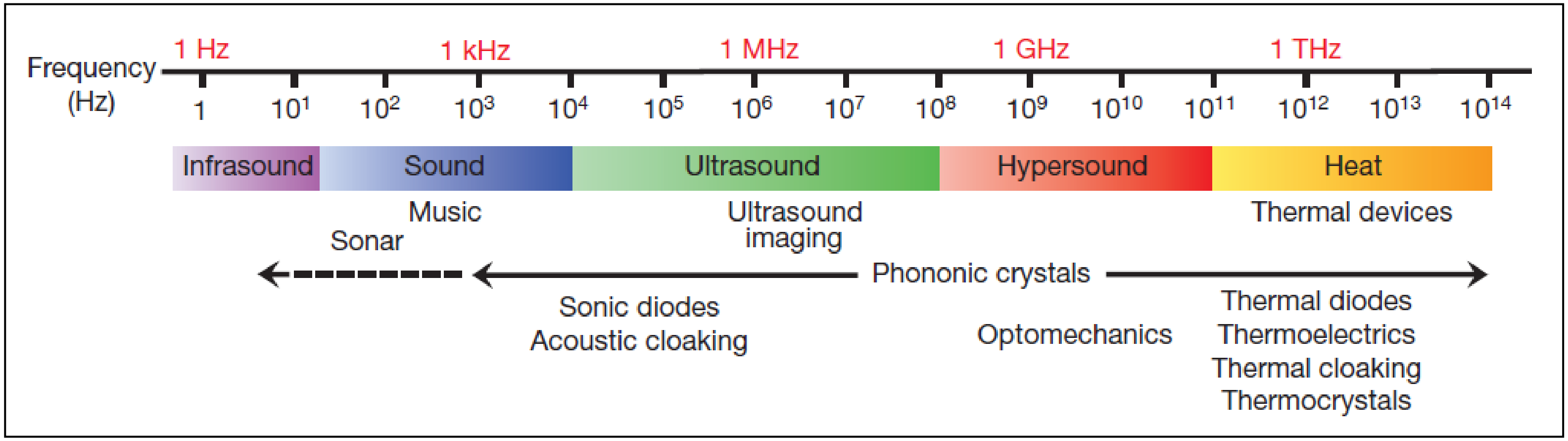
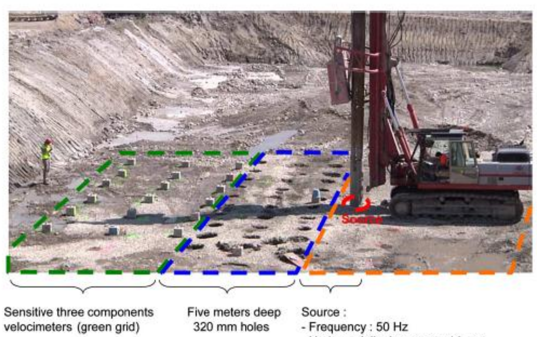
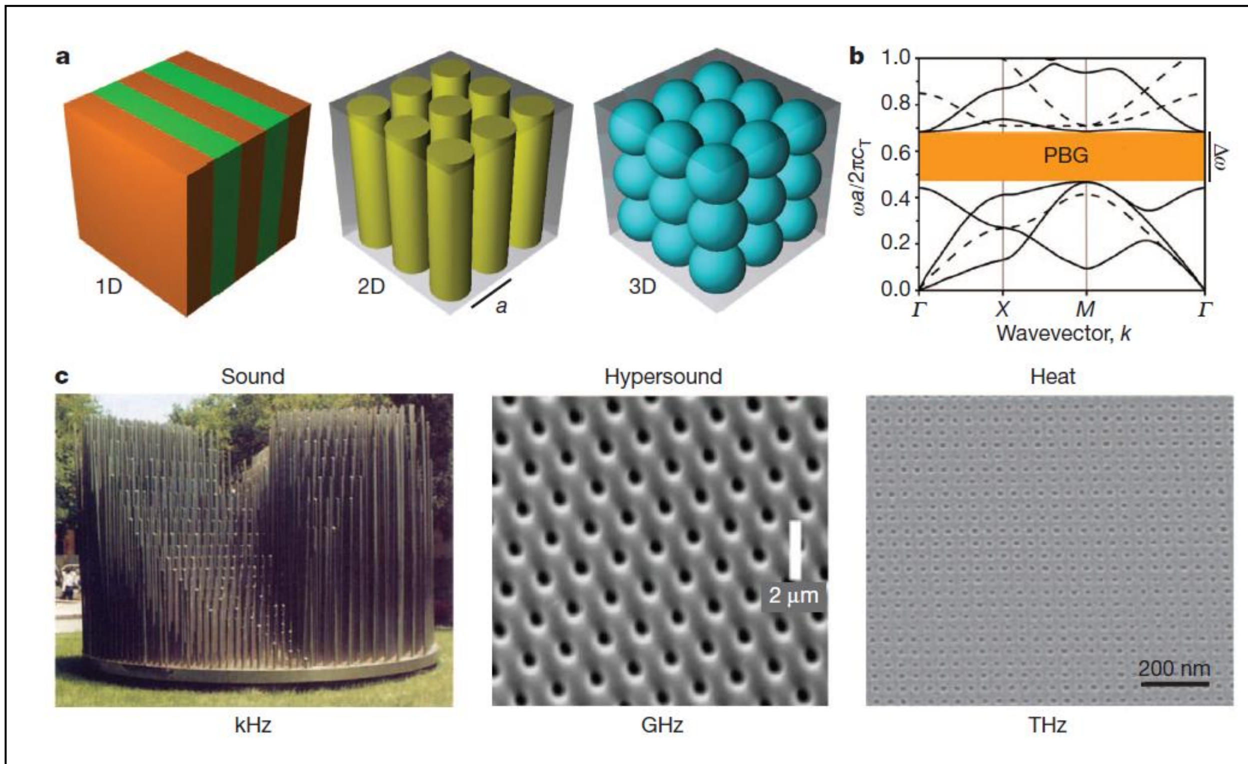




S. Brûlé et al, Phys. Rev. Lett. 112, 133901 (2014)



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Outline

1.Simple analytical models to introduce basic notions

- Band gaps and localized modes associated to defects
- Zeros of transmission and Fano resonances

2. One-dimensional (1D) multilayer structures

- Theoretical methods
- Dispersion curves, band gaps and localized modes
- Transmission coefficient: tunnelling (fast)transmission and resonant (slow) transmission

3. Two-dimensional (2D) Phononic crystals

- Theoretical methods
- Dispersion curves and complete band gaps (Bragg gaps and hybridization gaps)
- Local resonances and low frequency gaps
- Waveguide and cavity modes

4. Phononic crystal slabs and nanobeams

- Array of holes in a Si membrane
- Array of pillars on a thin membrane
- Surface waves in semi-infinite phononic crystals
- Nanobeam waveguides

Outline

5. Brief overview of refractive properties

- Negative refraction and focusing
- Self-collimation and beam splitting

6. Subwavelength structures and applications of metamaterials

- Effective properties (positive and negative dynamic parameters)
- Focusing and imaging. Superlens and heperlens
- Cloaking
- GRIN devices
- Metasurfaces. Resonating units and space coiling. Absorption. Phase manipulation

7. Active materials and some emerging topics

Non reciprocal behaviors . Time-space periodicity. PT symmetry. Topological phononics.

8. Dual phononic-photonic crystals (phoXonic) and Optomechanics

- Simultaneous phononic-photonic band gaps.
- Waveguide modes. Slow and fast modes
- Enhanced phonon-photon interaction in a cavity. Comparison of photoelastic and optomechanic effects
- Phononic and Phoxonic sensors

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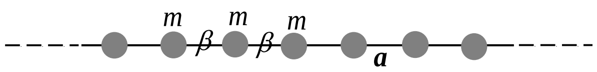
Analytical models with linear chains

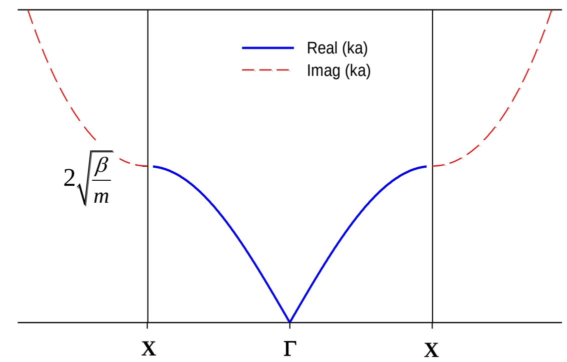
Monoatomic linear chain



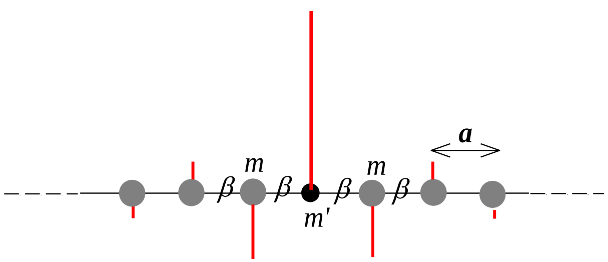
$$\frac{d^2u_n}{dt^2} = -m\,\omega^2 u_n = \beta(u_{n+1} + u_{n-1} - 2u_n)$$

$$u_n(t) = Ae^{i(k\,n\,a\,-\,\omega\,t)}$$

$$\cos(ka) = 1 - \frac{m\omega^2}{2\beta} \quad \Rightarrow \quad \text{k} = \text{k}' + \text{i k}''$$

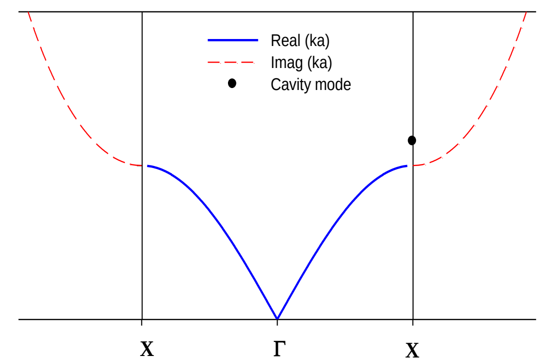


Monoatomic linear chain with a cavity

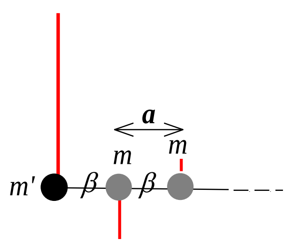


$$m' < m \;, \quad \omega_d = \sqrt{\frac{\beta}{m}}\frac{2m/m'}{\sqrt{2m/m'-1}}$$

$$m' = 0.5m \;, \quad \text{k}''\text{a} = \ln(3)$$

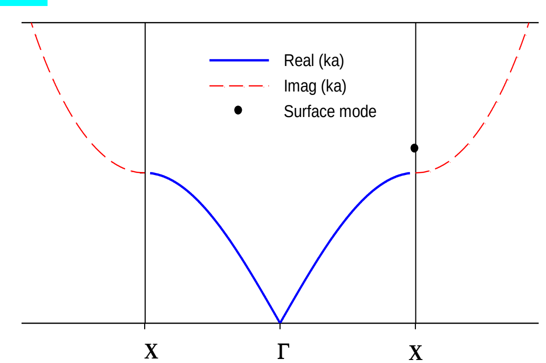


Monoatomic linear chain with a defect atom at the surface



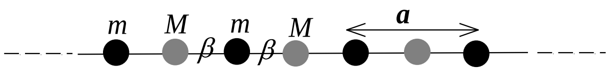
$$m' < m/2 \;, \quad \omega_s = \sqrt{\frac{\beta}{m}}\frac{m/m'}{\sqrt{m/m'-1}}$$

$$m' = 0.25m \;, \quad \text{k}''\text{a} = \ln(3)$$



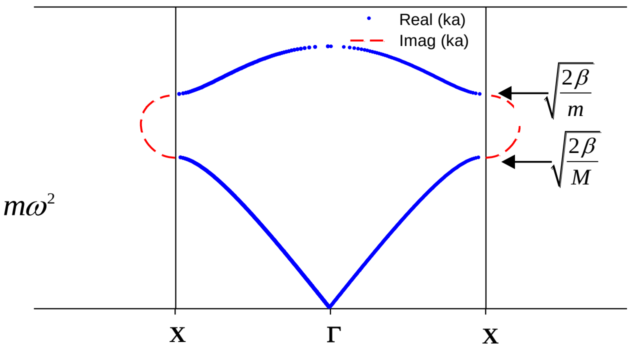
Analytical models with linear chains

Diatomic linear chain



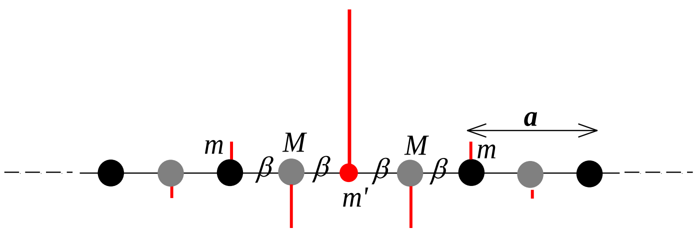
$$\cos(ka) = \frac{\gamma_1\gamma_2}{2\beta^2} - 1$$

$$\gamma_1 = 2\beta - M\omega^2 \quad , \quad \gamma_2 = 2\beta - m\omega^2$$

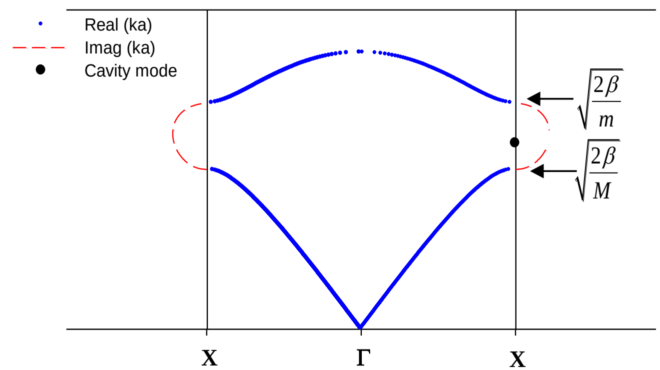


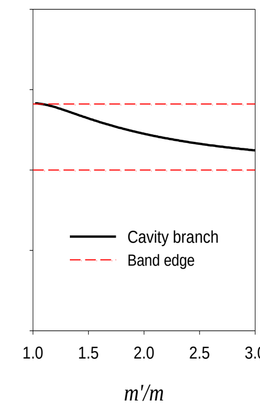
Diatomic linear chain with cavity

$$t - \frac{1}{t} + (m' - m)\omega^2 \frac{\gamma_M}{\beta^2} = 0 \quad , \quad t = e^{ika}$$

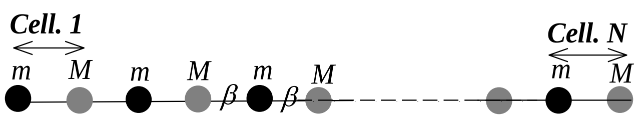


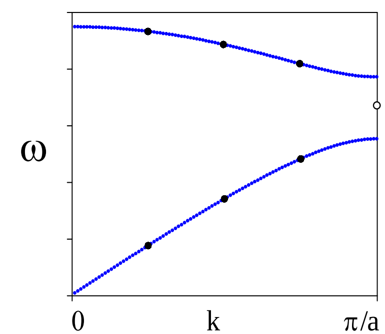
M = 2m , m' = 2.5m





Finite chain composed of N = 4 cells





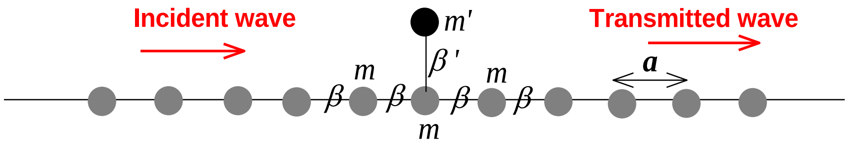
N-1 modes in each band : k = mπ/Na (m = 1, 2, …N-1)

+

One surface mode by gap

Analytical models with linear chains

Linear chain with attached stub



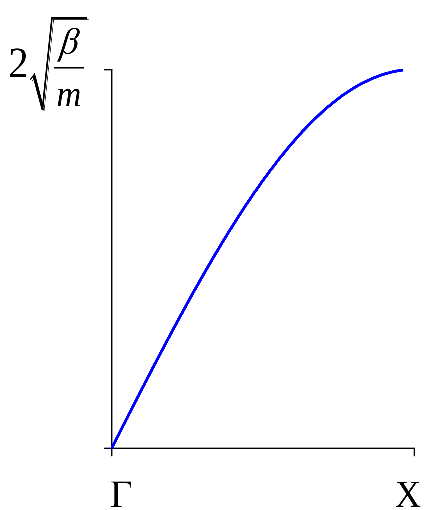
$$-m\omega^2 u_0 = \beta(u_1+u_{-1}-2u_0)+\beta'(v-u_0)$$
$$-m'\omega^2 v = \beta'(u_0-v)$$

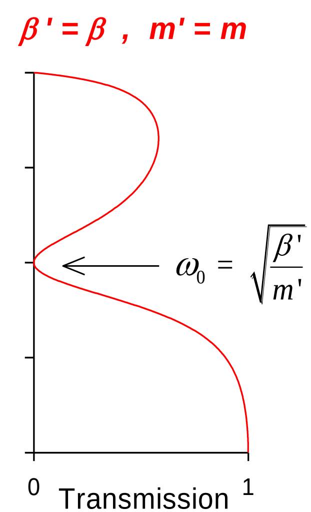
$$M(\omega)\,\omega^2 u_0 = \beta(u_1+u_{-1}-2u_0)$$

where $M(\omega) = m + \beta' m'/(\beta' - m'\omega^2)$

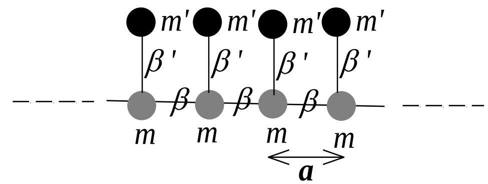
The system becomes equivalent to a linear chain
With a **dynamical mass defect** $M(\omega)$

$$\textit{Transmission}:\ t = \frac{\beta(t-1/t)}{2\beta(t-1)+M(\omega)\omega^2},\quad t=e^{ika}$$



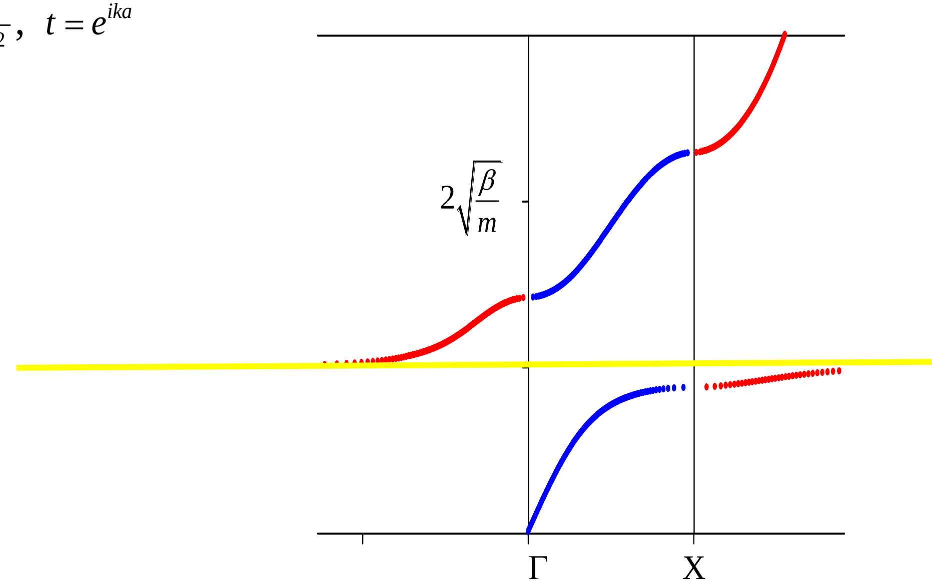


Periodic array of stubs



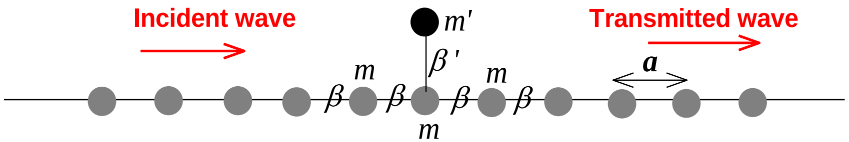
$$\cos(ka) = 1 - \frac{M(\omega)\omega^2}{2\beta}$$

Opening of a gap around ω_0
due to the local resonance



Analytical models with linear chains

Linear chain with attached stub



$$-m\omega^2 u_0 = \beta(u_1+u_{-1}-2u_0)+\beta'(v-u_0)$$
$$-m'\omega^2 v = \beta'(u_0-v)$$

$$M(\omega)\,\omega^2 u_0 = \beta(u_1+u_{-1}-2u_0)$$

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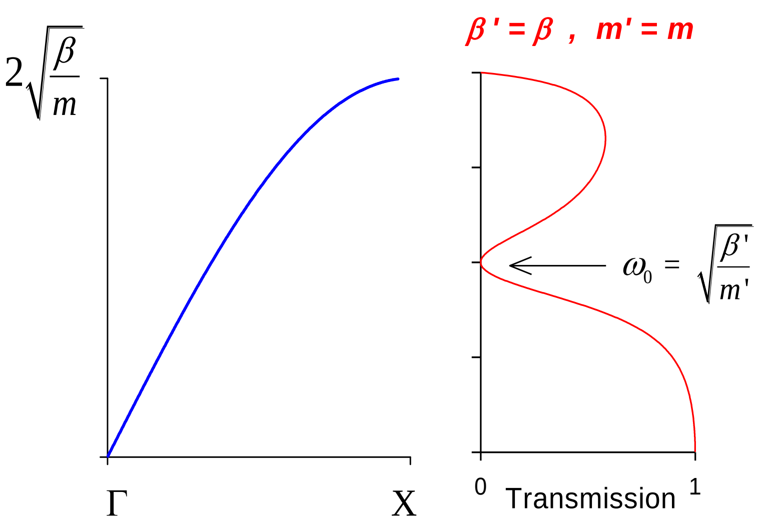
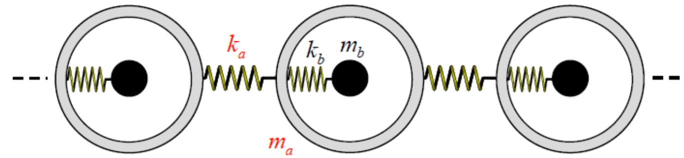


Periodic array of stubs



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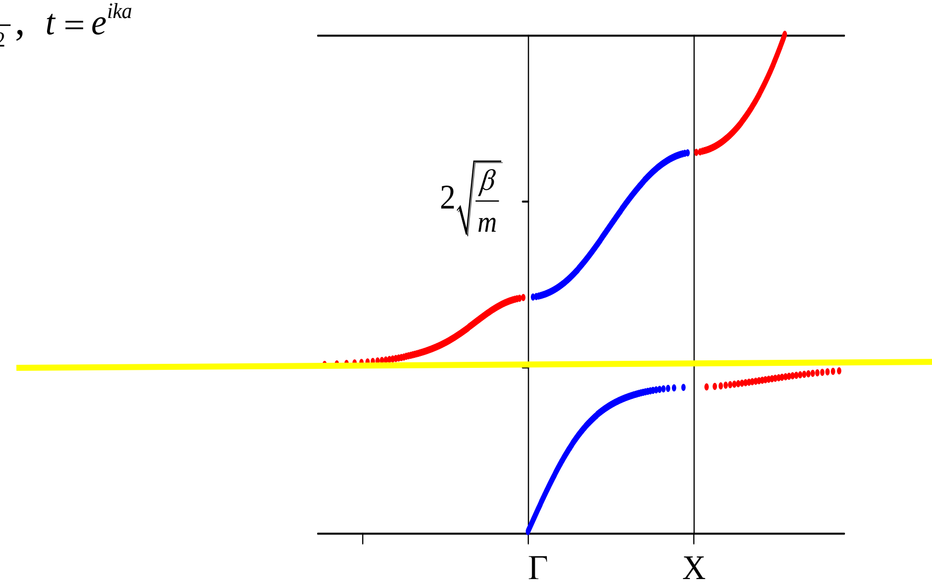
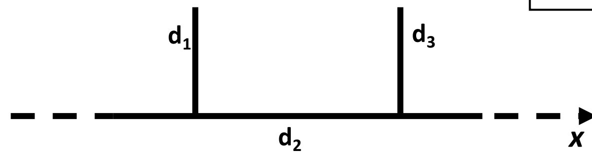
Opening of a gap around ω_0
due to the local resonance



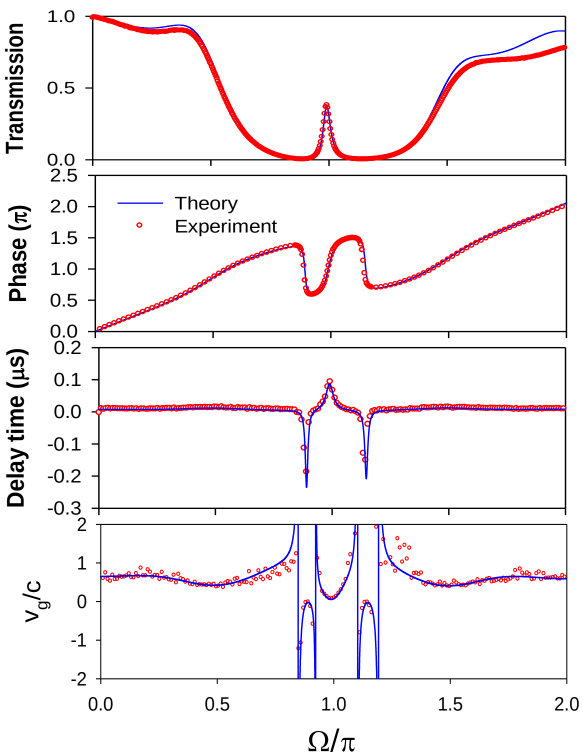
Fano resonances in a stubbed waveguide

Fano and EIT resonances



Symmetric (EIT-like) Fano resonnace

$d_1=0.44d_2$, $d_3 = 0.56d_2$, $d_2=1m$



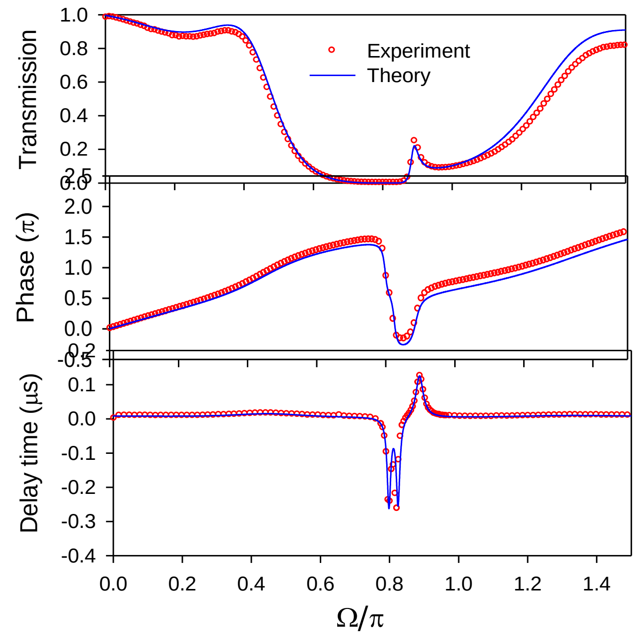
Transmission coefficient following the Fano lineshape around the resonance

$$\Omega = \omega d_2 n / c = \pi + \varepsilon$$

$$T = A\frac{(\varepsilon + q_1\Gamma)^2(\varepsilon - q_2\Gamma)^2}{\varepsilon^2 + \Gamma^2}$$

Asymmetric Fano resonance

$d_1=0.605d_2$, $d_3 = 0.625d_2$, $d_2=1m$



Transmission coefficient following the Fano lineshape around the resonance

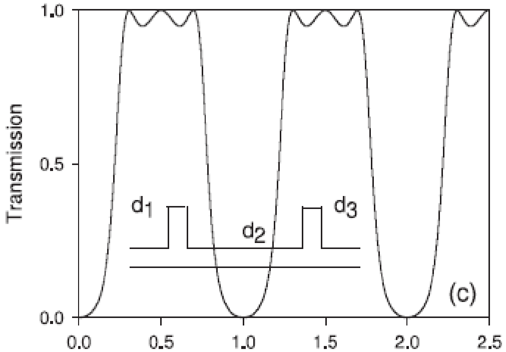
$$\Omega = \omega d_2 n / c = \pi + \varepsilon$$

$$T = B\frac{(\varepsilon - \varepsilon_R + q\Gamma)^4}{(\varepsilon - \varepsilon_R)^2 + \Gamma^2}$$

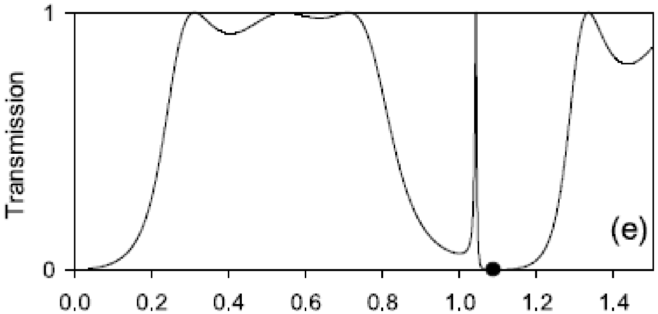
Γ, q, ε_R, q_1 and q_2 depend on the geometrical paremeters d_1, d_2 and d_3

Mouadili et al., JPCM 26, 505901 (2014)

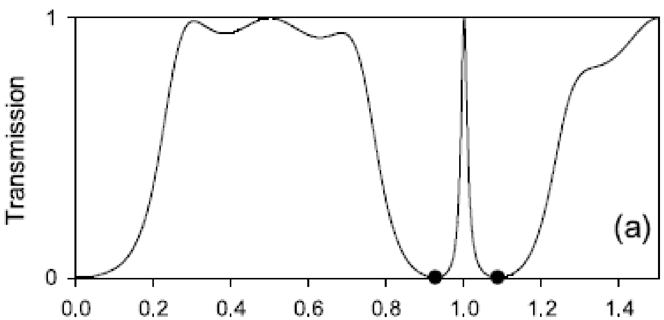
Fano and EIT resonances



$d_1 = d_3 = 0.5 d_2$

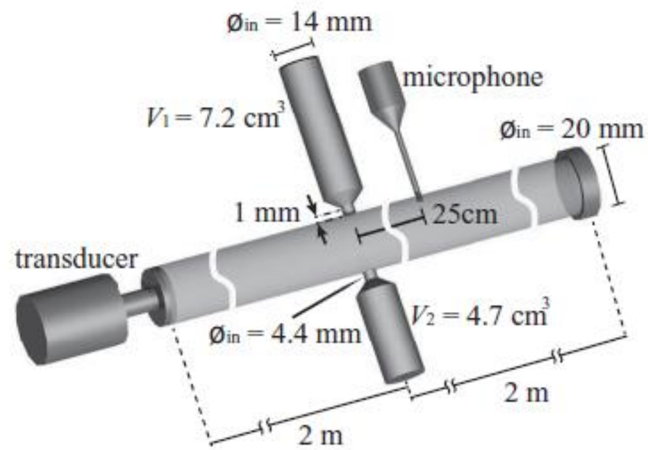


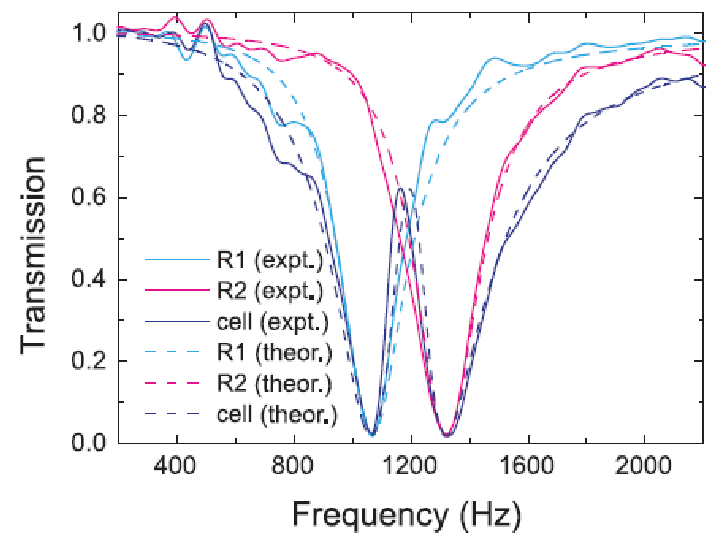
$d_1 = d_3 = 0.46 d_2$



$d_1 = 0.46 d_2$, $d_3 = 0.54 d_2$

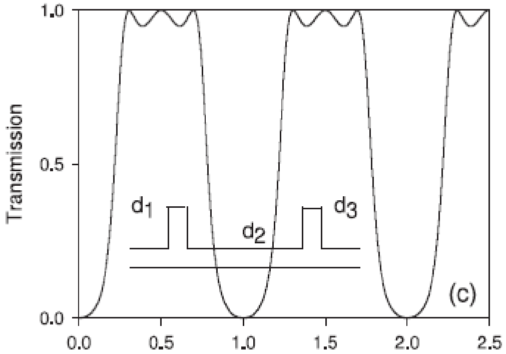
Transmission gaps and Fano resonances in an acoustic waveguide: Analytical model,
E.H. El Boudouti et al., JPCM 20, 255212 (2008)



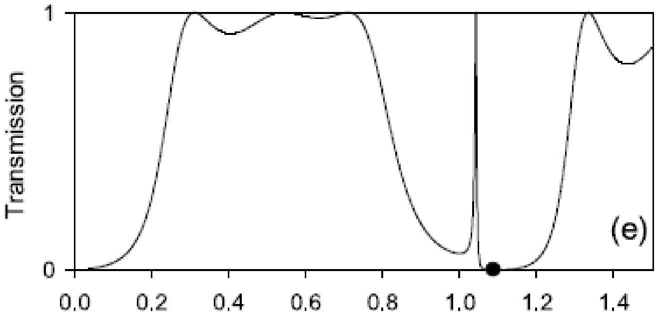


Acoustic transparency and slow sound using detuned acoustic resonators
A. Santillan et al, Phys. Rev. B 84, 064304 (2011)

Fano and EIT resonances



$d_1=d_3=0.5d_2$



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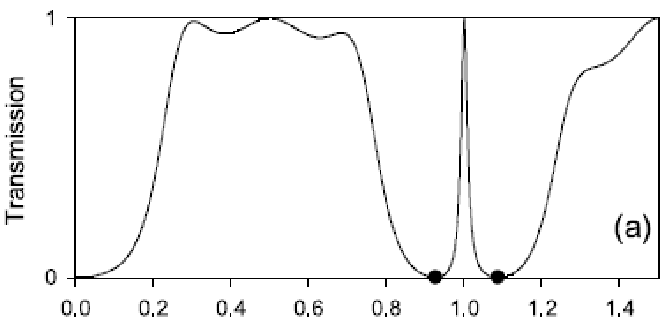

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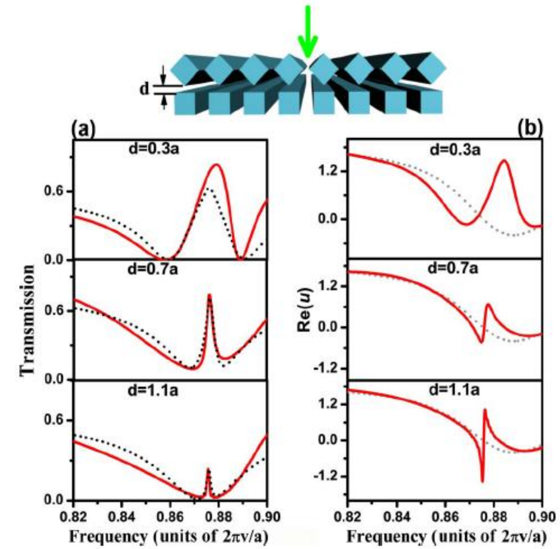
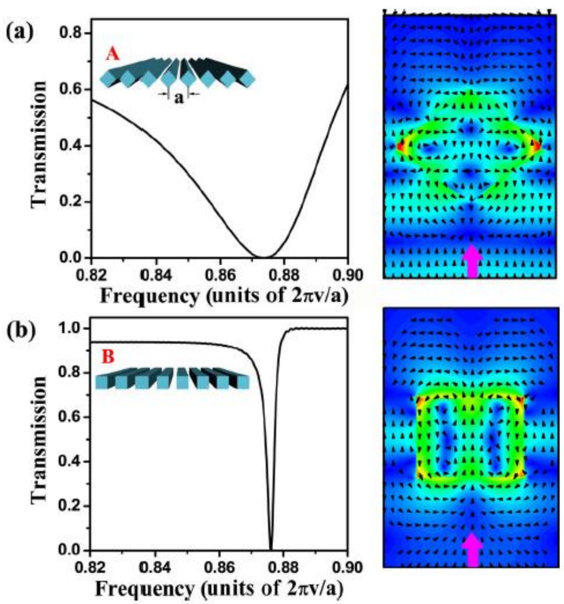
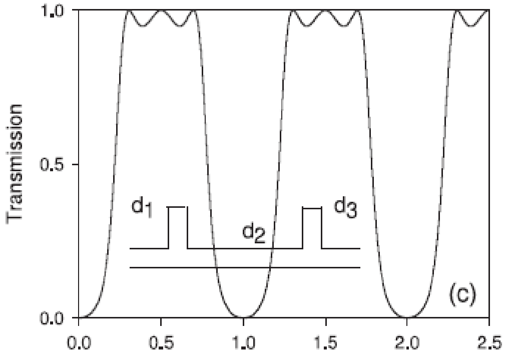


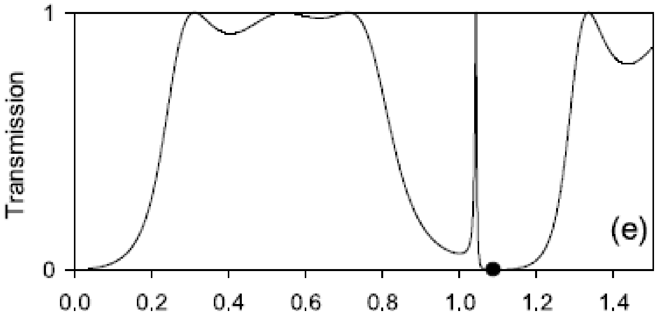


Acoustic analog of EIT in periodic arrays of square rods
F. Liu et al, Phys. Rev. B 82, 026601 (2010)

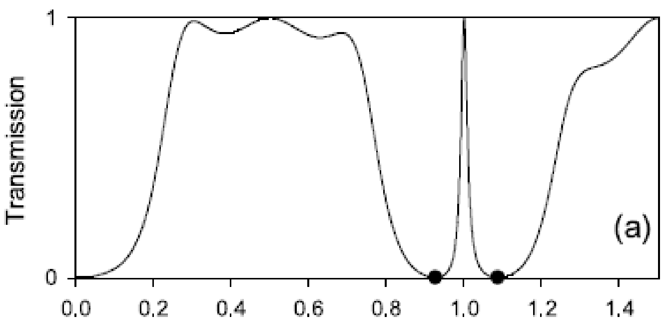
Fano and EIT resonances



$d_1=d_3=0.5d_2$

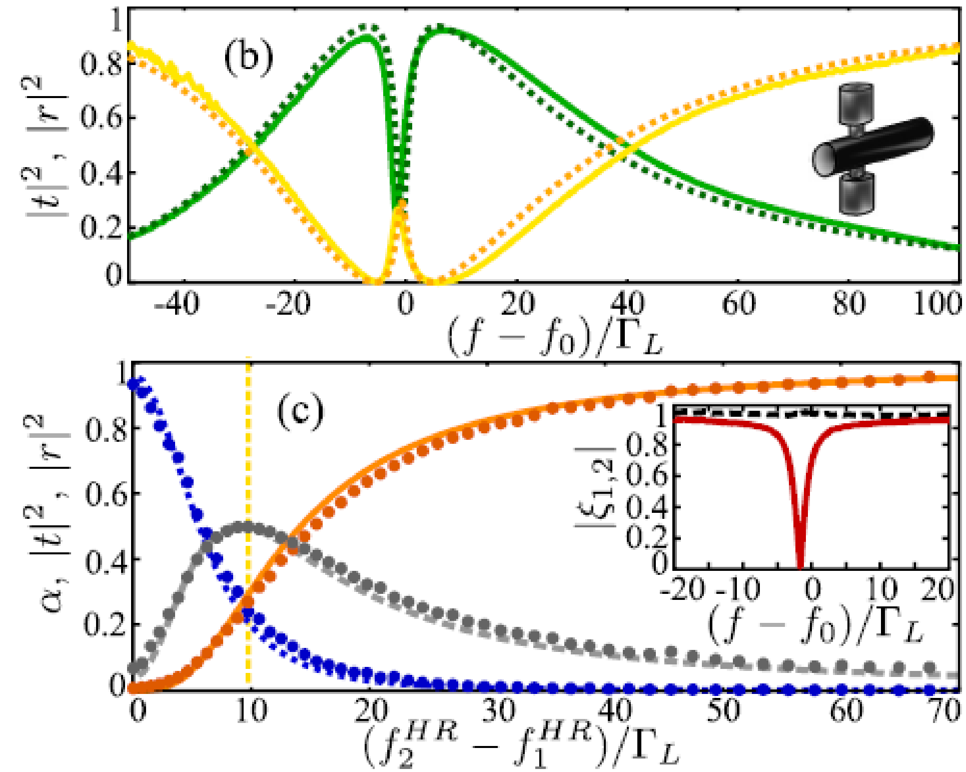


$d_1=d_3=0.46d_2$



$d_1=0.46d_2\,,\ d_3=0.54d_2$

Transmission gaps and Fano resonances in an acoustic waveguide: Analytical model,
E.H. El Boudouti et al., JPCM 20, 255212 (2008)



Control of acoustic absorption in 1D scattering by resonant scatterers

A. Merkel, G. Theocharis, O. Richoux, V. Romero-García, and V. Pagneux, Appl. Phys. Lett. **107**, 244102 (2015)

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- Band gaps and localized modes associated to defects
- Zeros of transmission and Fano resonances

2. One-dimensional (1D) multilayer structures

- Theoretical methods
- Dispersion curves, band gaps and localized modes
- Transmission coefficient: tunnelling (fast)transmission and resonant (slow) transmission

3. Two-dimensional (2D) Phononic crystals

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- Local resonances and low frequency gaps
- Waveguide and cavity modes

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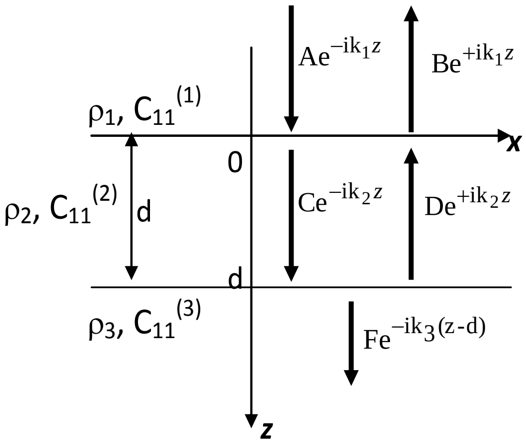
- Array of holes in a Si membrane
- Array of pillars on a thin membrane
- Surface waves in semi-infinite phononic crystals
- Nanobeam waveguides

Transfer Matrix Method

Transmission across a layer 2 inserted between two substrates 1 and 3



Continuity of the displacement field u(z) at z = 0 and z = d :

$$A + B = C + D$$
$$Ce^{-ik_2d} + De^{+ik_2d} = F$$

Continuity of the stress $C_{11}\dfrac{du(z)}{dz}$ at z = 0 and z = d :

$$C_{11}^{(1)}k_1(A-B) = C_{11}^{(2)}k_2(C-D)$$
$$C_{11}^{(2)}k_2\left(Ce^{-ik_2d} - De^{+ik_2d}\right) = C_{11}^{(3)}k_3$$

$$\begin{pmatrix}1 & 1\\ k_1 & -k_1\end{pmatrix}\begin{pmatrix}A\\ B\end{pmatrix} = F\,\boldsymbol{T}\begin{pmatrix}1\\ k_3\end{pmatrix}$$

where

$$\boldsymbol{T} = \begin{pmatrix}\cos(k_2d) & \dfrac{i\sin(k_2d)}{Z_2}\\ i\,Z_2\sin(k_2d) & \cos(k_2d)\end{pmatrix}$$

T is the transfer matrix

wave vector $k_i = \dfrac{\omega}{v_i}$

longitudinal velocity of sound $v_i = \sqrt{\dfrac{C_{11}^{(i)}}{\rho_i}}$

acoustic impedance $Z_i = \rho_i v_i$

Transmission coefficient

$$t = \frac{F}{A} = \frac{4Z_1Z_2\,e^{-ik_2d}}{(Z_1+Z_2)(Z_1-Z_2)+(Z_1-Z_2)(Z_2-Z_3)e^{-i2k_2d}}$$

Reflection coefficient

$$r = \frac{B}{A} = \frac{(Z_1-Z_2)(Z_2+Z_3)+(Z_1+Z_2)(Z_2-Z_3)e^{-i2k_2d}}{(Z_1+Z_2)(Z_2+Z_3)+(Z_1-Z_2)(Z_2-Z_3)e^{-i2k_2d}}$$

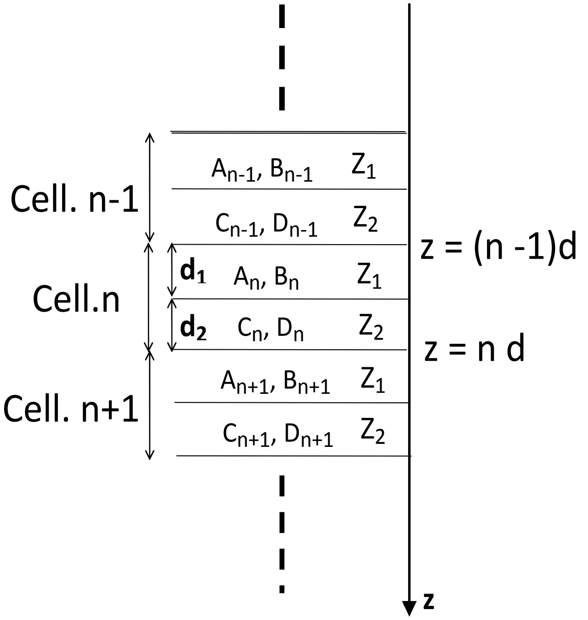
Transfer Matrix Method

Theoretical methods

Periodic structure made of two layers 1 and 2

1) A transfer matrix can relate the amplitudes A_n, B_n of layer 1 in the cell n to the amplitudes A_{n+1}, B_{n+1} of layer 1 in the cell n+1

$$\begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} A_n \\ B_n \end{pmatrix}$$



2) Bloch theorem: $\begin{pmatrix} A_n \\ B_n \end{pmatrix} = e^{-inkD} \begin{pmatrix} A_0 \\ B_0 \end{pmatrix}$

3) The dispersion relation are obtained by writing the boundary conditions at two consecutive interfaces

$$\cos(kD) = \cos(k_1 d_1)\cos(k_2 d_2) - 0.5\left(\frac{Z_1}{Z_2} + \frac{Z_1}{Z_2}\right)\sin(k_1 d_1)\sin(k_2 d_2)$$

k is the Bloch wave vector and $D = d_1 + d_2$ is the period.

Allowed bands → k real → $-1 < \cos(kD) < 1$

Forbidden bands → k complex → $\cos(kD) > 1$ or $\cos(kD) < -1$

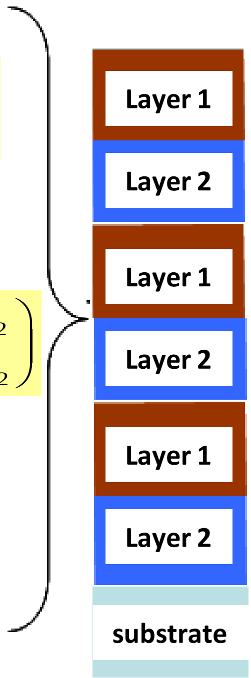
Green function Method
Interface response theory

Theoretical methods

Layer 1 $g_1^{-1}(M_m M_m) = \begin{pmatrix} a_1 & b_1 \\ b_1 & a_1 \end{pmatrix}$

Layer 2 $g_2^{-1}(M_m M_m) = \begin{pmatrix} a_2 & b_2 \\ b_2 & a_2 \end{pmatrix}$

substrate $g_{sub}^{-1}(0,0) = -F_s$



$$g_{tot}^{-1}(MM) = \begin{pmatrix} -F_{sub}+a_2 & b_2 & 0 & 0 & 0 & 0 & 0 \\ b_2 & a_1+a_2 & b_1 & 0 & 0 & 0 & 0 \\ 0 & b_1 & a_1+a_2 & b_2 & 0 & 0 & 0 \\ 0 & 0 & b_2 & a_1+a_2 & b_1 & 0 & 0 \\ 0 & 0 & 0 & b_1 & a_1+a_2 & b_2 & 0 \\ 0 & 0 & 0 & 0 & b_2 & a_1+a_2 & b_1 \\ 0 & 0 & 0 & 0 & 0 & b_1 & a_1 \end{pmatrix}$$

M represents the space of interfaces between different layers

$a_i = -F_i C_i/S_i$
$bi = Fi/Si$
$F_i = -j\,\omega\,\rho_i\,C_{Li}$
$C_i = Cosh(-j\,\omega\,d_i/\,C_{Li})$
$S_i = Sinh(-j\,\omega\,d_i/\,C_{Li})$
$i = 1, 2$

*Local DOS = -(1/π) *Im* [g(MM)] on each interface M

*Total DOS (integrating the LDOS over the whole space)

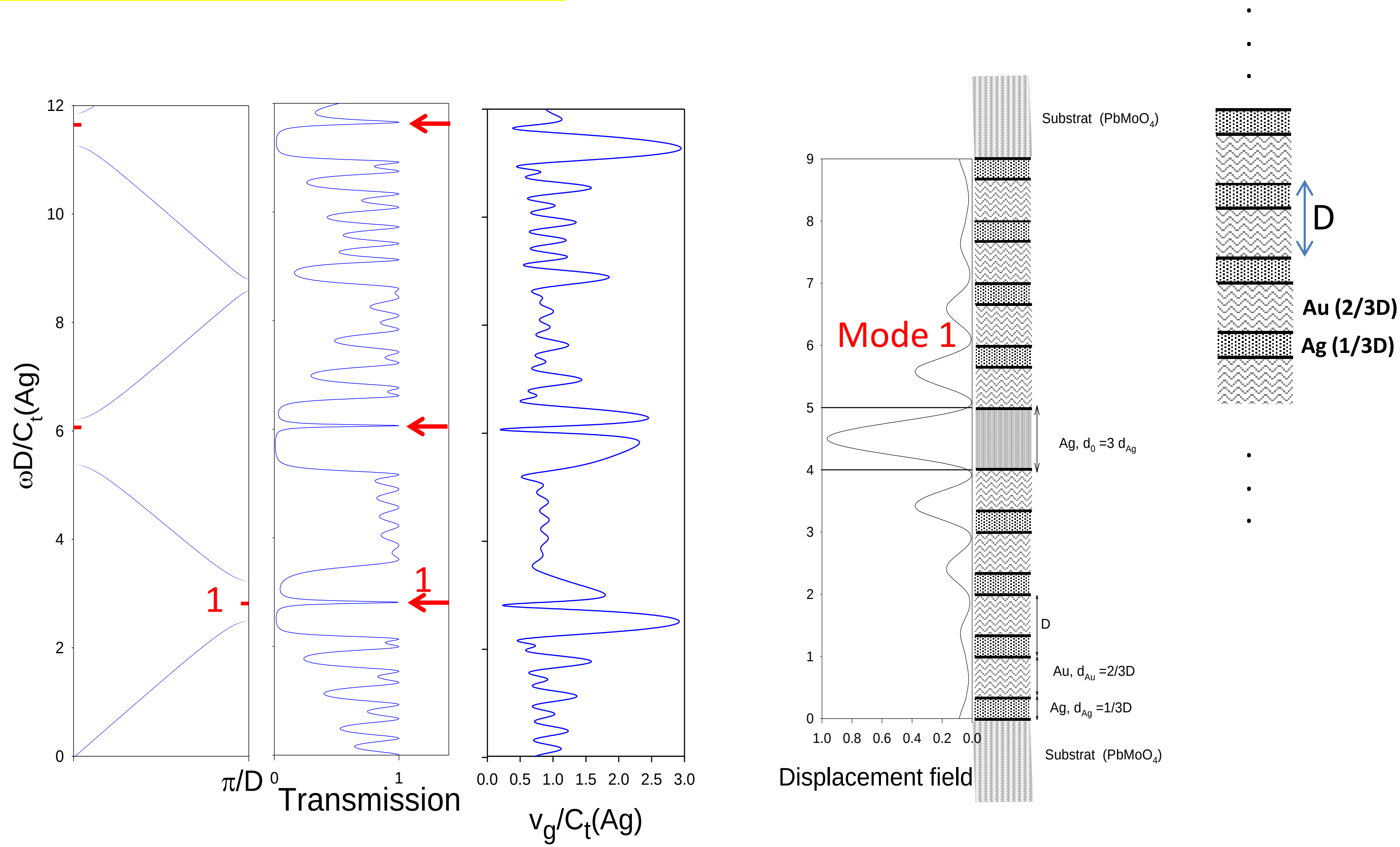
***Transmission and Reflection coefficients**

Longitudinal acoustic waves in a superlattice

-Example of a Ag/Au superlattice

- Effect of a cavity layer

Band structure and transmission



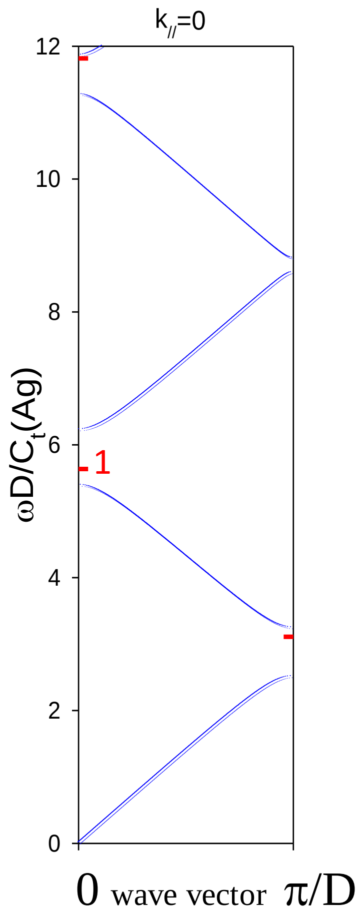
$$\cos(kD) = \cos(k_1 d_1)\cos(k_2 d_2) - 0.5\left(\frac{Z_1}{Z_2} + \frac{Z_1}{Z_2}\right)\sin(k_1 d_1)\sin(k_2 d_2)$$

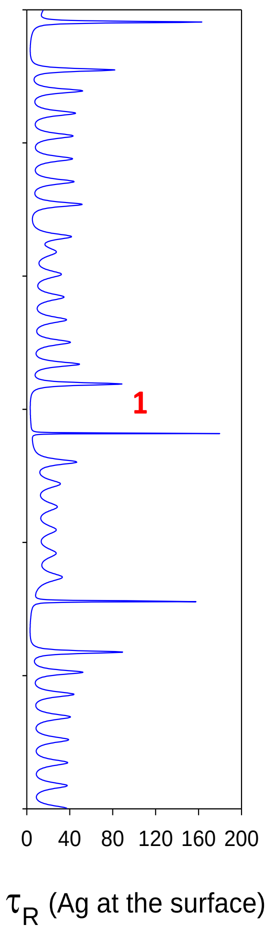
Dieleman et al. Phys. Rev. B 64, 174304 (2001)

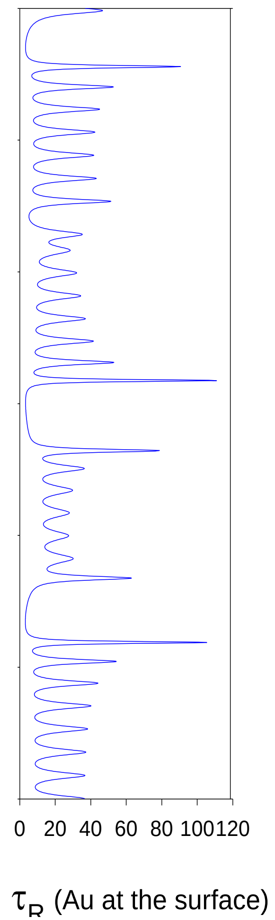
Longitudinal acoustic waves in a superlattice

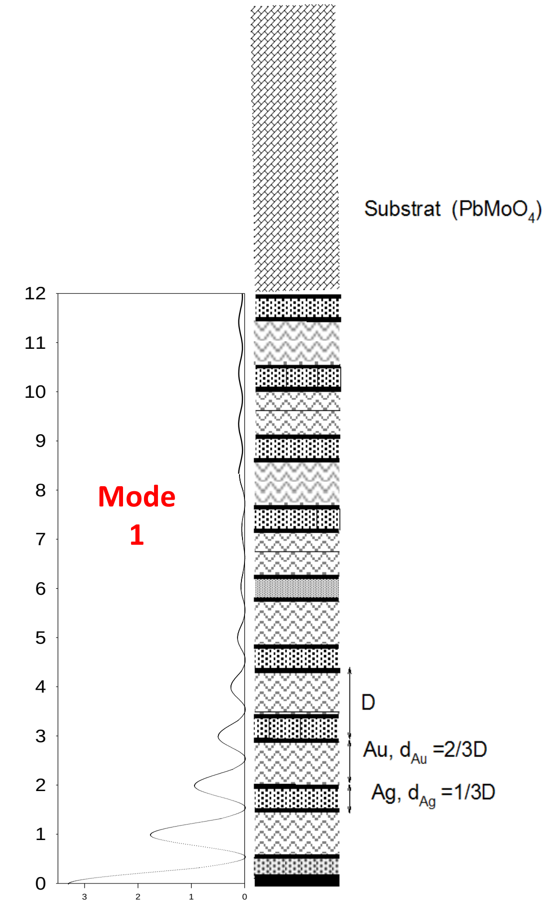
-Example of a Ag/Au superlattice
- Effect of a surface layer

Band structure and transmission





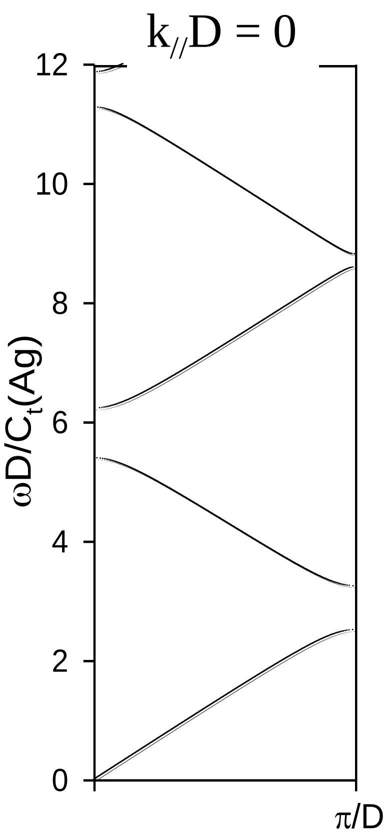


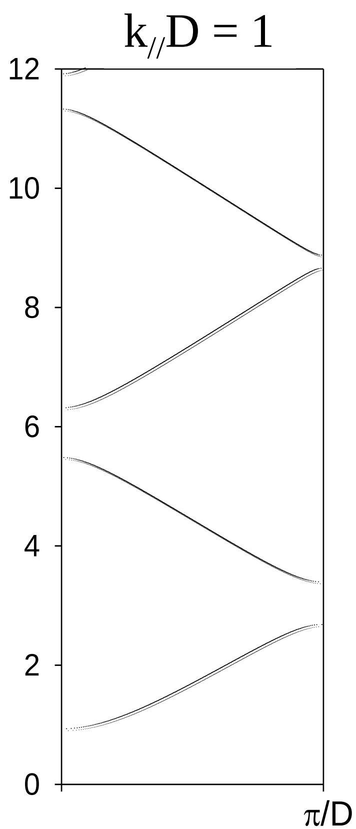


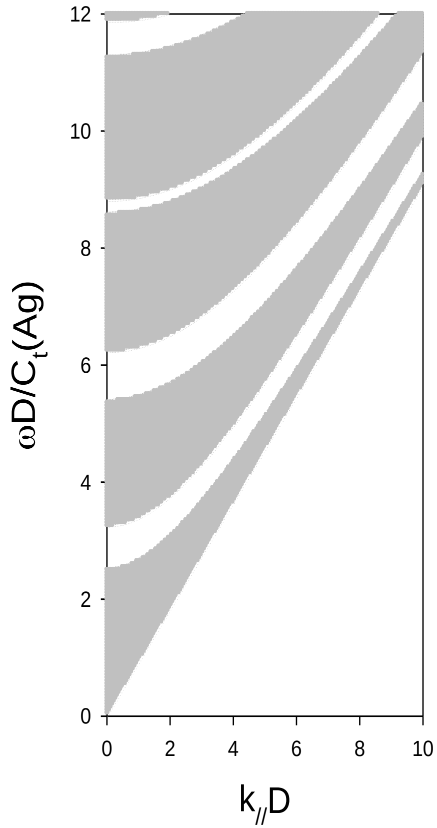
Band structure and transmission

Projected band structure

As a function of $k_{//}$







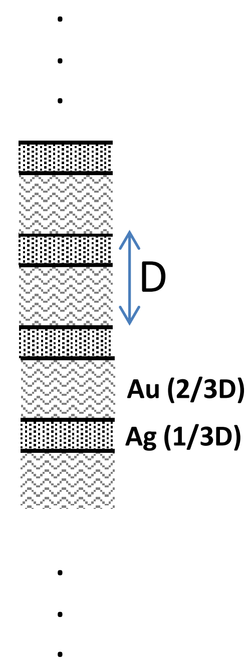
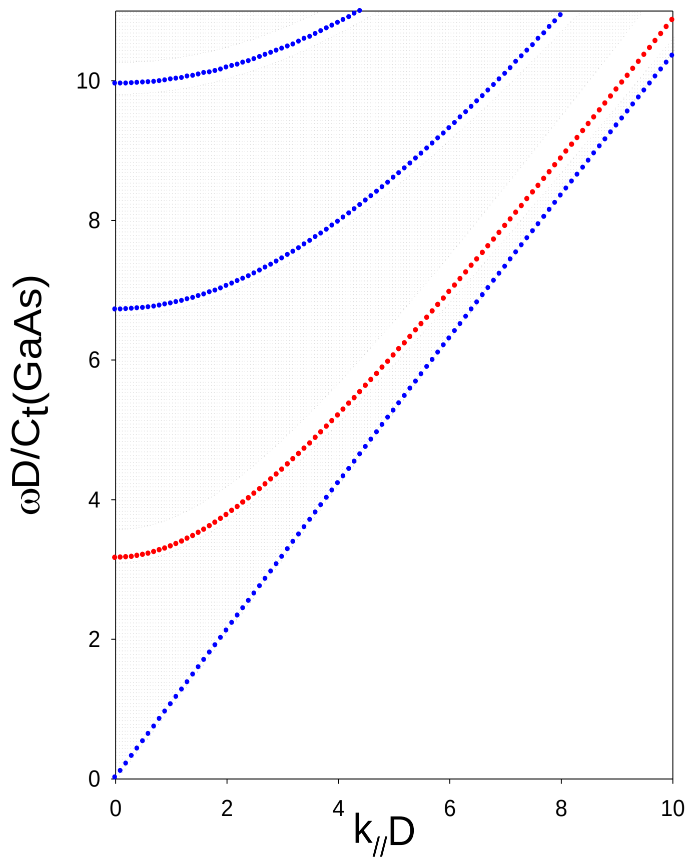


Illustration of surface modes

as a function of $k_{//}$ in a GaAs-AlAs superlattice

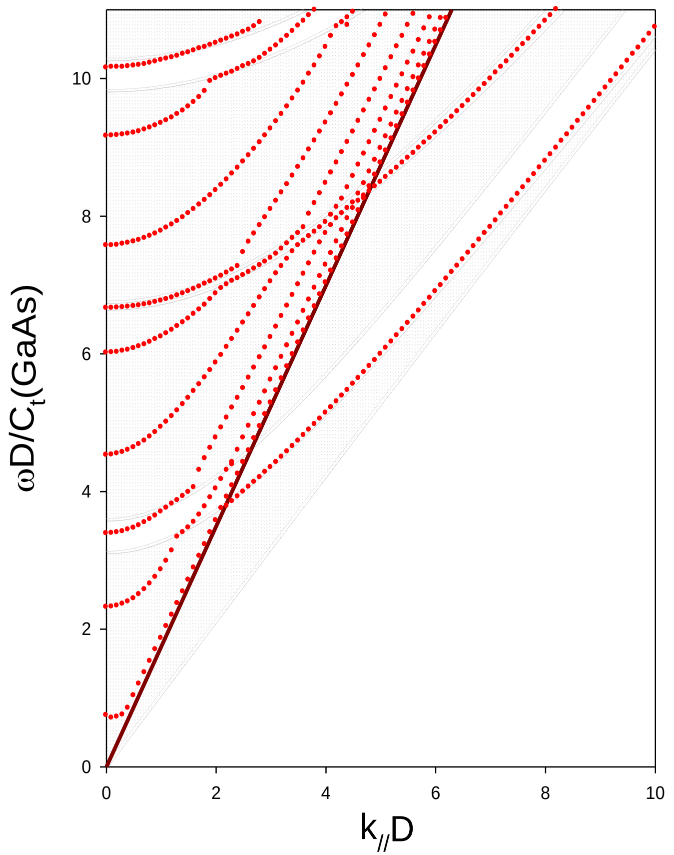
Band structure and transmission



GaAs at the surface

Surface layer with thickness ds=0.7 d(GaAs)

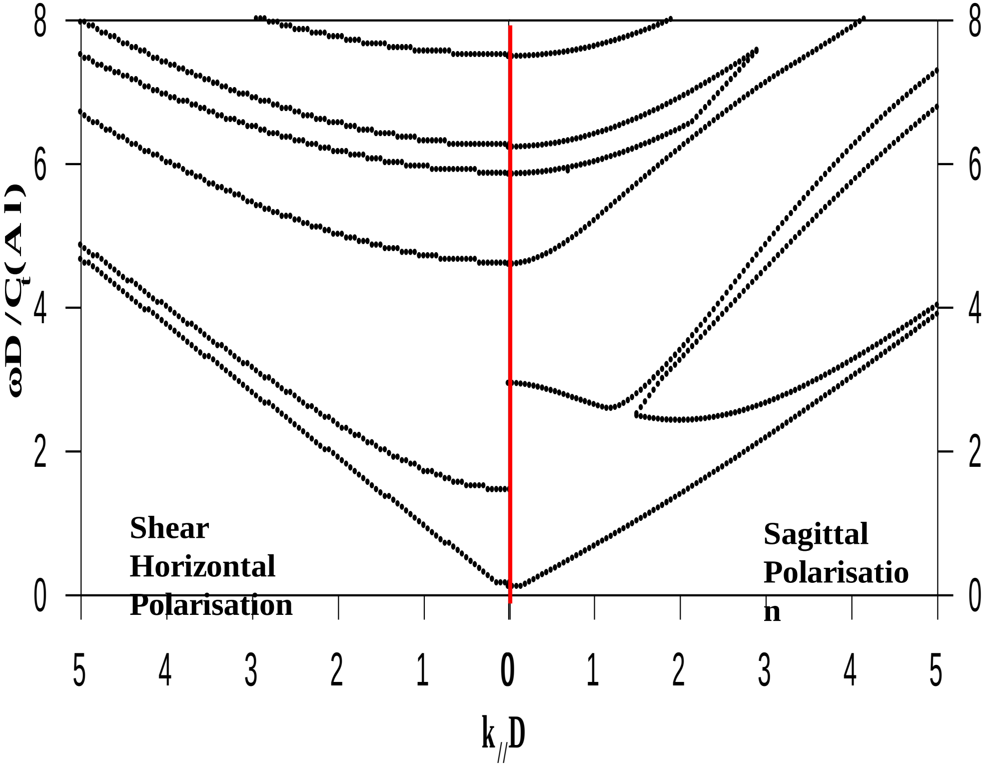
Surface layer with thickness ds=0.3 d(GaAs)



A Si layer of thickness d_{Si}= 3D at the surface

Projected band structure
Al-W superlattice

Band structure and transmission

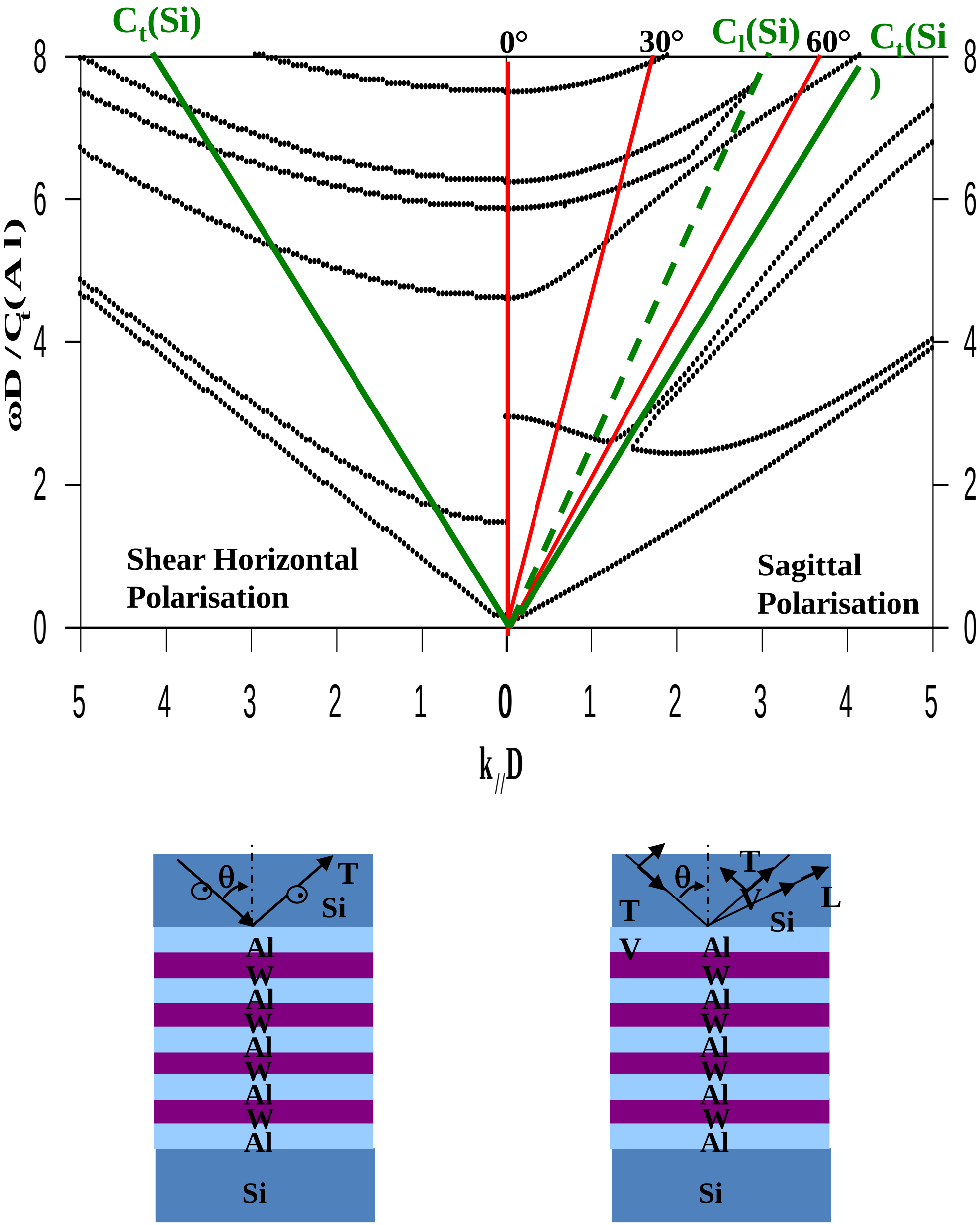


Looking for an omnidirectional transmission gap

Bousfia et al., Surf. Sci. 482-5, 1175 (2001)

Projected band structure
Al-W superlattice

Band structure and transmission



Bousfia et al., Surf. Sci. 482-5, 1175 (2001)

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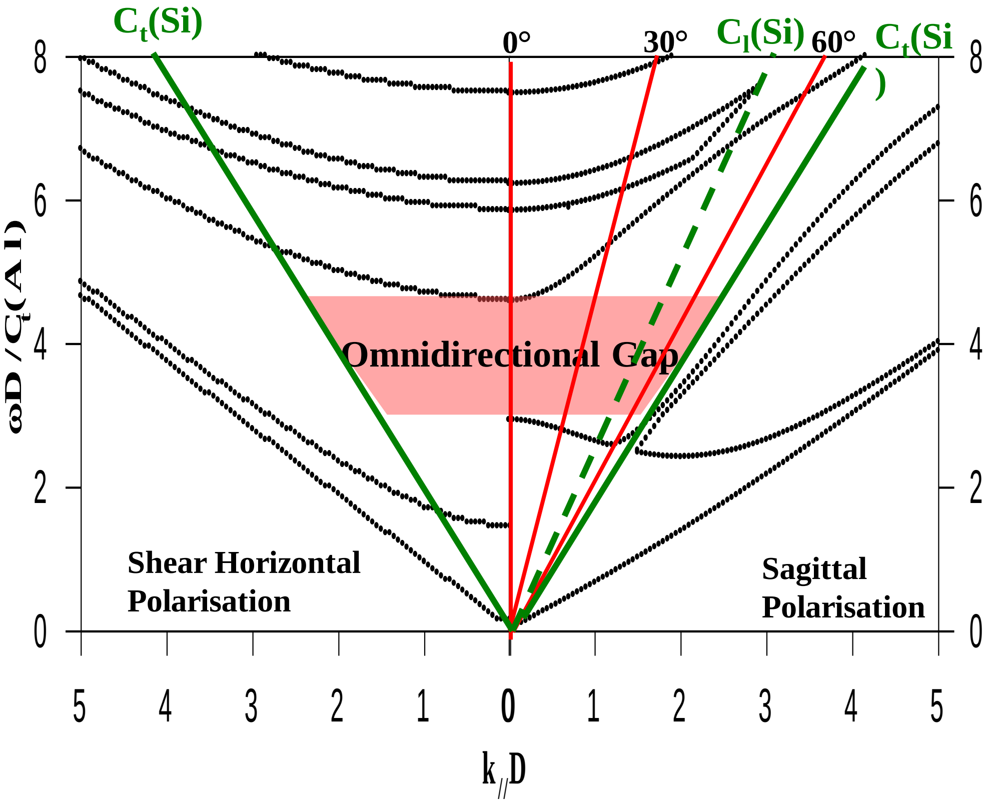

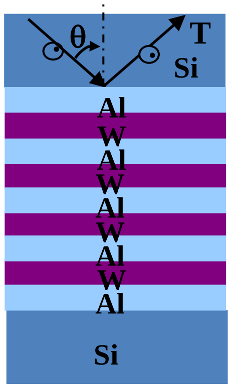

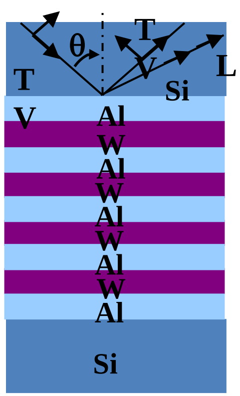

Bousfia et al., Surf. Sci. 482-5, 1175 (2001)

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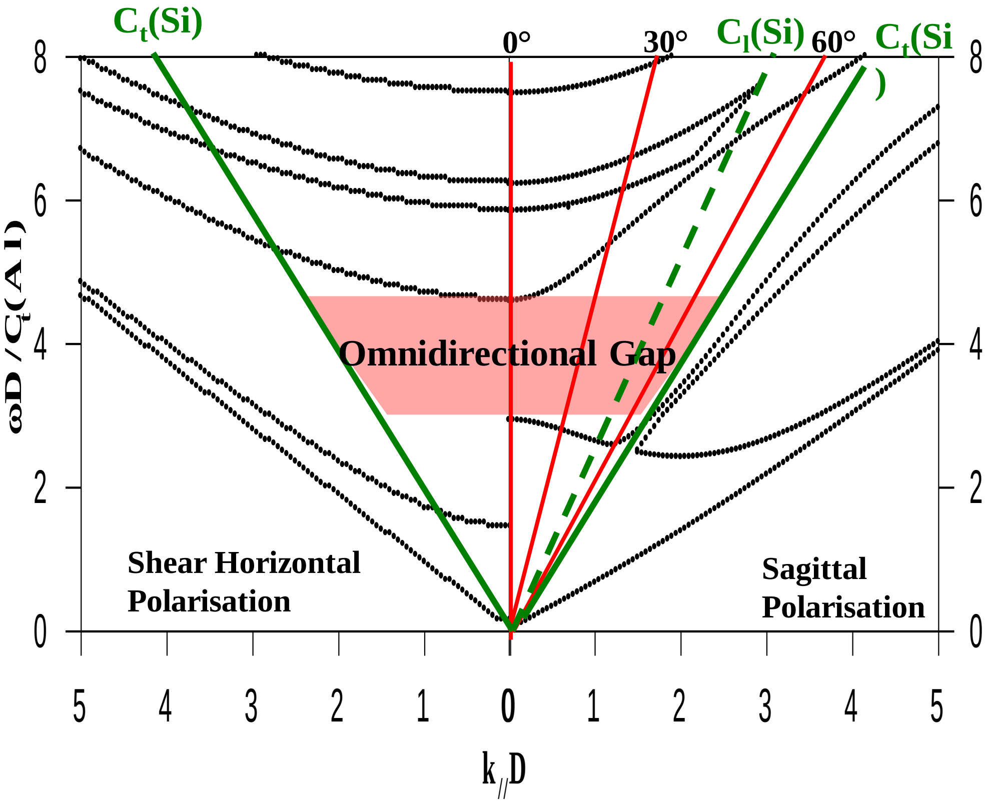

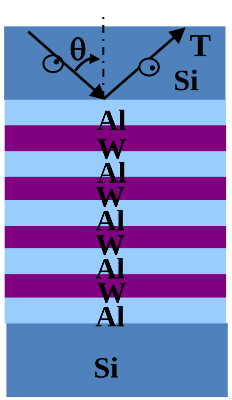

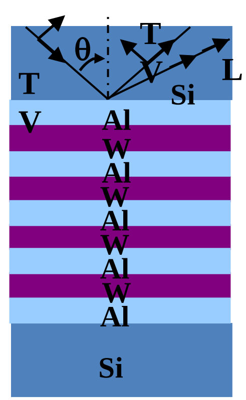

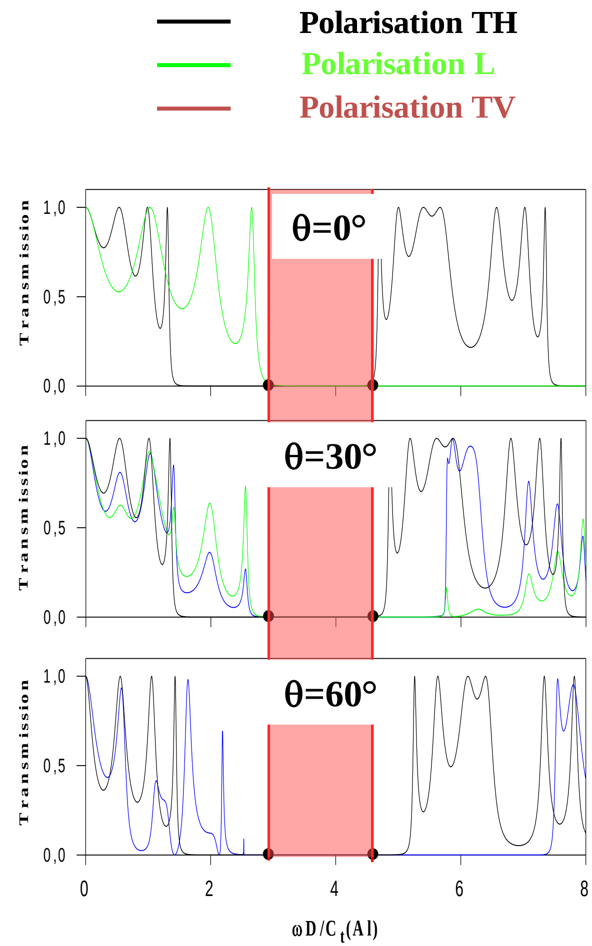

Transmission coefficients at different incidence angles

Bousfia et al., Surf. Sci. 482-5, 1175 (2001)

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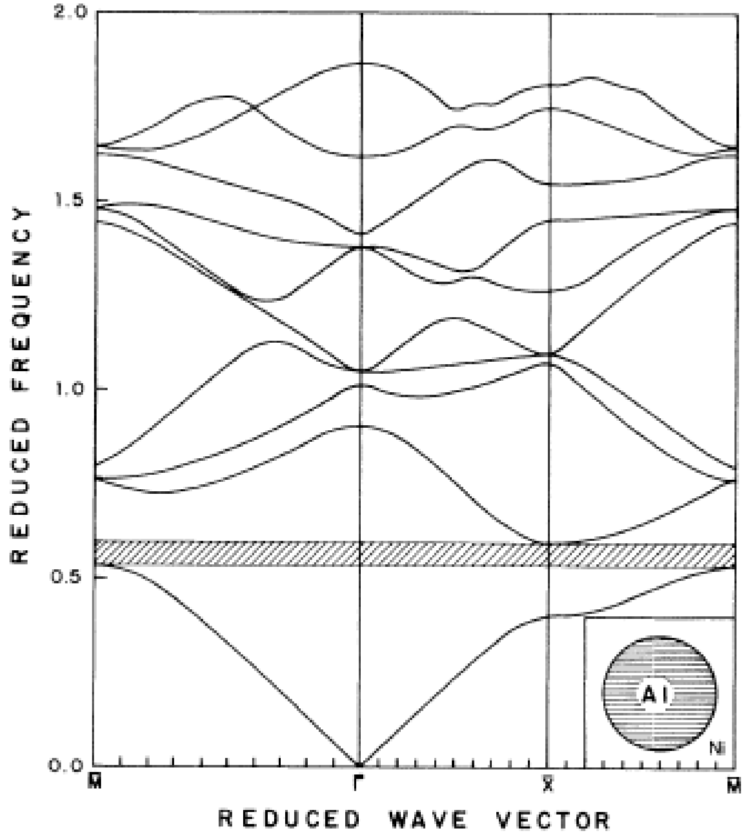
Introduction

Out-of_plane propagation in a 2D phononic crystal of Al cylinders in a Ni matrix. Filling fraction=40%



M.S. Kushwaha, P. Halevi, L. Dobrzynski, and B. Djafari Rouhani, Phys. Rev. Lett, 71, 2022 (1993)

Eusebio Sempere's sculpture in Madrid

R. Martínez-Sala, J. Sancho, J. V. Sánchez, V. Gómez, J. Llinares & F. Meseguer, Nature, 378,241 (1995)

Equations of motion:

$$\rho \frac{\delta^2 u_i}{\delta t^2} = \sum_j \frac{\delta \sigma_{ij}}{\delta x_j} = \sum_{j,kl} C_{ijkl} \frac{\delta u_k}{\delta x_l}$$

Most Usual Methods

- ✓PWE (Plane Wave Expansion)
- ✓FDTD (Finite Difference Time Domain)
- ✓FEM (Finite Element Method)
- ✓MST (Multiple Scattering Theory)

Main calculated properties

- ✓Dispersion
- ✓Transmission
- ✓Reflection
- ✓Map of the fields

Symbolic equations in 1D:

$$\rho \frac{\delta^2 u}{\delta t^2} = \frac{\delta \sigma}{\delta x} = \frac{\delta}{\delta x}\left(C \frac{\delta u}{\delta x} \right)$$

PWE (Plane Wave Expansion)

Theoretical Methods

Equation of motion

$$\rho \frac{\delta^2 u}{\delta t^2} = \frac{\delta}{\delta x}\left(C \frac{\delta u}{\delta x} \right)$$

$$\rho(x) = \sum_{G = m\frac{2\pi}{a}} \rho_G e^{iGx}$$

$$C(x) = \sum_{G = m\frac{2\pi}{a}} C_G e^{iGx}$$

$$u_k(x,t) = U_k(x) e^{i(kx - \omega t)} = \sum_G U_G e^{i((k+G)x - \omega t)}$$

Fourier series of periodic functions
+
Bloch theorem

Inserting the functions into the equation of motion, we obtain for each Fourier component:

$$\forall G: \quad -\omega^2 \sum_{G'} \rho_{G-G'} U_{G'} = \sum_{G'} C_{G-G'} (k+G)(k+G') U_{G'}$$

This can be written in the following matrix form:

$$\vec{U} = \begin{pmatrix} \cdots \\ U_G \\ \cdots \end{pmatrix} \rightarrow -\omega^2 \overset{\leftrightarrow}{M} \vec{U} = \overset{\leftrightarrow}{C} \vec{U} \longrightarrow$$

Eigenvalue equation $\longrightarrow$

$$\omega^2 = f(k)$$

PWE Plane Wave Expansion

Another formulation where k is calculated as a function of ω

$$k^2 \overleftrightarrow{N}_1 \vec{U} + k \overleftrightarrow{N}_2 \vec{U} + N_3(\omega)\vec{U} = 0$$

Construct a double-sized matrix:

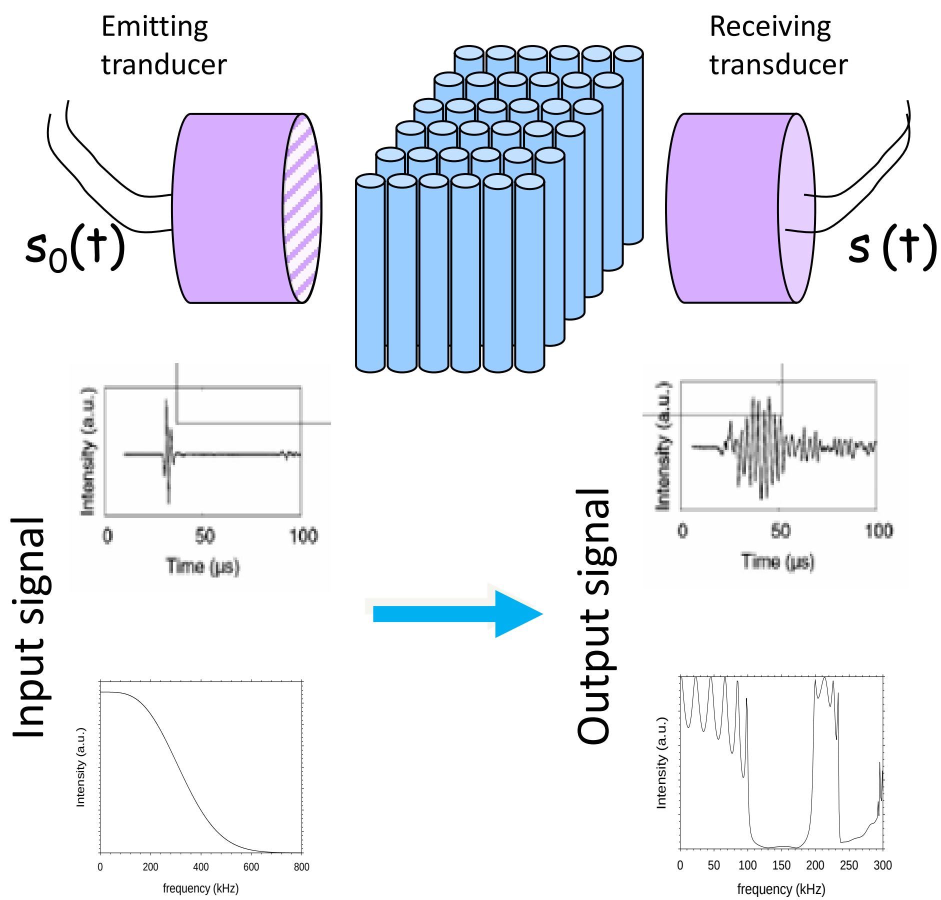
$$\vec{W} = \begin{pmatrix} k\vec{U} \\ \vec{U} \end{pmatrix} : \qquad k\begin{pmatrix} \overleftrightarrow{N}_1 & \overleftrightarrow{N}_2 \\ 0 & \overleftrightarrow{1} \end{pmatrix}\begin{pmatrix} k\vec{U} \\ \vec{U} \end{pmatrix} = \begin{pmatrix} 0 & \overleftrightarrow{N}_3 \\ \overleftrightarrow{1} & 0 \end{pmatrix}\begin{pmatrix} k\vec{U} \\ \vec{U} \end{pmatrix}$$

Or: $k\overleftrightarrow{M}\vec{W} = \overleftrightarrow{M}'\vec{W}$ ⟶ Eigenvalue equation for k ⟶ $k = k' + ik'' = f(\omega)$

Advantage: obtaining complex values of k=k'+ik" for each ω:

- Complex band structure ⟶ Decay factor when the frequency belongs to a gap
- Taking account of the acoustic absorption (complex and frequency dependent elastic constants)

Finite Difference Time Domain (FDTD) Method



$$s(t) = \int s(\omega) e^{i\omega t} d\omega$$

$$t(\omega) = \frac{s(\omega)}{s_0(\omega)}$$

FDTD Finite Difference Time Domain

Theoretical Methods

$$u, v, \sigma$$

$$\sigma = C\frac{\delta u}{\delta x}$$

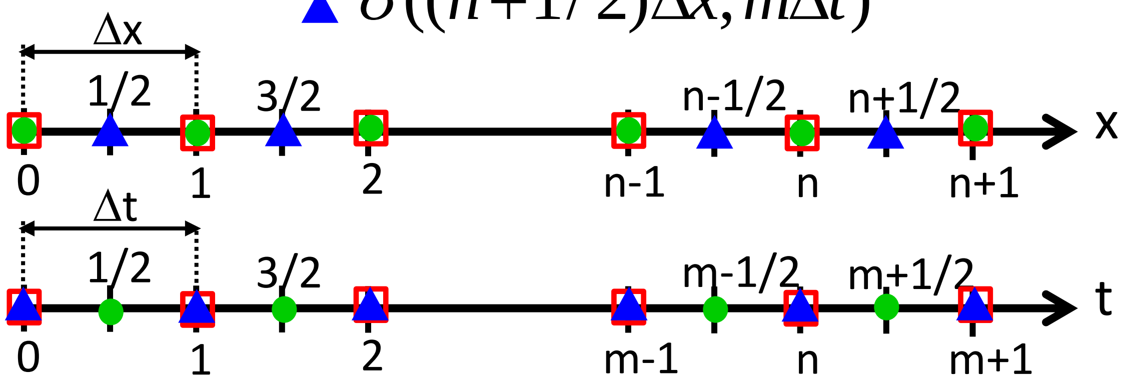
$$\rho\frac{\delta v}{\delta t} = \frac{\delta \sigma}{\delta x}$$

$$v = \frac{\delta u}{\delta t}$$

□ $u(n\Delta x, m\Delta t)$

● $v(n\Delta x, (m+1/2)\Delta t)$

▲ $\sigma((n+1/2)\Delta x, m\Delta t)$



Transformation of the differential equations into difference equations

$$\sigma(n+1/2, m) = C(n+1/2, m)\frac{u(n+1, m) - u(n, m)}{\Delta x}$$

$$\rho(n, m+1/2)\left(\frac{v(n, m+1/2) - v(n, m-1/2)}{\Delta t}\right) = \frac{\sigma(n+1/2, m) - \sigma(n-1/2, m)}{\Delta x}$$

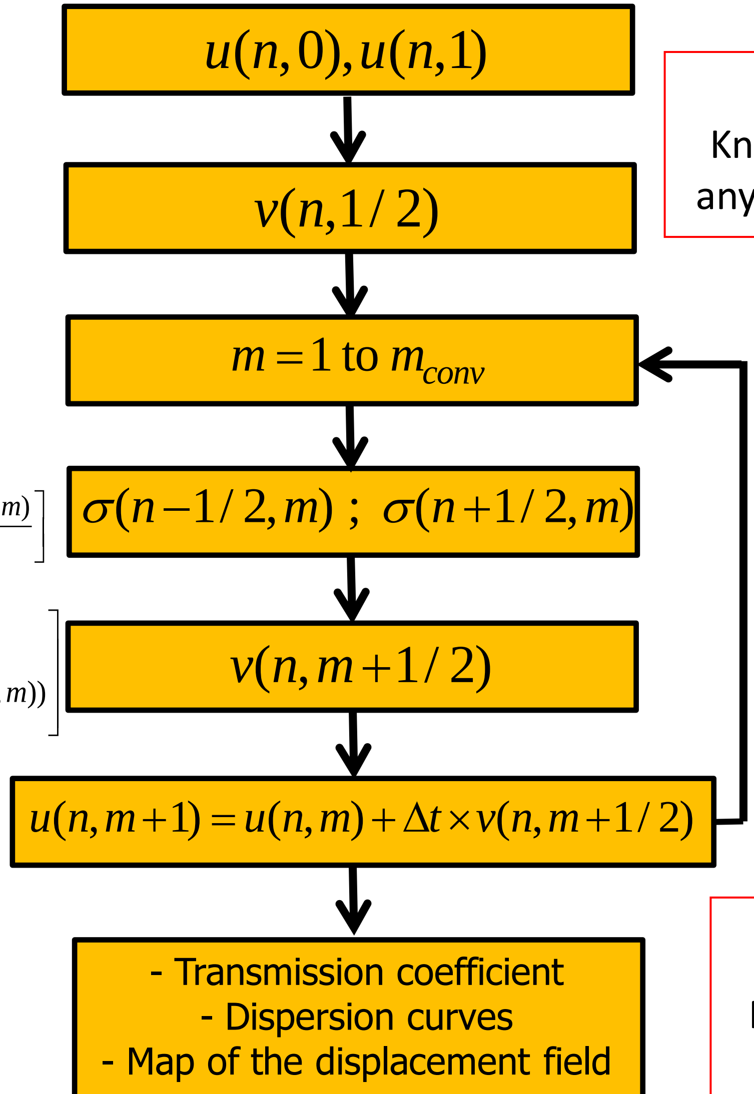
$$v(n, m+1/2) = \frac{u(n, m+1) - u(n, m)}{\Delta t}$$

FDTD Finite Difference Time Domain

$$\left[v(n,1/2) = \frac{u(n,1) - u(n,0)}{\Delta t} \right]$$

$$\left[\sigma\left(n+1/2,m\right) = C(n+1/2,m)\frac{u(n+1,m) - u(n,m)}{\Delta x} \right]$$

$$\left[v(n,m+1/2) = v(n,m-1/2) + \frac{\Delta t}{\rho(n,m+1/2)\Delta x}(\sigma(n+1/2,m) - \sigma(n-1/2,m)) \right]$$



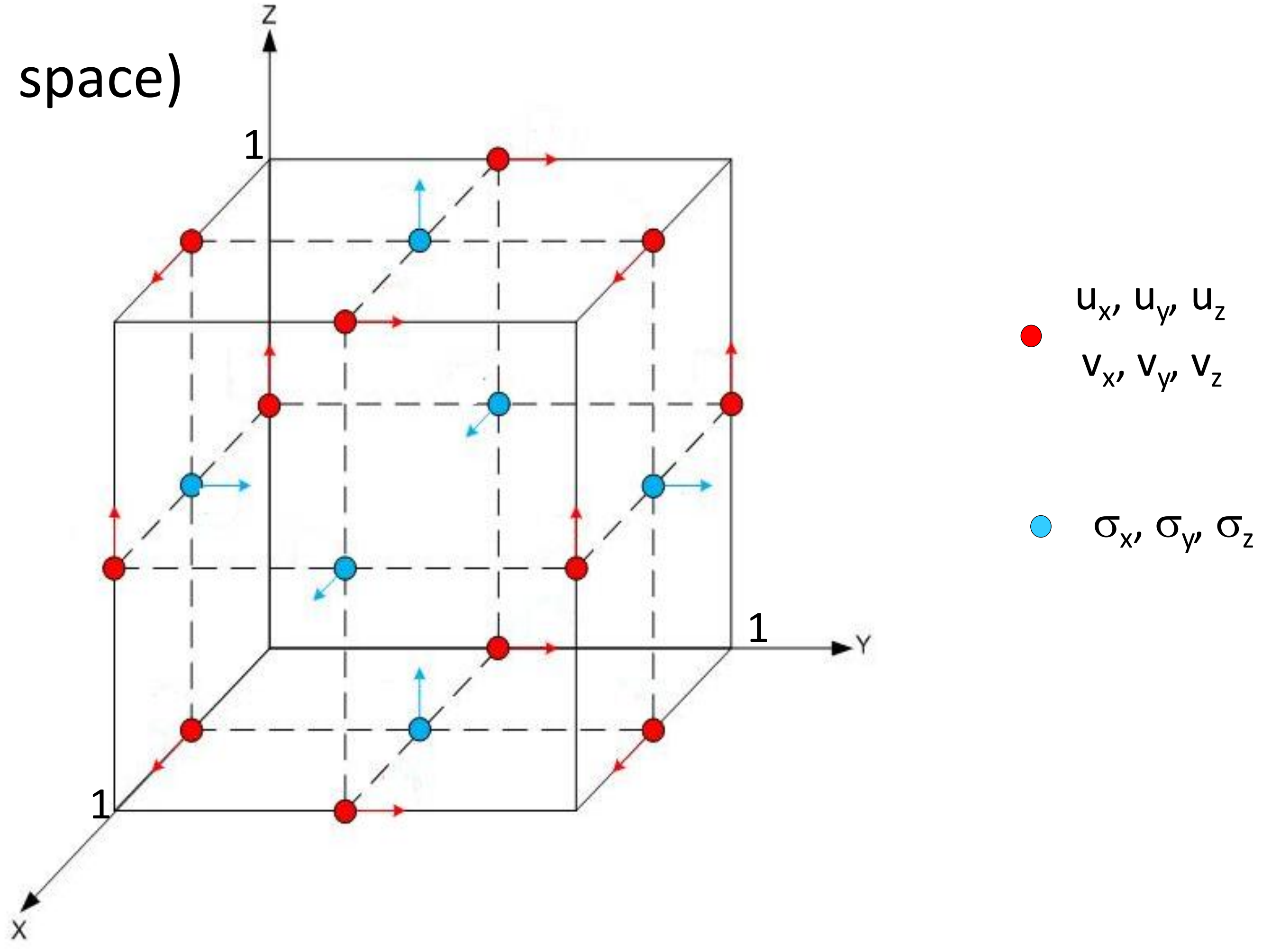
1. Initial conditions:
Knowledge of the displacements at any spatial position at times t_0 and t_1

2. Recurrence relations:
Obtain the results at time $(m+1)\Delta t$ from those at $m\Delta t$

3. Frequency analysis:
Fourier transform of the time dependent fields

FDTD Finite Difference Time Domain

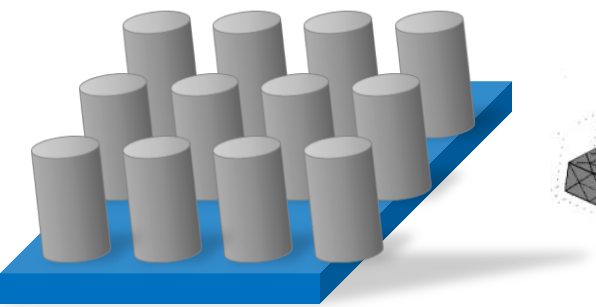
Yee cell (in space)

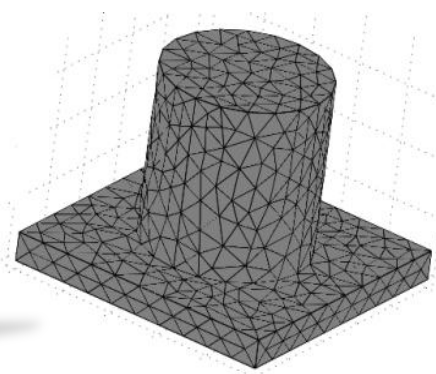


Finite Element Method (FEM)

Equations of motion: $$\rho \frac{\delta^2 u_i}{\delta t^2} = \sum_j \frac{\delta \sigma_{ij}}{\delta x_j} = \sum_{j,kl} C_{ijkl} \frac{\delta u_k}{\delta x_l}$$

-The structure is divided into small elements.
-The displacements are developped on basis functions, where the variables are the values of the displacement at the nodes.
- A variational method is applied which yields an eignevalue problem



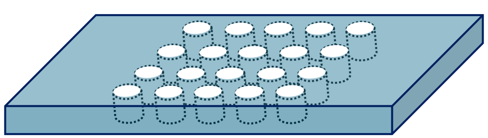


Unit cell of an array of pillars on a membrane

$$-\omega^2 \overleftrightarrow{M} \vec{U} = \overleftrightarrow{K} \vec{U}$$

↑ Density matrix

↑ Stifness matrix



Multiple Scattering Theory:

Based on the KKR (Kohn-Korringa-Rostocker) theory in electronic structures of solids

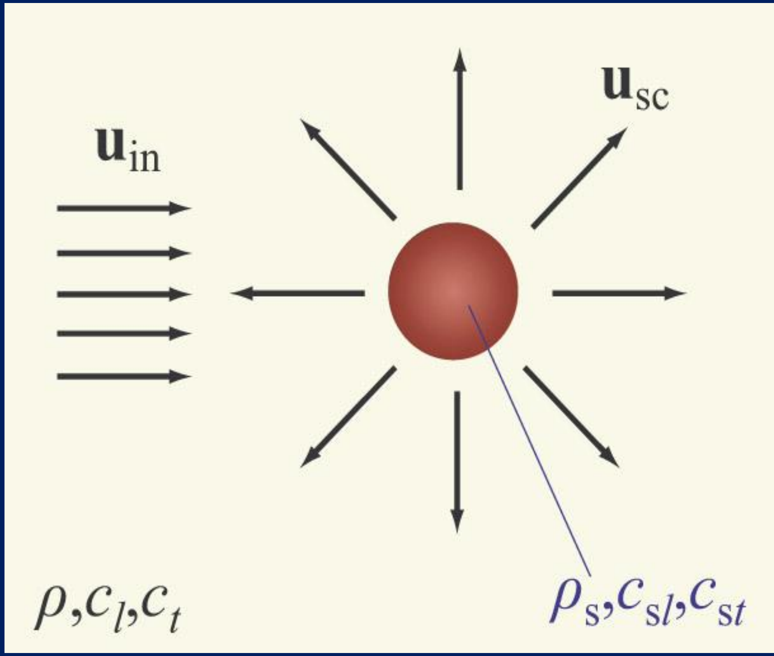
Scattering by an elastic sphere

$$c_l^2\nabla(\nabla\cdot\mathbf{u})-c_t^2\nabla\times\nabla\times\mathbf{u}+\omega^2\ \mathbf{u}=0$$

$$\mathrm{L}=P\ell m$$

$P=L$ **Longitudinal**

$P=M,N$ **Transverse**



$$\mathbf{u}_{\rm in}(\mathbf{r})=\sum_{\mathrm{L}} a_{\mathrm{L}}^{0}\mathbf{J}_{\mathrm{L}}(\mathbf{r})$$

$$\mathbf{u}_{\rm sc}(\mathbf{r})=\sum_{\mathrm{L}} a_{\mathrm{L}}^{+}\mathbf{H}_{\mathrm{L}}(\mathbf{r})$$

Boundary Conditions →

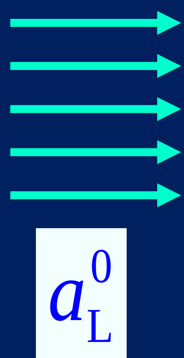
Scattering Matrix $\mathbf{T}$

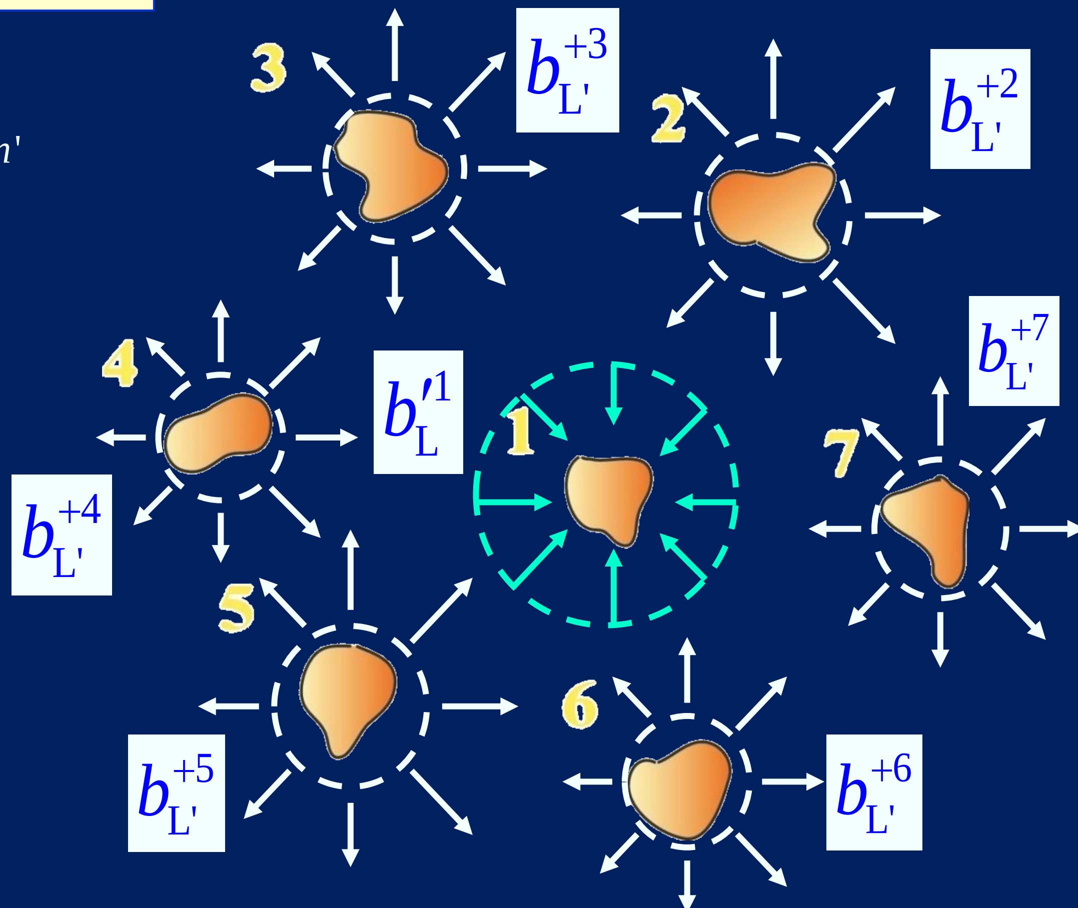
$$a_{\mathrm{L}}^{+}=\sum_{\mathrm{L}'} T_{\mathrm{LL}'}\, a_{\mathrm{L}'}^{0}$$

Courtesy of R. Sainidou

Multiple Scattering

$$b'^{1}_{L} = \sum_{n'L'} \Omega^{1n'}_{LL'} b^{+n'}_{L'}$$



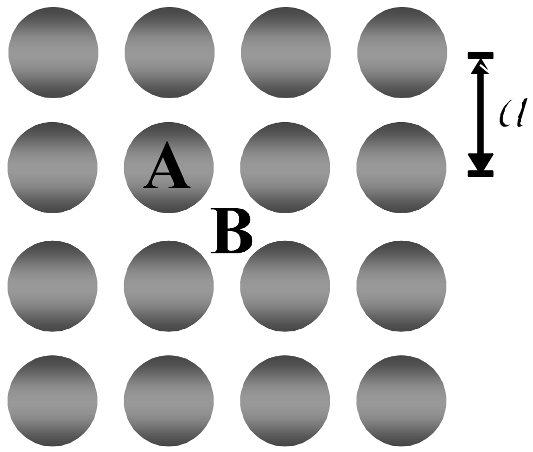


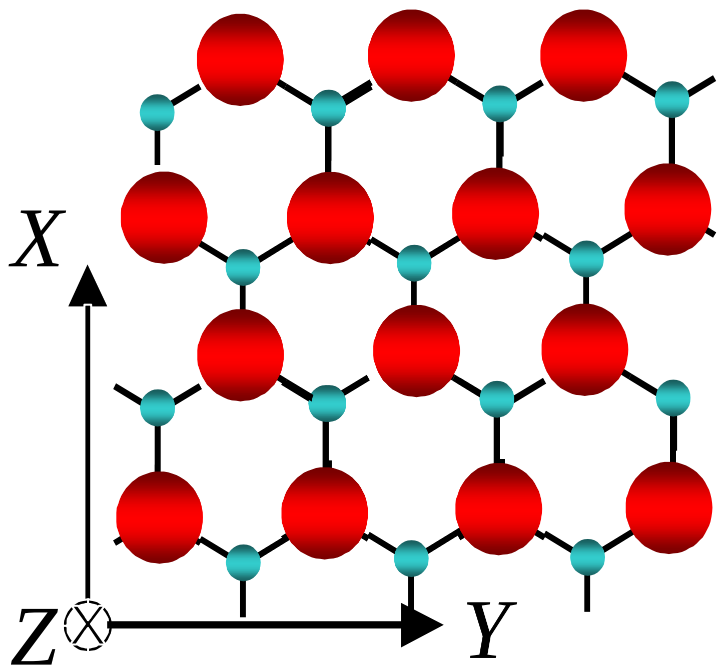
$$\sum_{n'L'} \left(\delta_{1n'} \delta_{LL'} - \sum_{L''} T^{1}_{LL''} \Omega^{1n'}_{L''L'} \right) b^{+n'}_{L'} = \sum_{L'} T^{1}_{LL'} a^{01}_{L'}$$

Courtesy of R. Sainidou

2D PHONONIC CRYSTALS

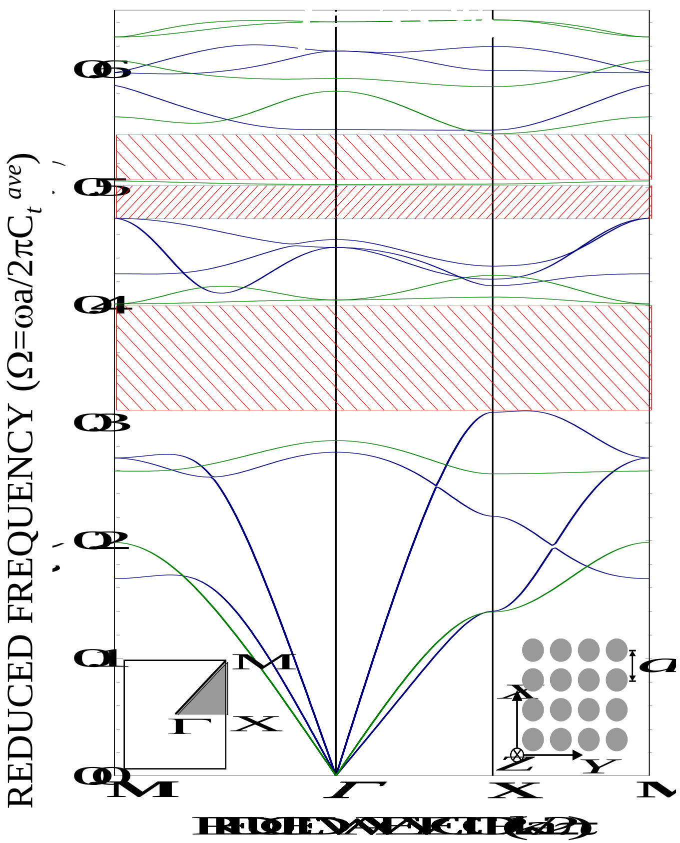
- Constituents : Solid/solid, fluid/fluid, mixed solid/fluid composites
- Structure : Square array and boron-nitride structure (BN)
- Shape of the inclusions: circle, square,…
- Composition : $f \equiv$ filling factor of inclusions





Band structure
Solid/solid systems

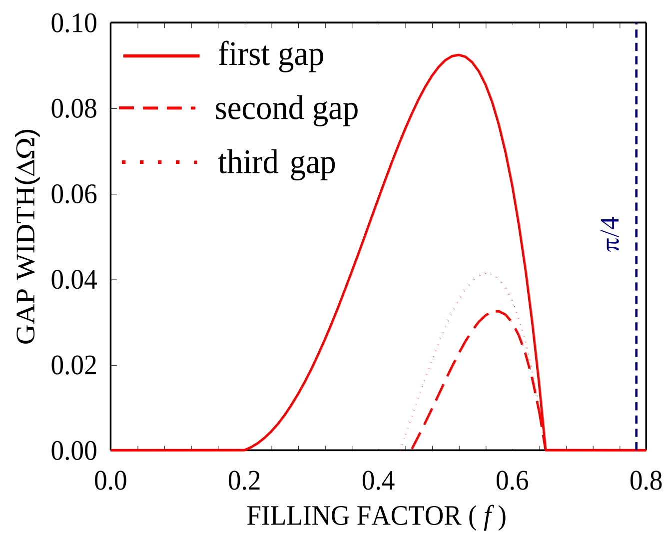
Square array of Carbon cylinders (circular cross section) embedded in an epoxy matrix



Propagation limited to the *XOY* plane

$\vec{k}(k_X, k_Y, k_Z = 0)$ → **XY modes** $\Leftrightarrow \vec{u} \in XOY$

$\vec{k}(k_X, k_Y, k_Z = 0)$ → **Z modes** $\Leftrightarrow \vec{u} // OZ$



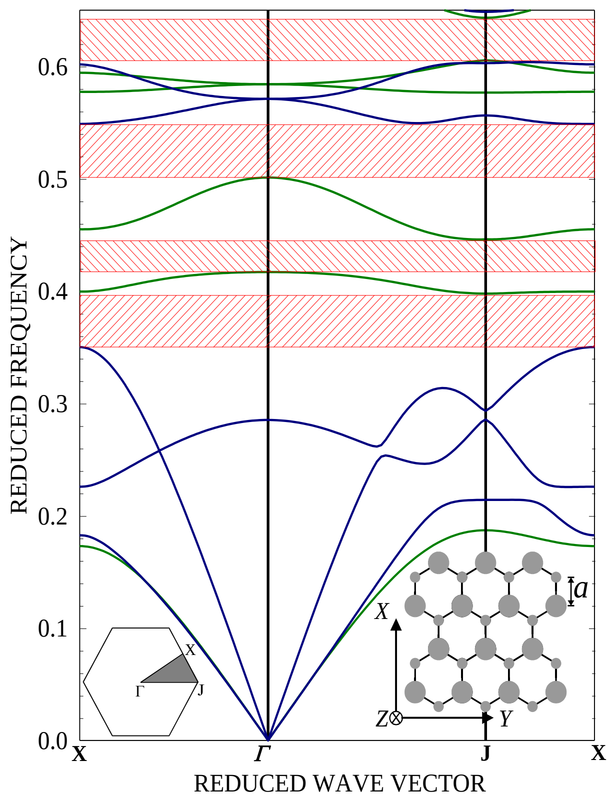
J.Vasseur, B. Djafari Rouhani, L. Dobrzynski, M.S. Kushwaha, P. Halevi, J. Phys: Condensed Matter (1994)

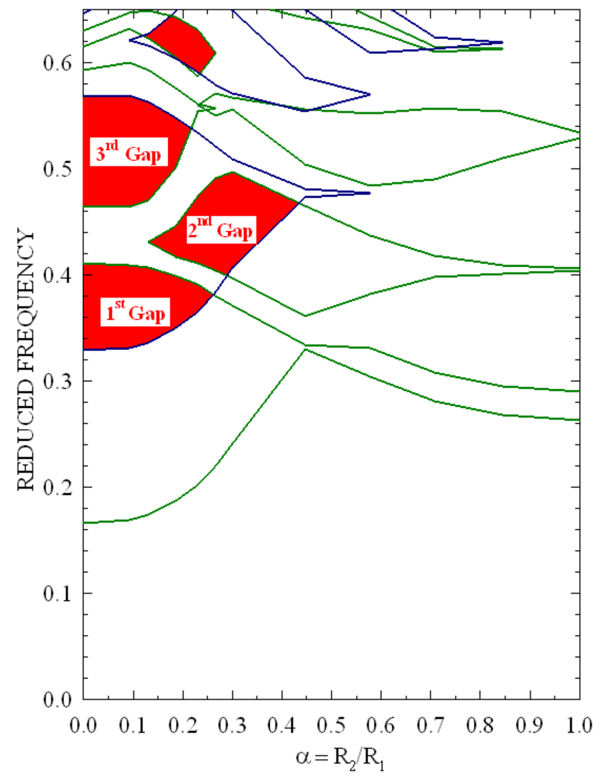
Band structure
Solid/solid systems

BN array of Carbon cylinders (circular cross section) embedded in an epoxy resin matrix

$f = f_1 + f_2 = 60\ \%$

$\alpha = R_1 / R_2 = 0.19$





$\alpha = 0 \Leftrightarrow$ Triangular array

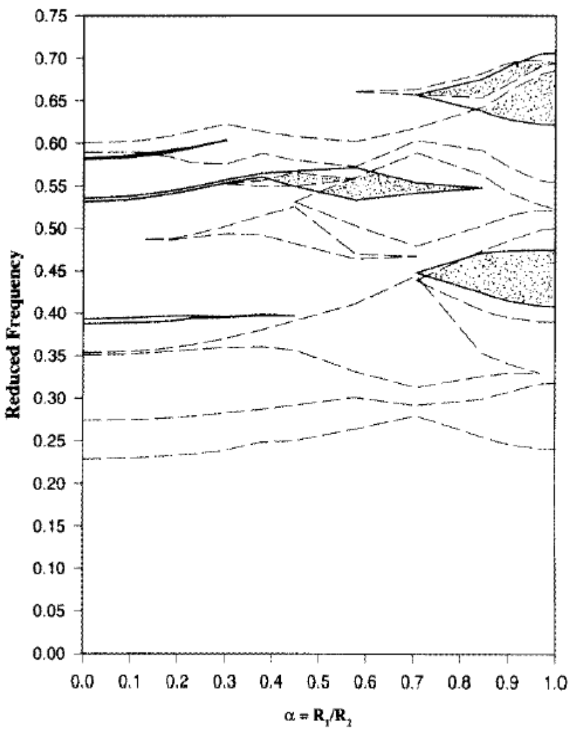
$\alpha = 1 \Leftrightarrow$ Graphite array

J.Vasseur, B. Djafari Rouhani, L. Dobrzynski, P. Deymier, J. Phys: Condensed Matter (1997)

Band structure
Solid/solid systems

BN array of epoxy cylinders embedded in a Carbon matrix

$$f_1 + f_2 = 0.60$$

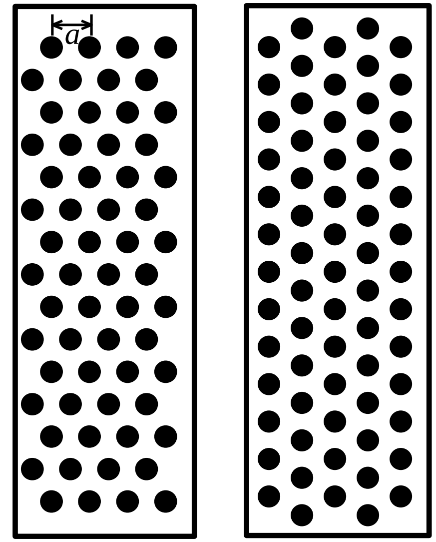


J.Vasseur, B. Djafari Rouhani, L. Dobrzynski, P. Deymier, J. Phys: Condensed Matter (1997)

Band structure
Solid/solid systems

EXPERIMENTAL RESULTS :

Triangular array of steel cylinders embedded in an epoxy resin matrix

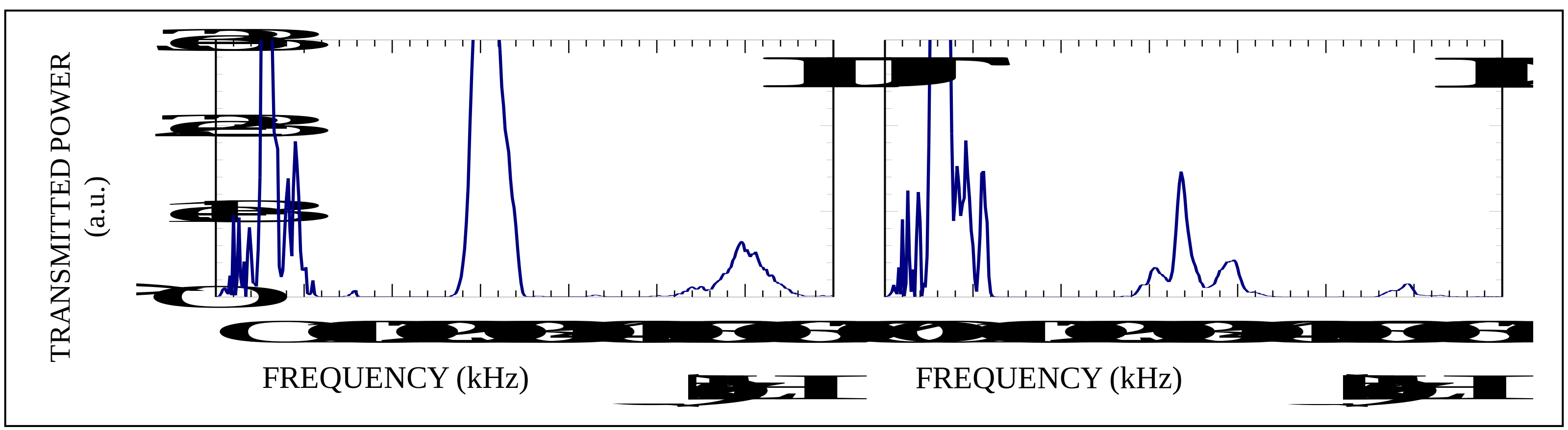


ΓJ ΓX

R = 2 mm, a = 6.02 mm ⇒ f = 40 %

<u>Dimensions</u> : 80 mm x 80 mm x 27 mm

→ Transmission spectra of longitudinal waves

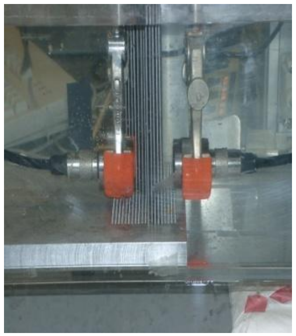


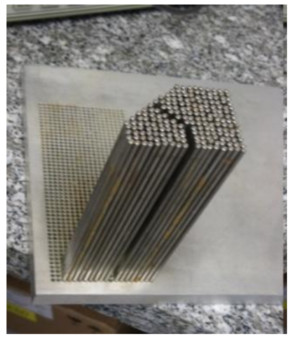
J.O. Vasseur *et al.*, Phys. Rev. Lett. **86**, 3012 (2001)

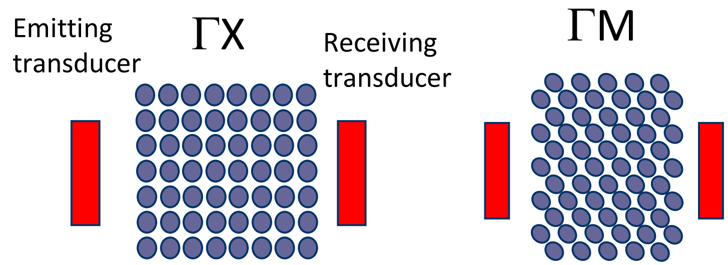
Band structure
Solid/fluid systems

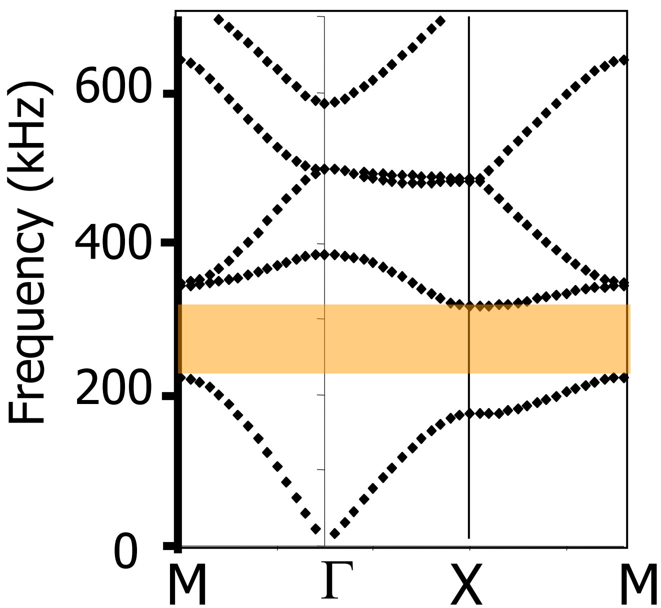
Ultrasonic 2D Phononic crystals: steel cylinders in water

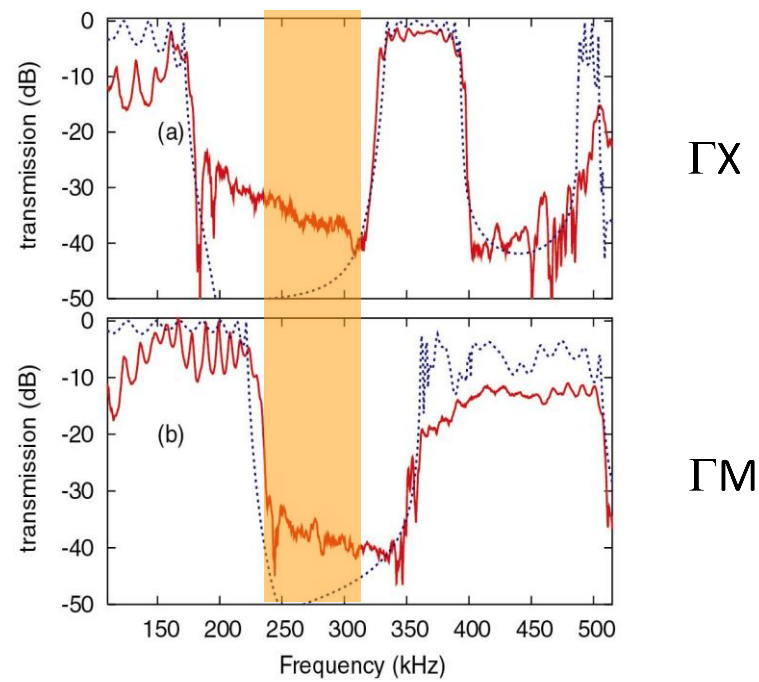
a = 3 mm; D = 2.5 mm $\Rightarrow f = 54{,}5\ \%$







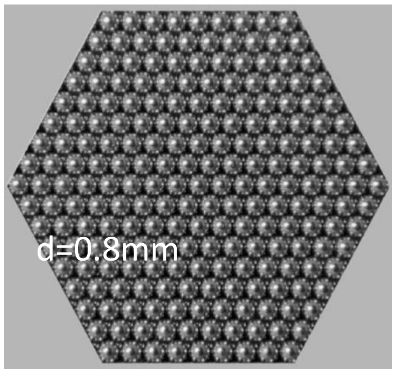




A. Khelif, A. Choujaa, B. Djafari Rouhani, M. Wilm, S. Ballandras, V. Laude, Phys. Rev. B **68**, 214301 (2003)

Band structure
Solid/fluid systems

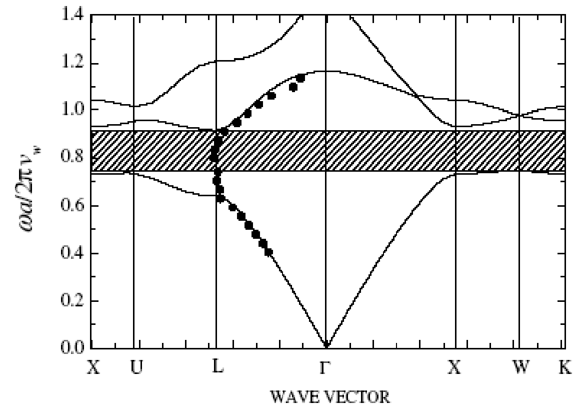
Ultrasonic 3D Phononic crystals:
fcc lattice of tungsten carbide beads in water

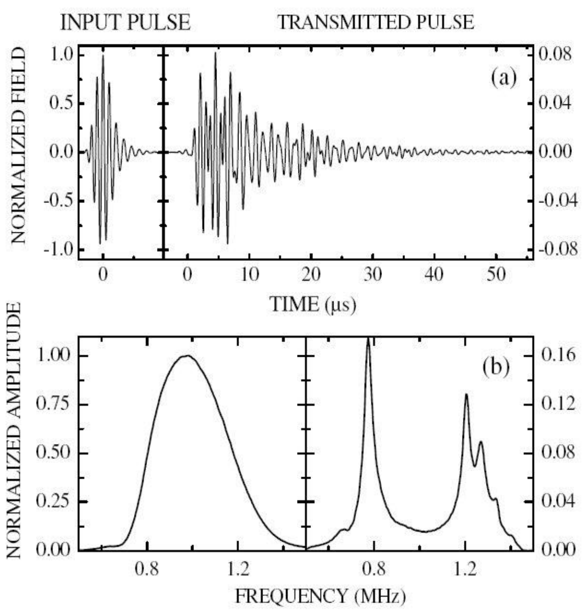


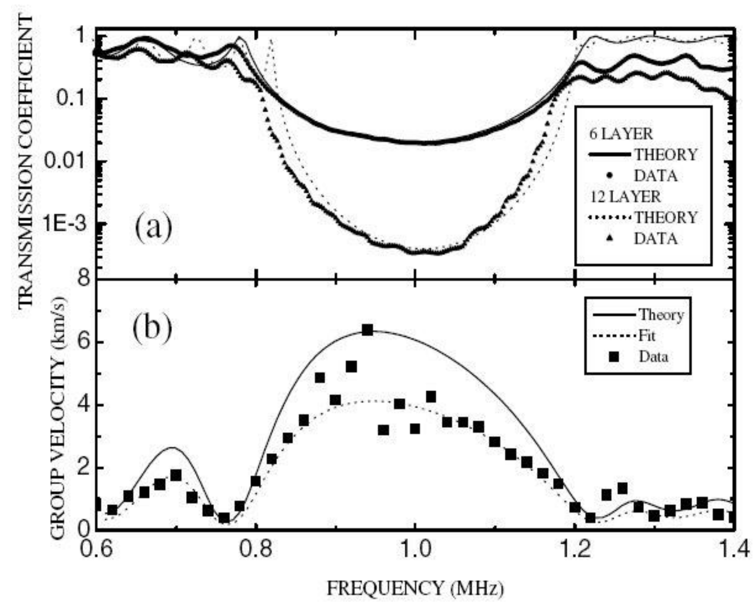
$$T(L,\omega) = A(L,\omega)\exp[i\phi(L,\omega)]$$

$$v_p(\omega) = \frac{\omega}{k} - L\frac{\omega}{\phi}$$

$$v_g(\omega) = \frac{d\omega}{dk} = L\frac{d\omega}{d\phi}$$







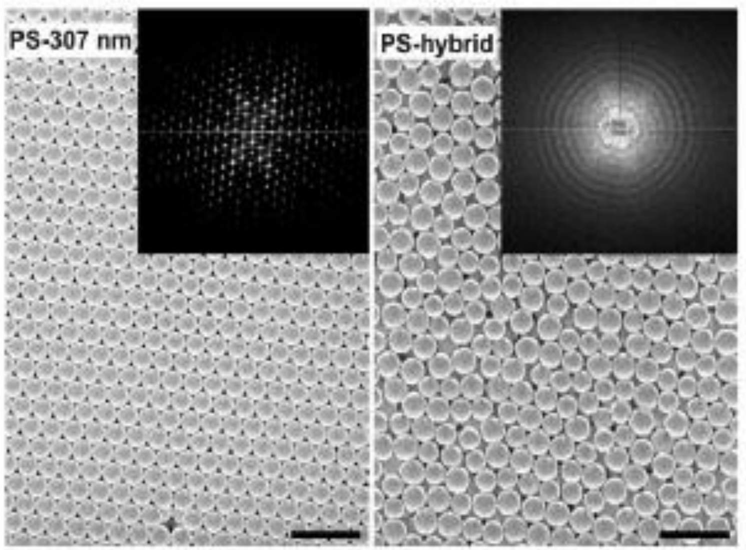
S. Yang et al., Phys. Rev. Lett. 88, 104301 (2002)

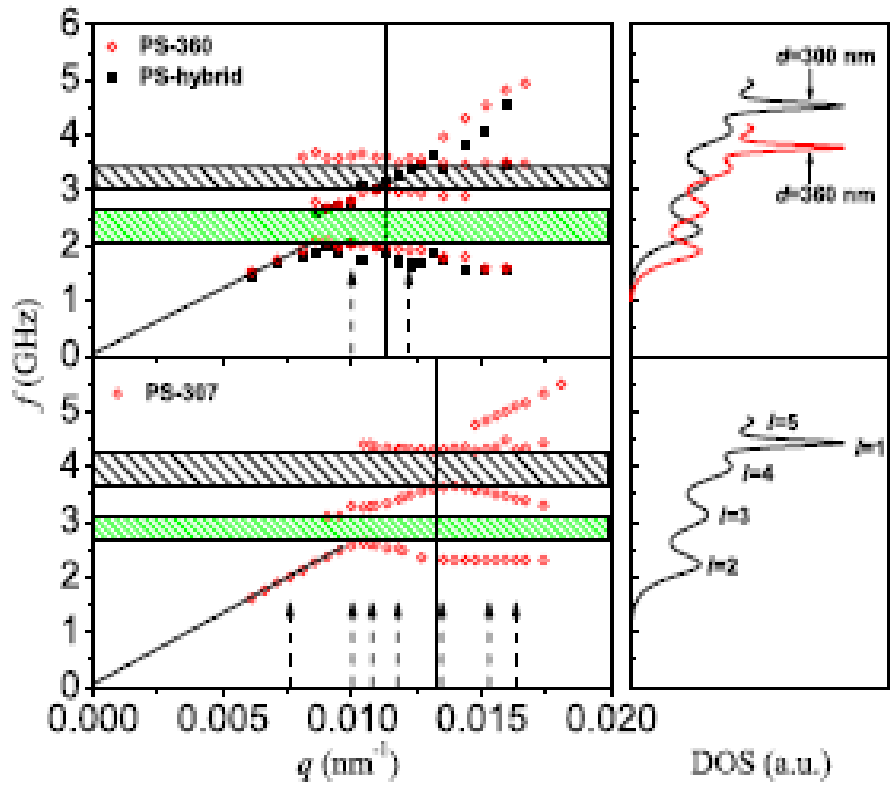
THE WIDTH OF THE ABSOLUTE FORBIDDEN BANDS IS VERY SENSITIVE TO:

- ⌧ the nature (solid, fluid) of the constituent materials
- ⌧ the contrast between the physical characteristics (density and elastic constants) of the constituent materials
- ⌧ the filling factor of inclusions
- ⌧ the symmetry of the lattice
- ⌧ the shape of the inclusions → Circular, square, ...

Bragg gap and hybridization gap

Colloidal films of polystyrene (PS) infiltrated with PDMS
FCC structures with PS diameters of 307 and 360 nm

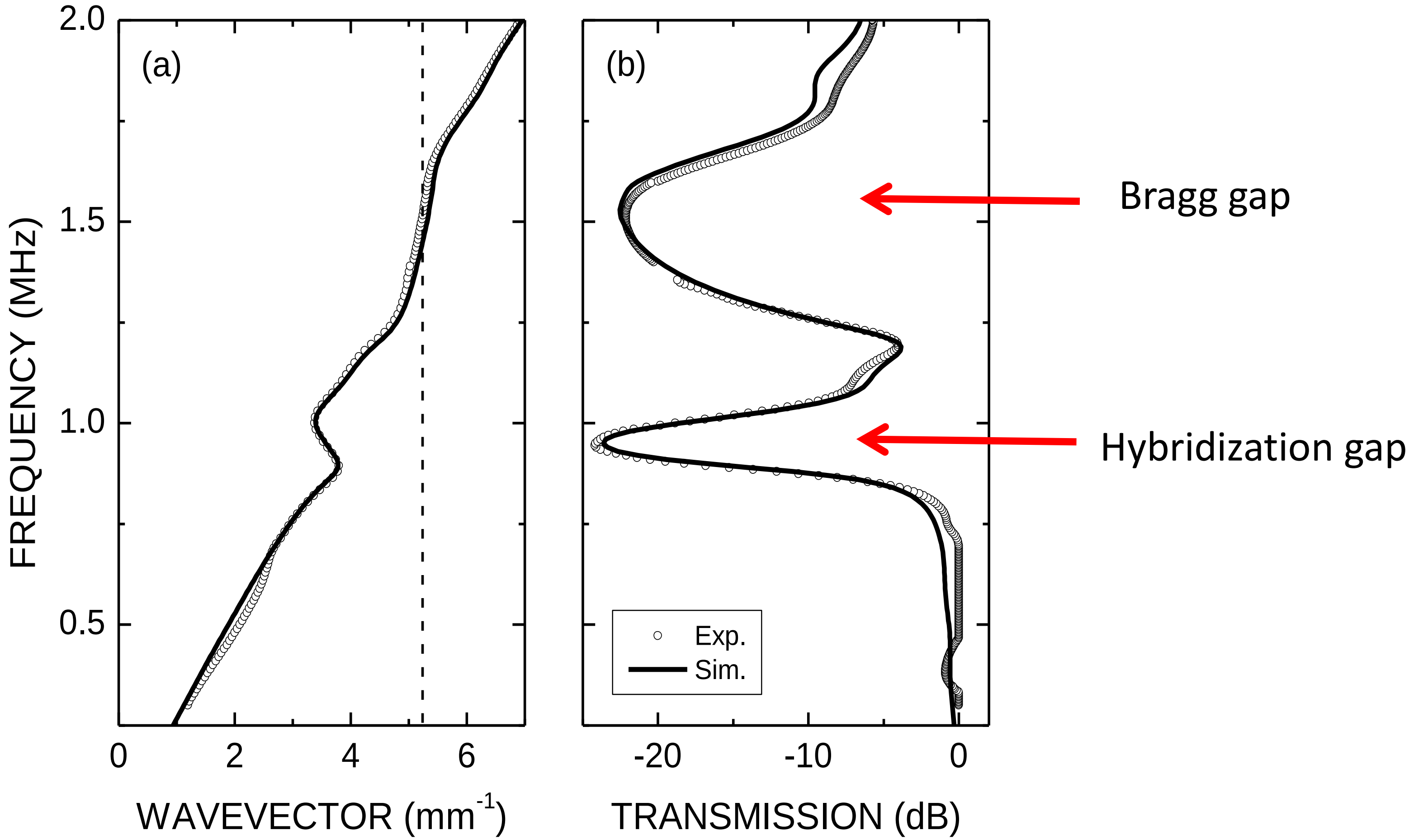




T. Still, W. Cheng, M. Retsch, R. Sainidou, J. Wang, U. Jonas, N. Stefanou, G. Fytas,
Phys. Rev. lett., 100, 194301 (2008)

Bragg gap and hybridization gap

Phononic crystal of nylon rods in water, Hexagonal lattice
Rods diameter=0.46mm; filling fraction=40%



C. Croënne, E. J. S. Lee, Hefei Hu and J. H. Page, AIP Advances, **1**(4), (2011).

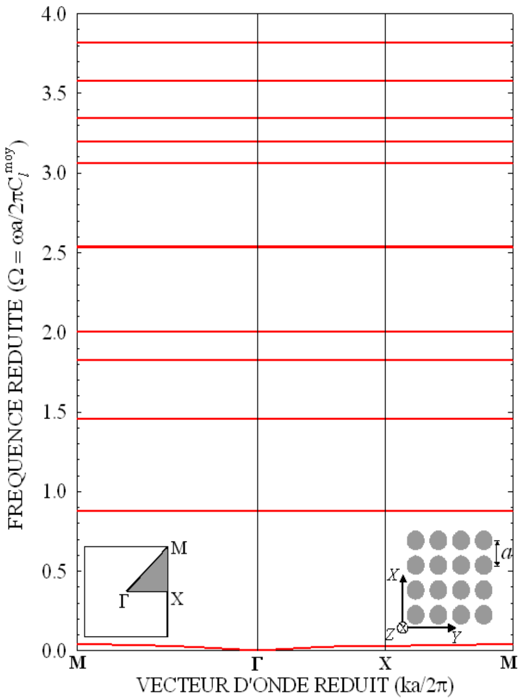
Overlapping hybridization and Bragg gaps

E. Psarobas et al, Phys. Rev. B 65, 064307 (2002)
T. Still et al., Phys. Rev. Lett. 100, 194301 (2008)
C. Croënne et al., AIP Adv. 1, 041401 (2011)
Y. Achaoui et al. Phys. Rev. B 83, 104201 (2011)
A. Bretagne et al., AIP Conf. Proc. 1433, 317 (2012)
N. Kaina et al. Scientific reports 3, 3240 (2013)

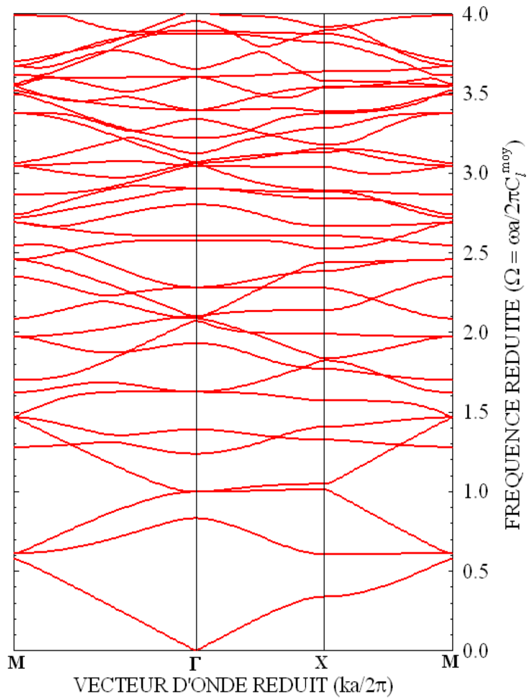
Band structure
Fluid/fluid systems

Square array of cylinders (circular cross section) of:

air in water



water in air



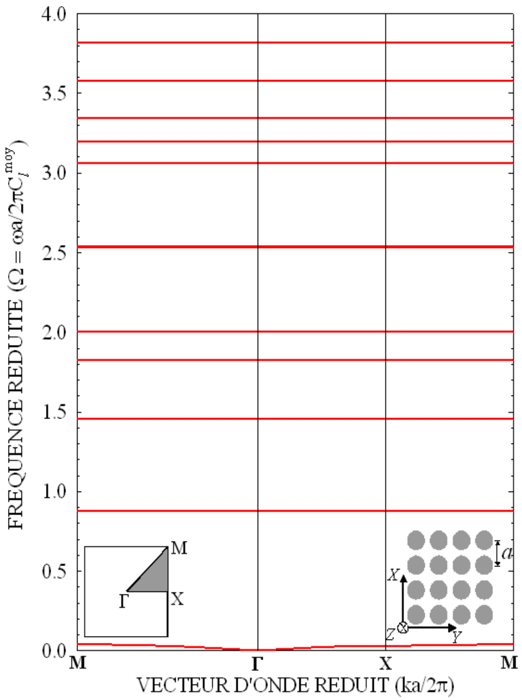
Flat bands: resonances of a single cylinder of air in water

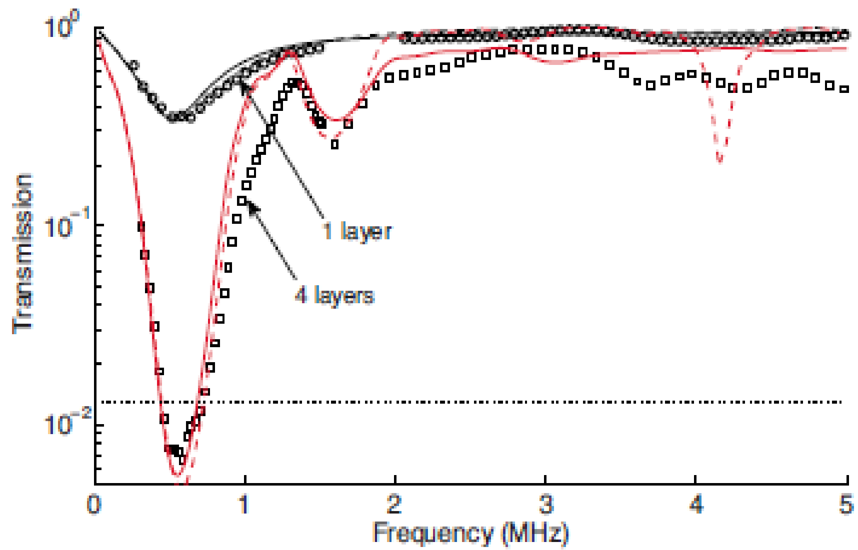
f = 35 %

M.S. Kushwaha, B. Djafari Rouhani, J. Appl. Physics (1998)

Band structure
Fluid/fluid systems

Square array of cylinders of air in water





Flat bands: resonances of a single cylinder of air in water

Ultrasonic measurement in a phononic crystal made of bubbles in PDMS. Period= 300μm , bubble radius=38 μm

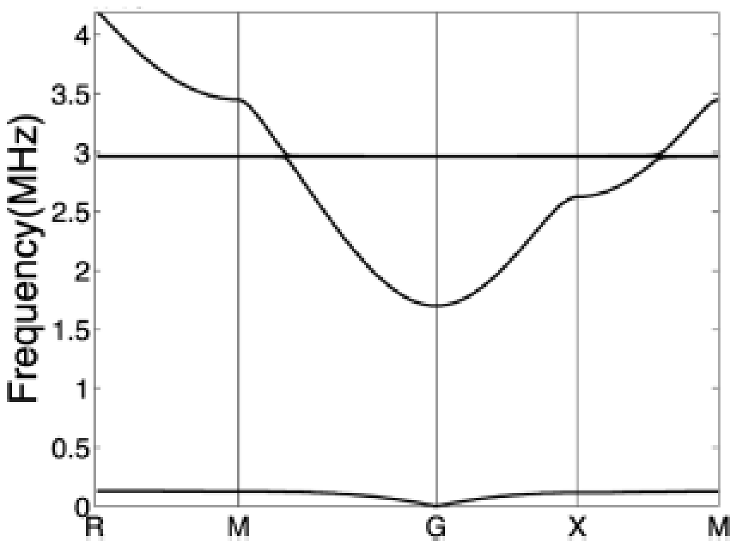
The first gap is attributed to the combined effect of Bragg reflections and bubble resonances

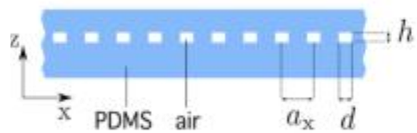
V. Leroy, , A. Bretagne, M. Fink, H. Willaime, P. Tabeling, and A. Tourin, APL 95, 171904 (2009)

M.S. Kushwaha, B. Djafari Rouhani, J. Appl. Physics (1998)

Band structure
Fluid/fluid systems

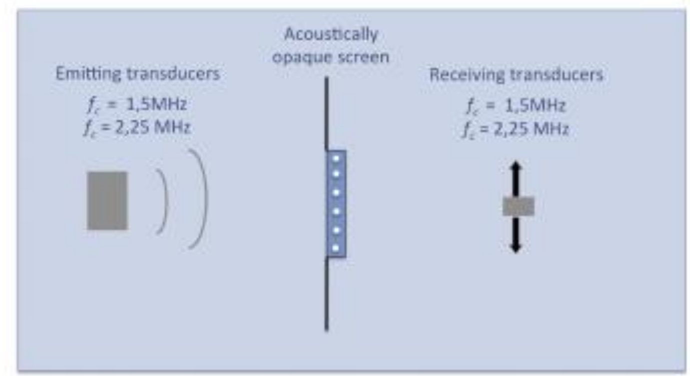
Air bubbles in PDMS. Period= 300μm ,
bubble radius=38 μm, Filling fraction <1%





(a)

(b)

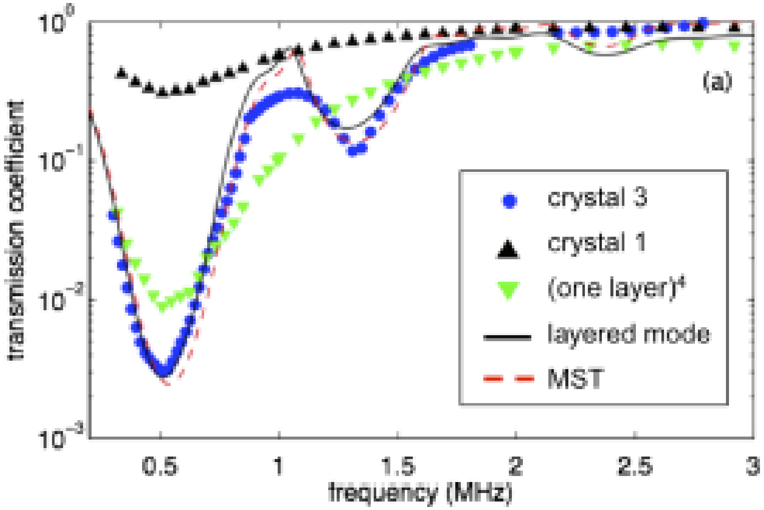


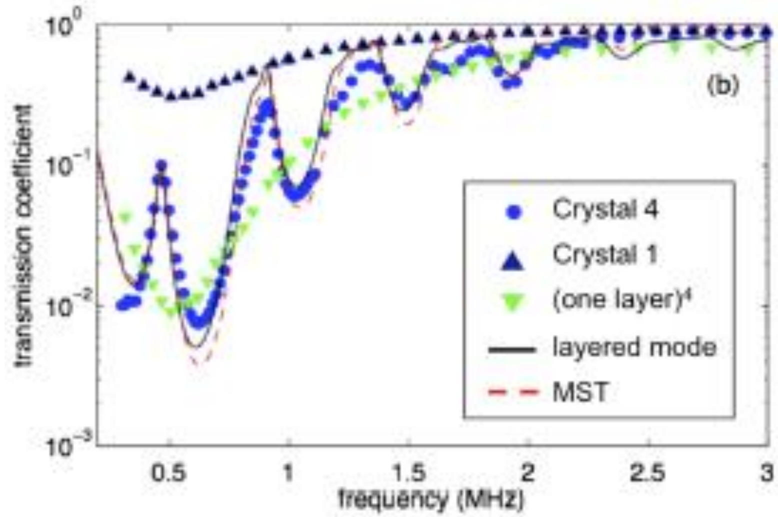
(c)

TABLE I. Lattice constants and bubble size for the 4 crystals used in this study.

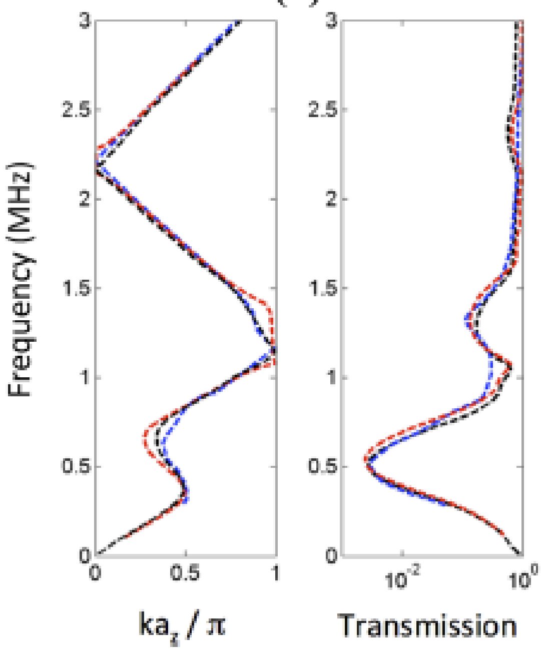
Sample	a_z (μm)	$a_x = a_y$ (μm)	h (μm)	d (μm)	R (μm) Radius of the equivalent sphere
Crystal 1	One single layer	300	50	78	38
Crystal 2	One single layer	200			
Crystal 3	475	300			
Crystal 4	1150	300			

V. Leroy, A. Bretagne, M. Lanoy, and A. Tourin, AIP Advances 6,121604 (2016)

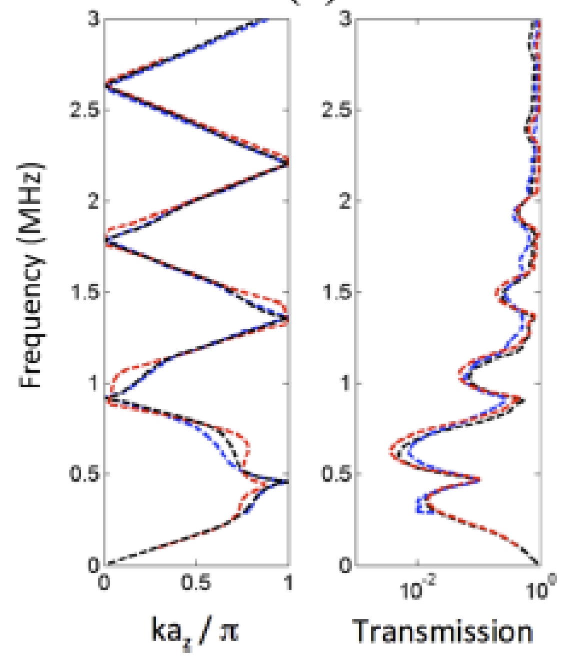




(a)



(b)

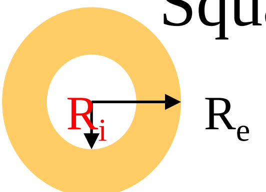


V. Leroy, A. Bretagne, M. Lanoy, and A. Tourin, AIP Advances 6,121604 (2016)

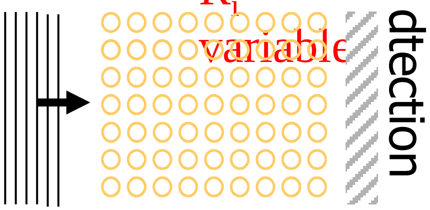
Band structure and
Local resonances

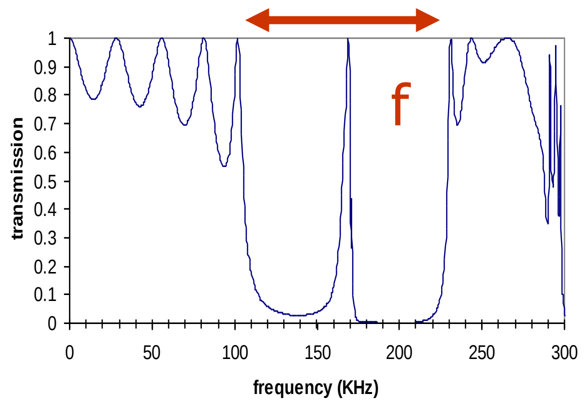
Frequency filtering with hollow scatterers

Square array of hollow steel cylinders in water



a = 5 mm
R_e = 2 mm
R_i variable





The frequency f is a function of the internal radius Ri
and the nature of the fluid inside and outside the cylinders

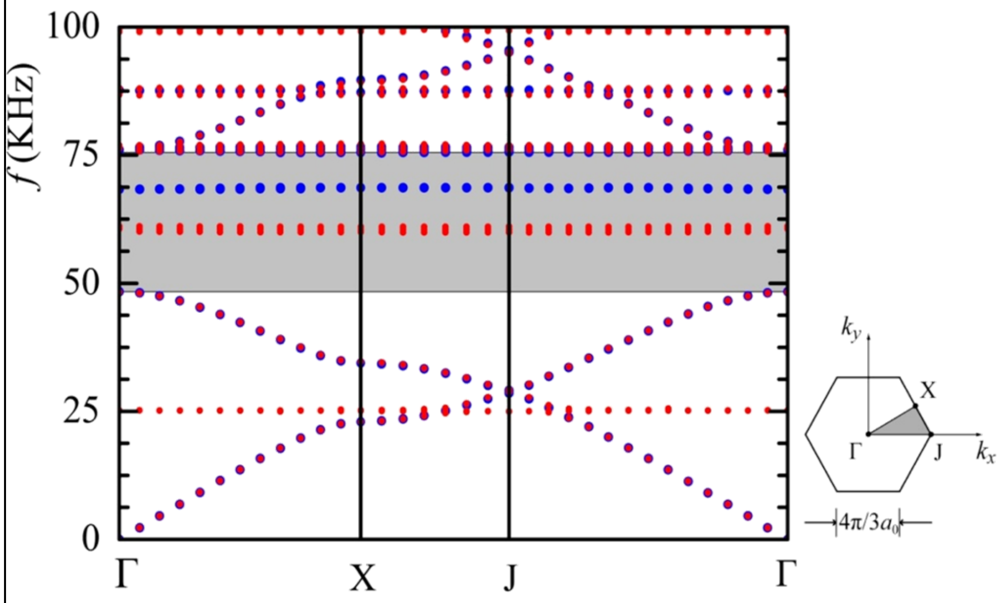
→ Tunable frequency filter

Y. Pennec, B. Djafari Rouhani, J. Vasseur, A. Khelif, P. Deymier, Phys. Rev. E69, 046608 (2004)

Band structure and Local resonances

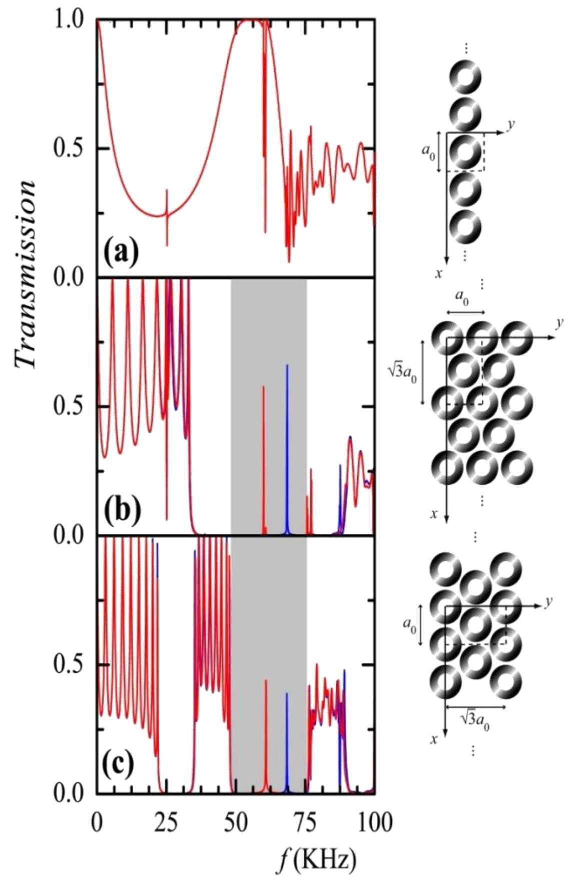
Frequency filtering with hollow scatterers: local resonances

Hollow polymer cylinders in air



Filled cylinders (radius S=2.35 mm)

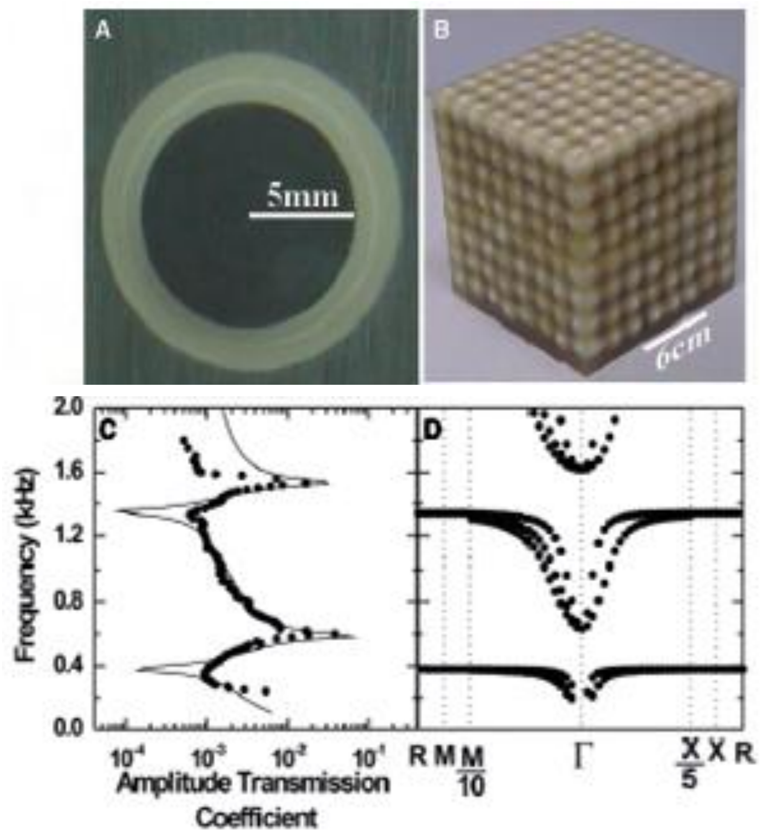
Hollow cylinders (S_{in}=1.15 mm, S_{out}=2.3 mm)



R. Sainidou, B. Djafari Rouhani, J. Vasseur, Y. Pennec, Phys. Rev. B 73, 024302 (2006)

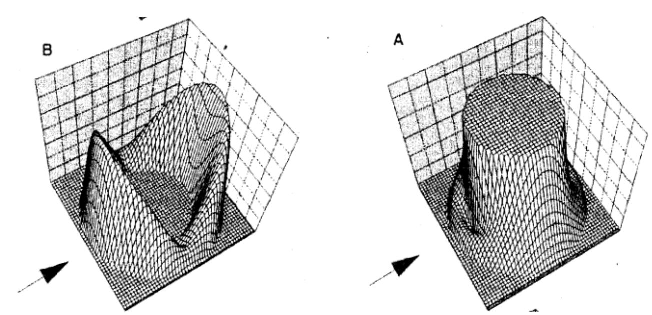
Locally resonant sonic materials (LRSM)

Simple cubic lattice of Pb spheres, coated with silicone rubber, in an epoxy background



Heavy core: d = 1 cm
Coating layer: δ = 0.25 cm
Period: a =1.55 cm

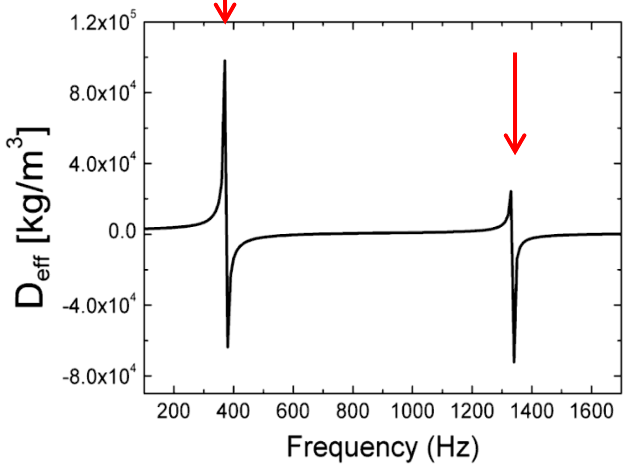
Opening of low frequency gaps in the audible range



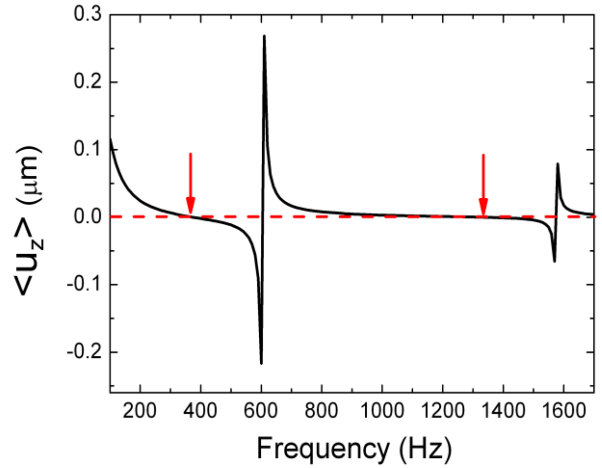
Liu et al., Science 289, 1734 (2000)

Locally resonant sonic materials
Negative mass density

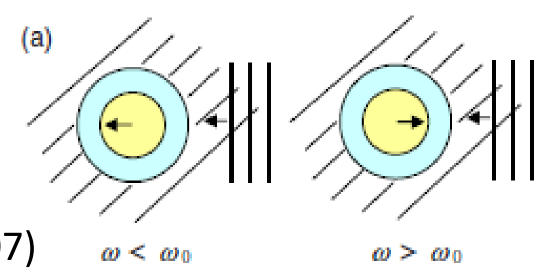
Dynamic effective mass:
The mass density changes sign at the **zeros of transmission**
(anti-resonances at 400 and 1350 Hz)



Average Normal Displacement $\langle u_z \rangle$:
- vanishes at the zeros of transmission
- becomes important at the maxima of transmission

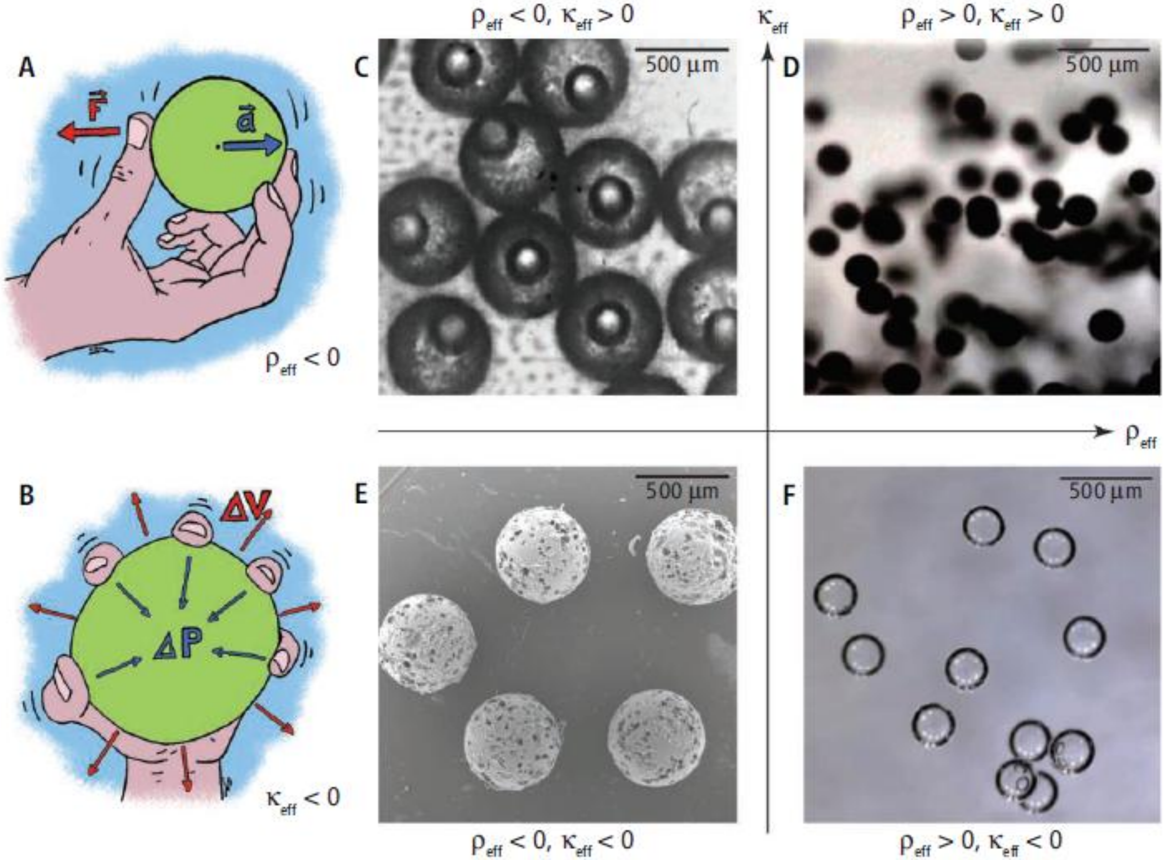


Schematic interpretation of positive and negative mass



J. Mei, Z. Liu, W. Wen and Ping Sheng, PRB 76, 134205 (2007)

Classification of various locally resonant materials in terms of the sign of effective mass density and bulk modulus



Features of acoustic metamaterials. (**A** and **B**) Schematic illustrations outline the dynamic behaviors of locally resonant materials with negative-valued effective parameters submitted to harmonic excitations. (A) The acceleration *a* of a material possessing a negative effective mass density ($\rho_{eff} < 0$) is opposite to the driving force *F*. (B) A material possessing a negative effective bulk modulus ($\kappa_{eff} < 0$) supports a volume expansion ($\Delta V > 0$) upon an isotropic compression ($\Delta P > 0$). (**C** to **F**) Classification of various locally resonant materials in the (ρ_{eff}, κ_{eff}) plane in terms of the sign of the effective mass density and the bulk modulus, made of various resonant inclusions: (C) core-shell particles, (D) "slow-oil" droplets, (E) polymer porous beads, and (F) air bubbles.

T. Brunet et al, Science, 342, 323 (2013)

Rubber spheres in water

Locally resonant sonic materials
Double Negativity

Change of sign in effective density

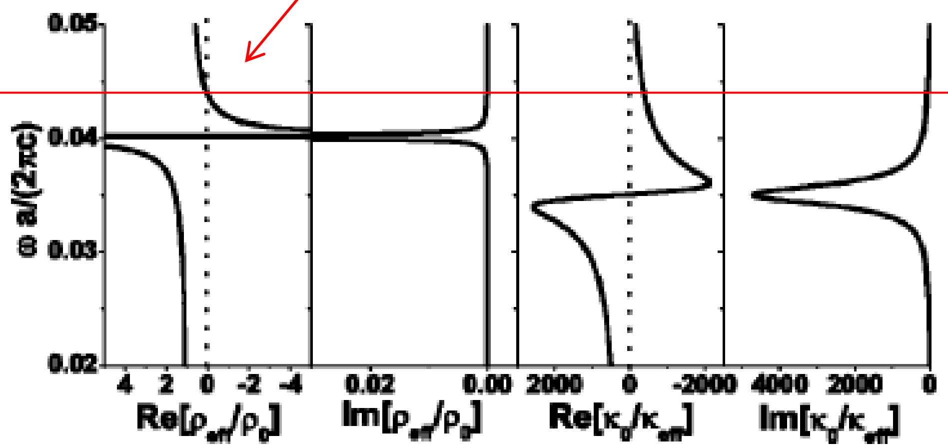
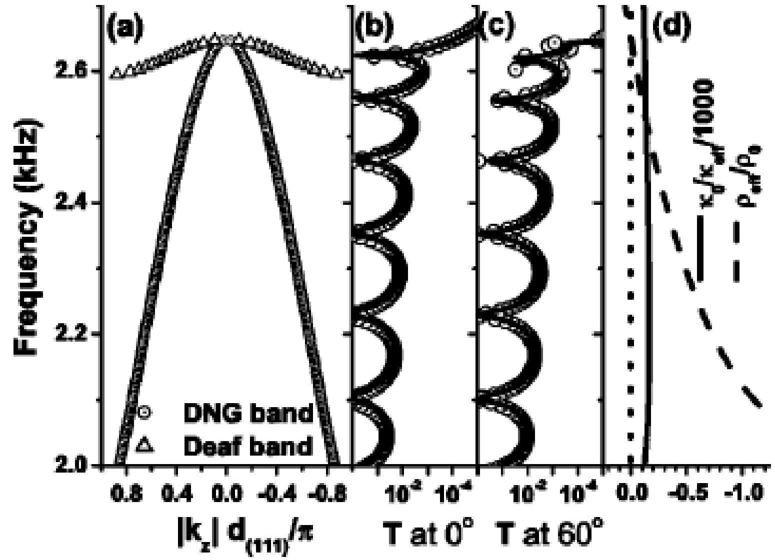


FIG. 1. Effective density and bulk modulus [using Eq. (5)] for rubber ($\rho=1300$ kg m^{-3}, $\kappa=6.27\times10^5$ Pa) spheres of filling ratio 0.1 within water ($\rho=1000$ kg m^{-3}, $\kappa=2.15\times10^9$ Pa).

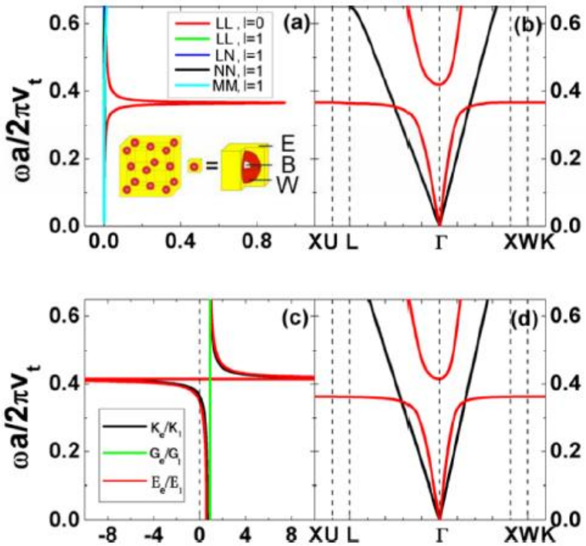


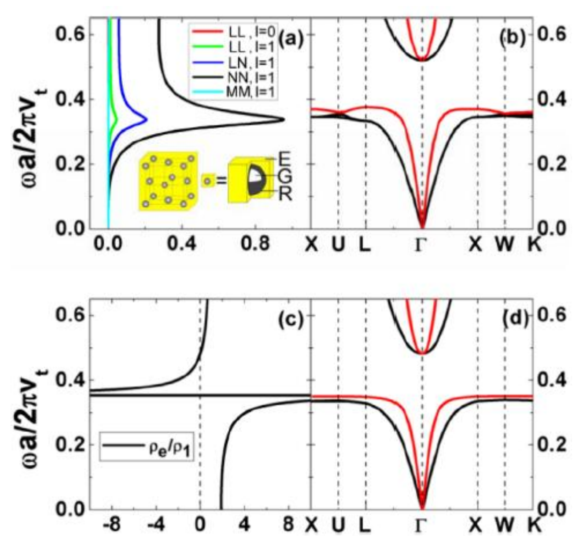
Rubber sphere radius=1cm

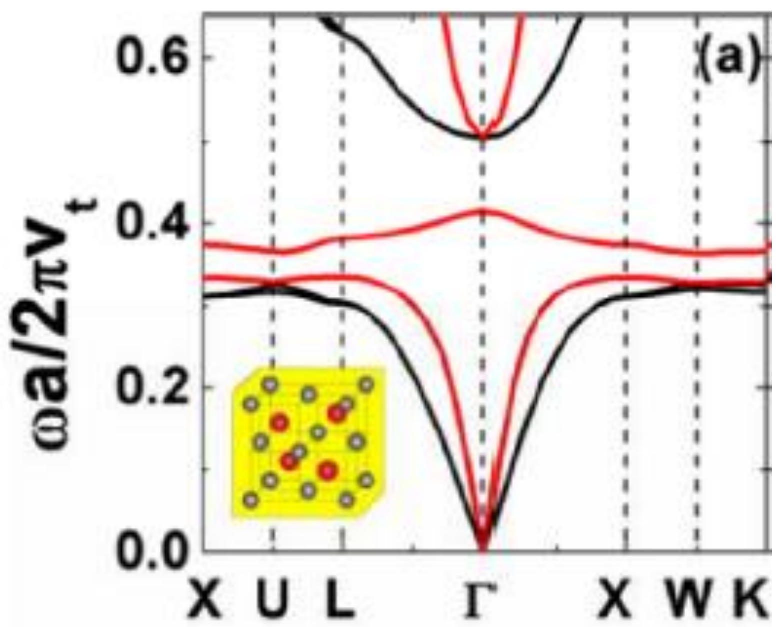
Jensen Li and C. T. Chan Phys. Rev. E 70, 055602 (2004)

Locally resonant sonic materials
Double Negativity

Metamaterial with Simultaneously Negative Bulk Modulus and Mass Density: Combination of air bubbles and rubber coated spheres



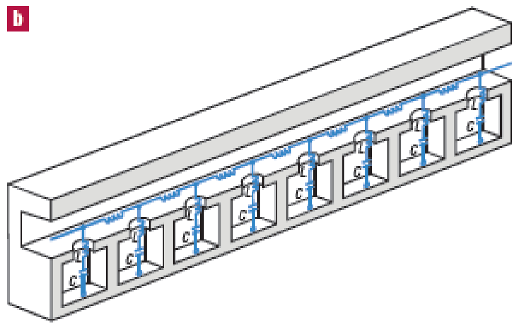


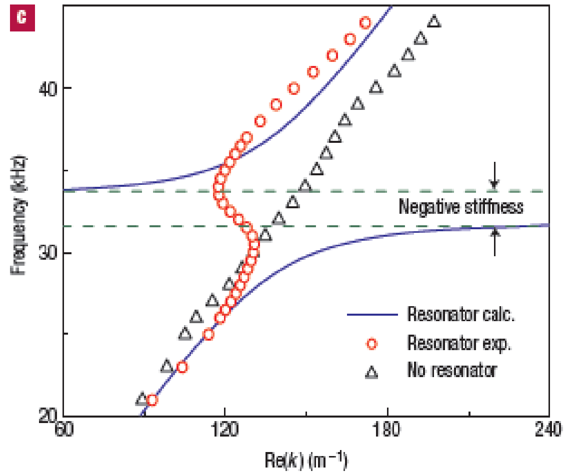


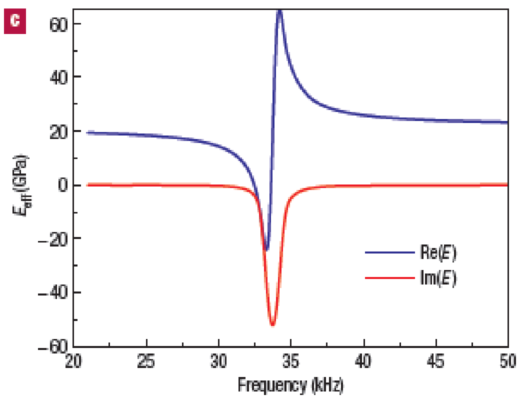
Yiqun Ding, Zhengyou Liu, Chunyin Qiu, and Jing Shi
Phys. Rev. Lett. 99, 093904 (2007)

Locally resonant sonic materials
Negative bulk modulus

Array of subwavelength Helmoltz resonators







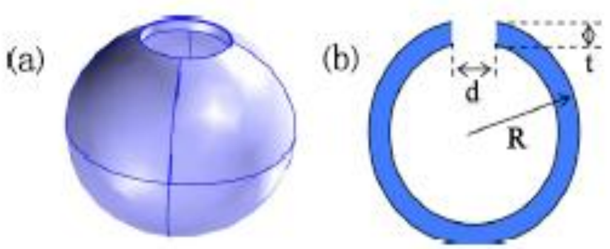
Structure

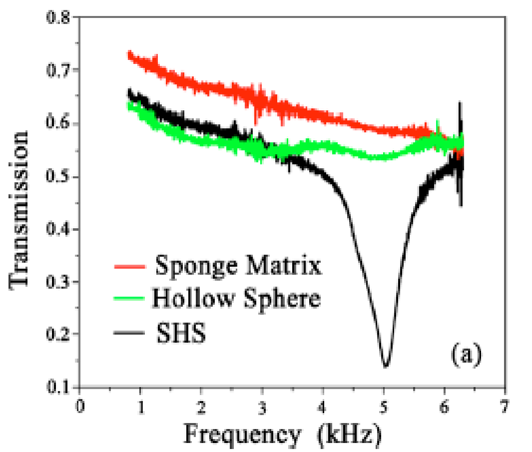
Dispersion curve

Effective bulk modulus

N. Fang, D. Xi, J. Xu, M. Ambati, W. Srituravanich, C. Sun, X. Zhang, Nature Materials, 5, 452 (2006)

Phononic crystal made of Split Hollow Sphere

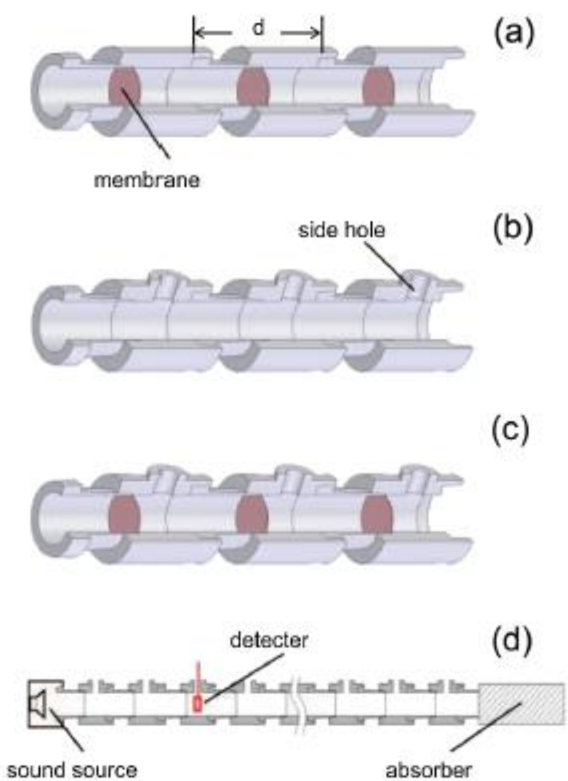


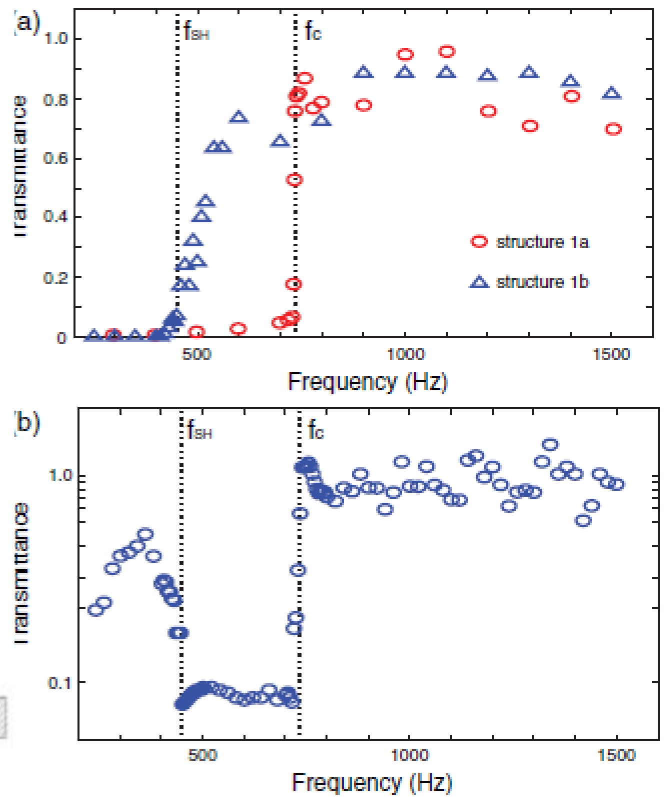


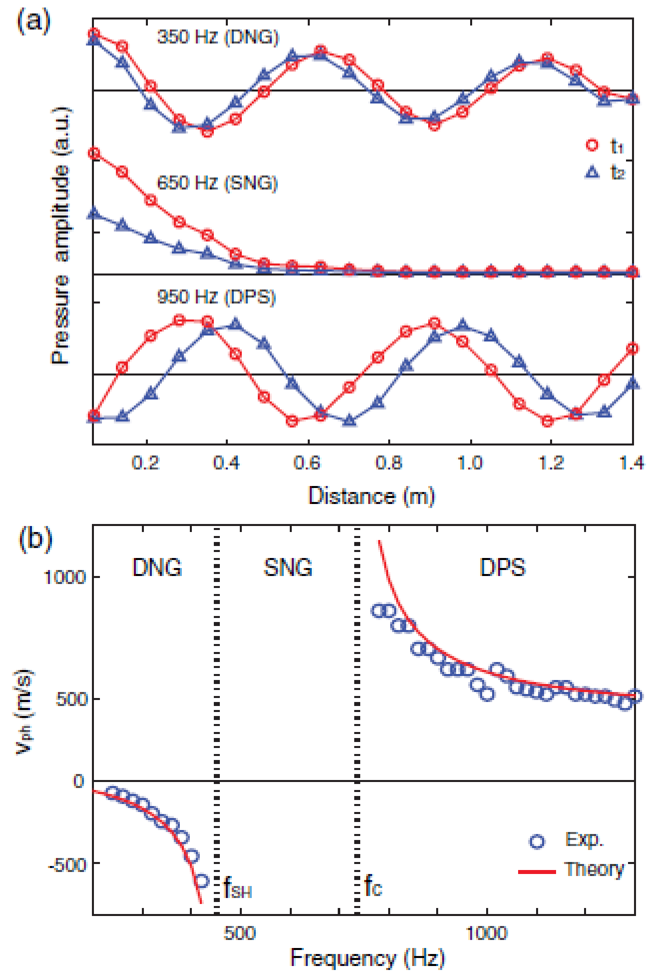
C. Ding, L. Hao, X. Zhao, J. Appl. Phys.108, 074911 (2010)

Locally resonant sonic materials
Double Negativity

Array of subwavelength membranes and side holes



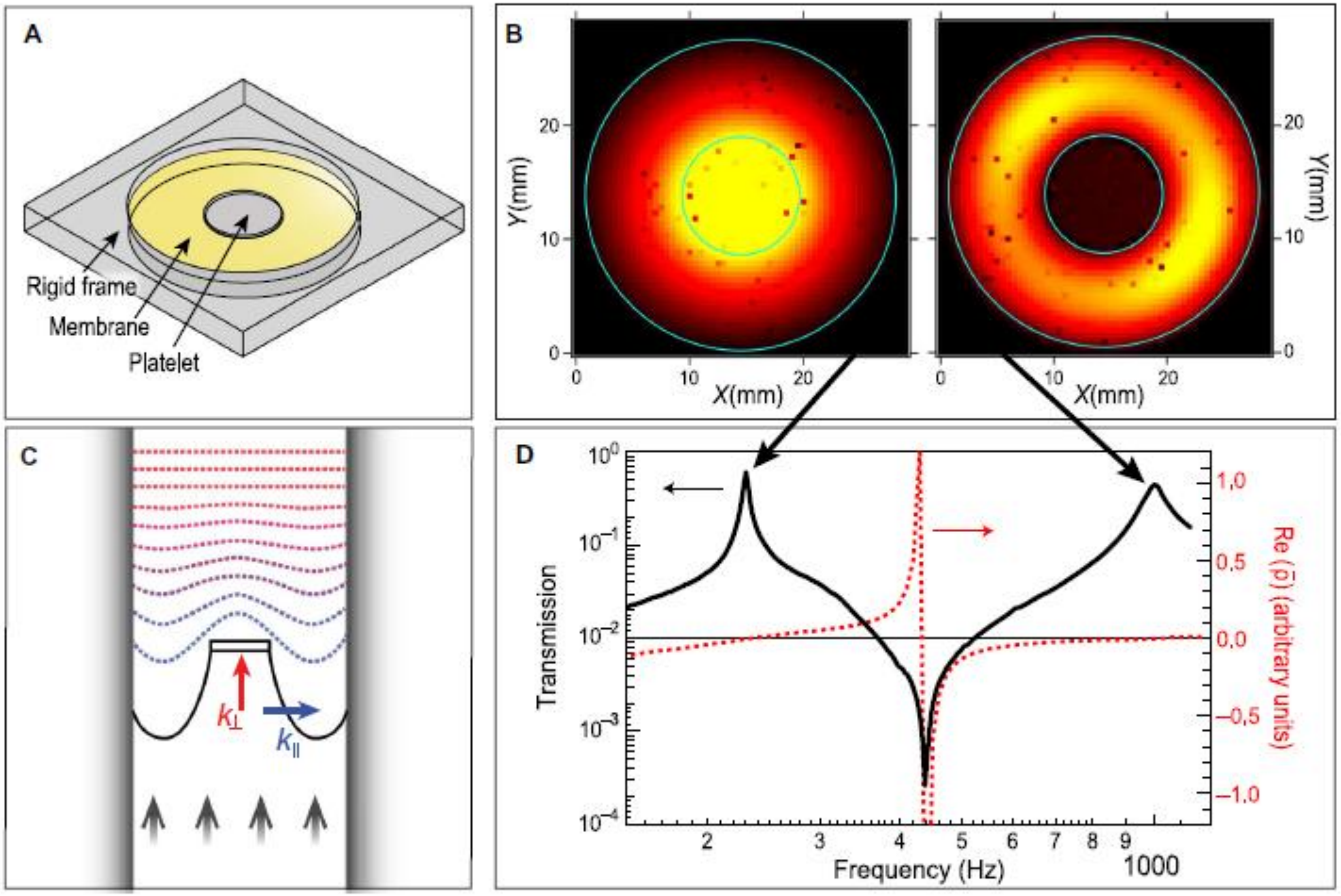




Sam Hyeon Lee, Choon Mahn Park, Yong Mun Seo, Zhi Guo Wang, and Chul Koo Kim,
Phys. Rev. Lett. ,104, 054301 (2010)

Single membrane with negative effective mass density

Decorated Membrane Resonators
Simple and Double Negativity



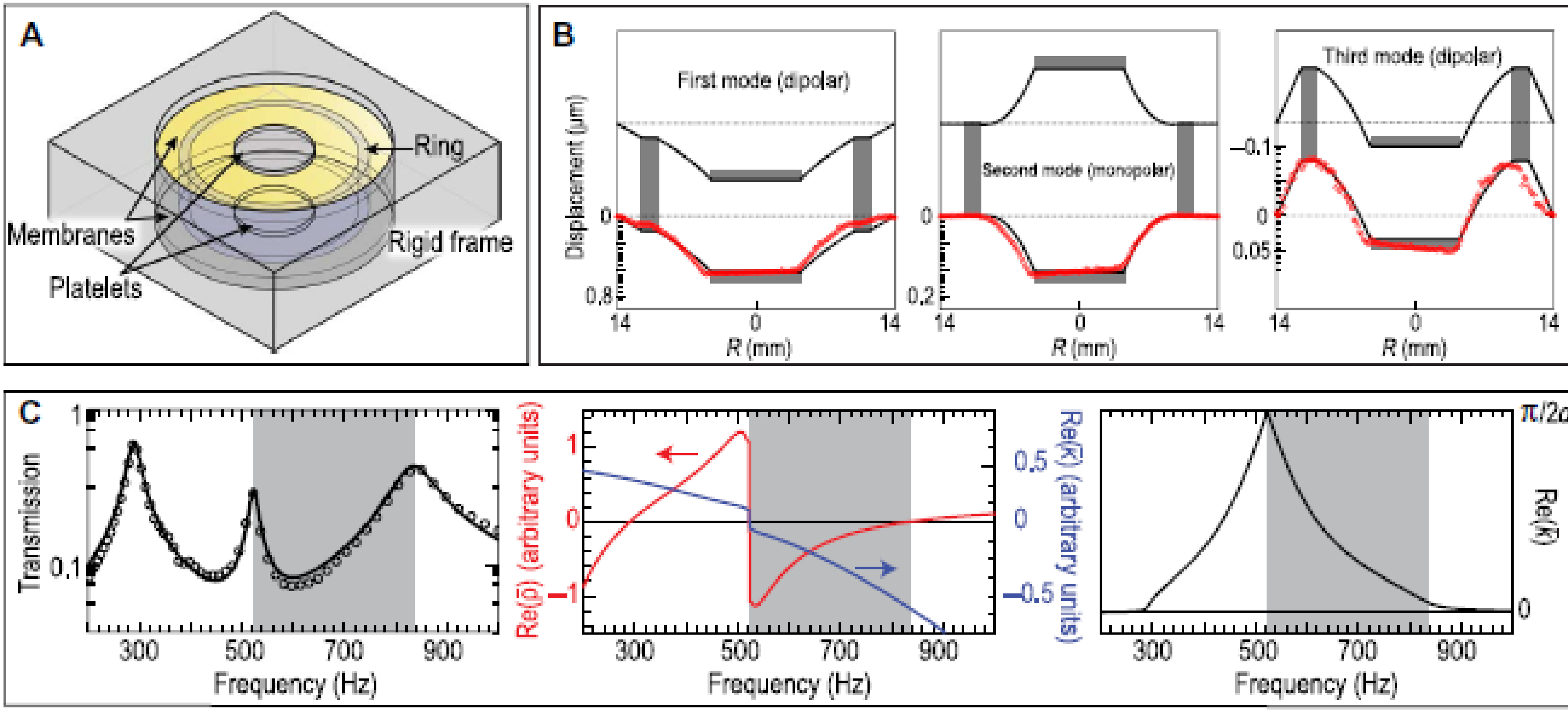
Two dipolar resonances

A rigid platelet is attached to the center of the membrane, whose mass is set by the desired resonant frequencies

Guancong Ma and Ping Sheng
Science Advances 2, 1501595 (2016)

Coupled membrane resonators with double negativity

Decorated Membrane Resonators
Simple and Double Negativity

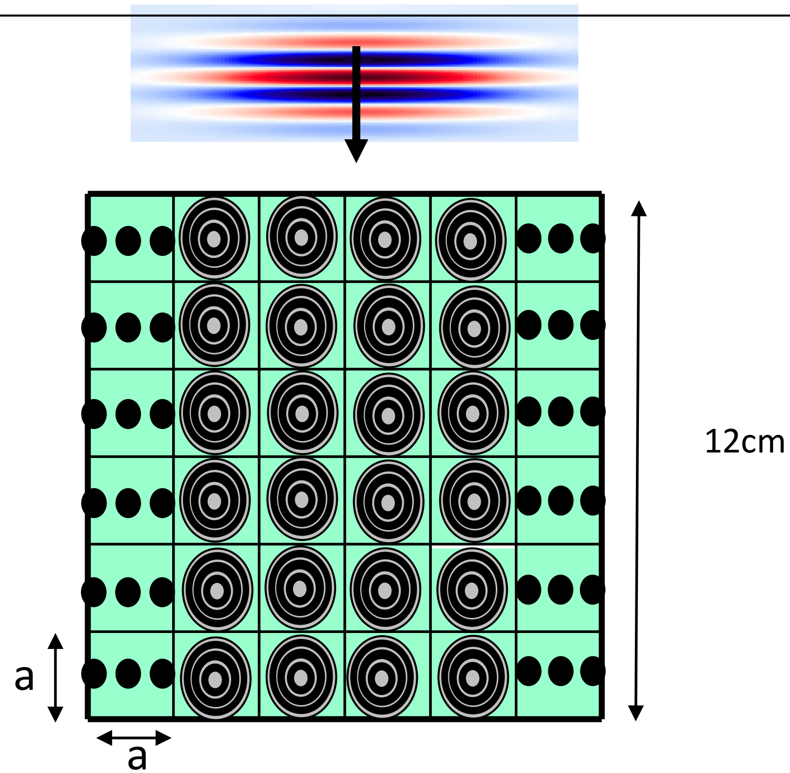


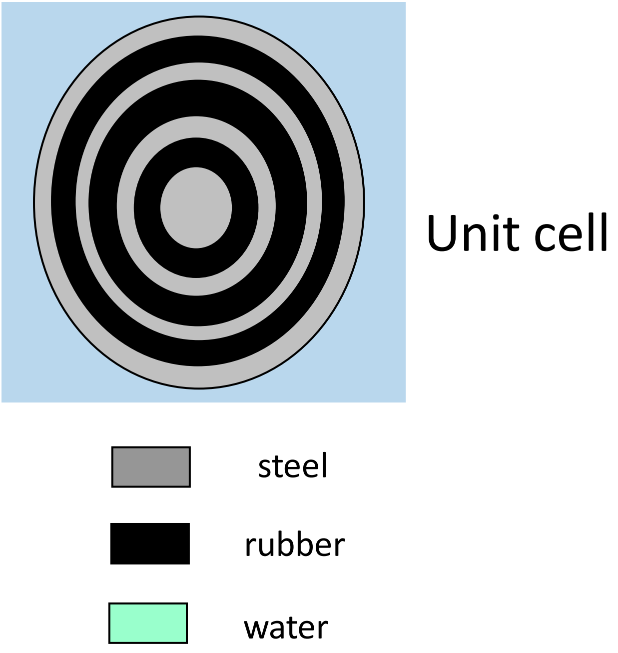
The dipolar and monopolar resonances are separately tunable

Guancong Ma and Ping Sheng
Science Advances 2, 1501595 (2016)

Locally resonant sonic materials and low frequency gaps

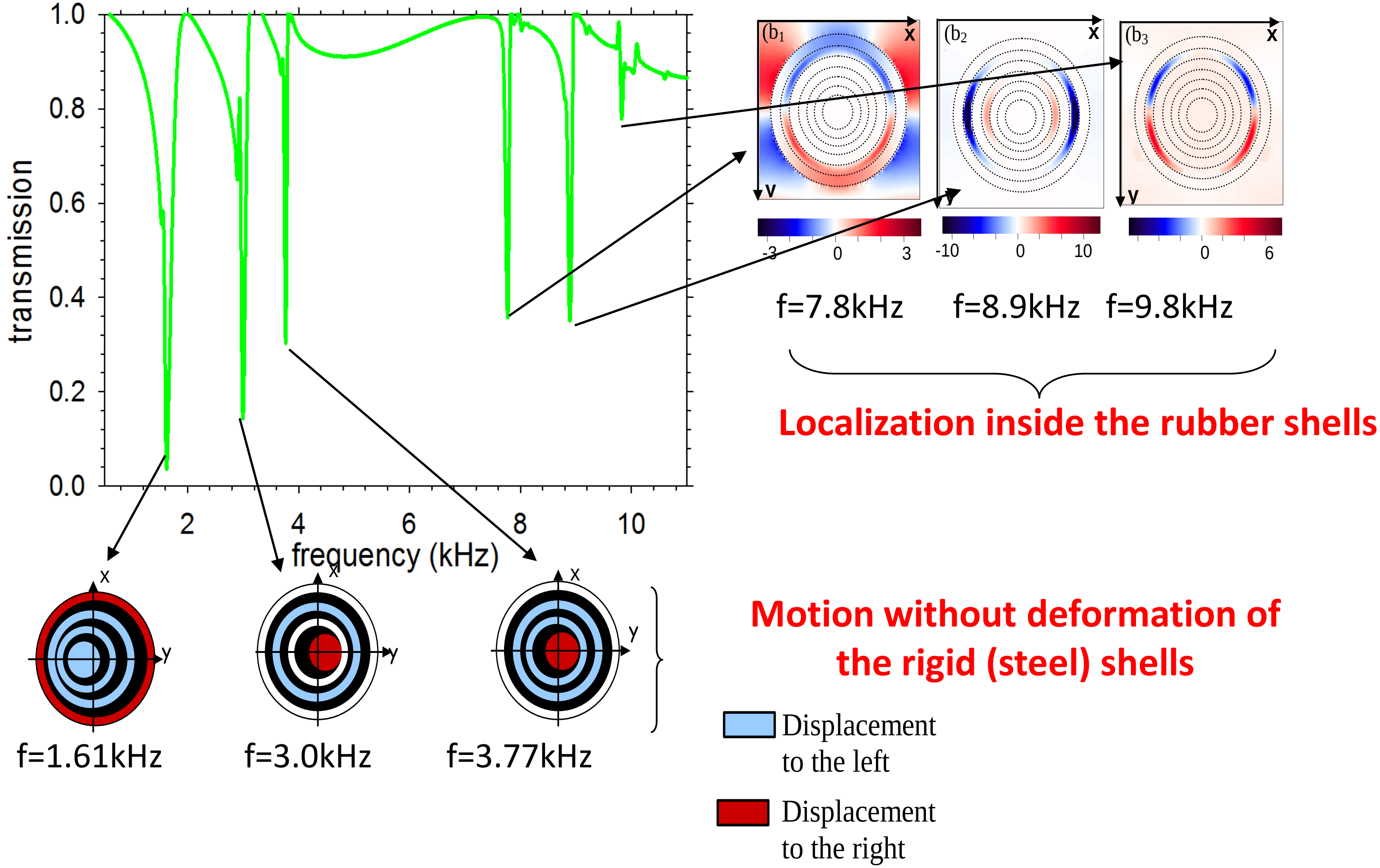
2D phononic crystal with multilayer inclusions made of alternate layers of steel and rubber immersed in water



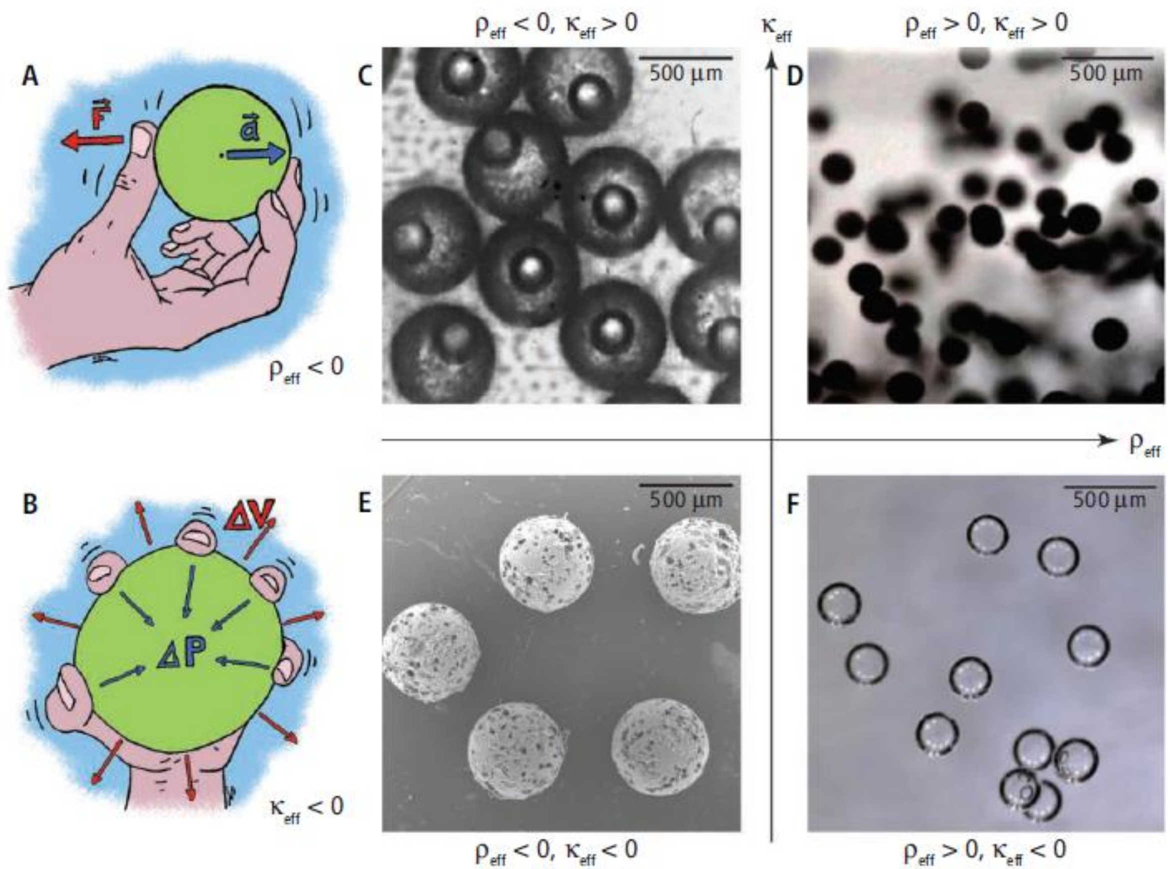


H. Larabi, Y. Pennec, B. Djafari Rouhani and J. Vasseur, Phys.Rev. B (2007)

Locally resonant sonic materials and low frequency gaps



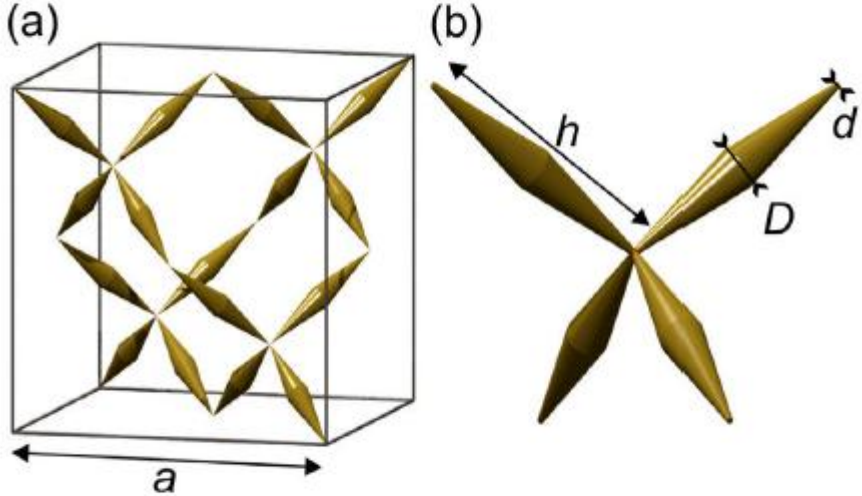
Classification of various locally resonant materials in terms of the sign of effective mass density and bulk modulus

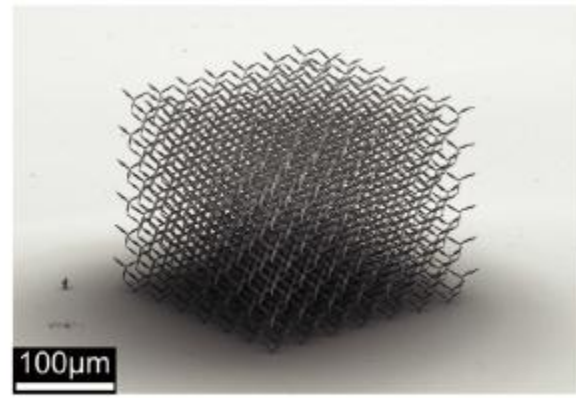


Features of acoustic metamaterials. (**A** and **B**) Schematic illustrations outline the dynamic behaviors of locally resonant materials with negative-valued effective parameters submitted to harmonic excitations. (A) The acceleration *a* of a material possessing a negative effective mass density ($\rho_{eff} < 0$) is opposite to the driving force *F*. (B) A material possessing a negative effective bulk modulus ($\kappa_{eff} < 0$) supports a volume expansion ($\Delta V > 0$) upon an isotropic compression ($\Delta P > 0$). (**C** to **F**) Classification of various locally resonant materials in the (ρ_{eff}, κ_{eff}) plane in terms of the sign of the effective mass density and the bulk modulus, made of various resonant inclusions: (C) core-shell particles, (D) "slow-oil" droplets, (E) polymer porous beads, and (F) air bubbles.

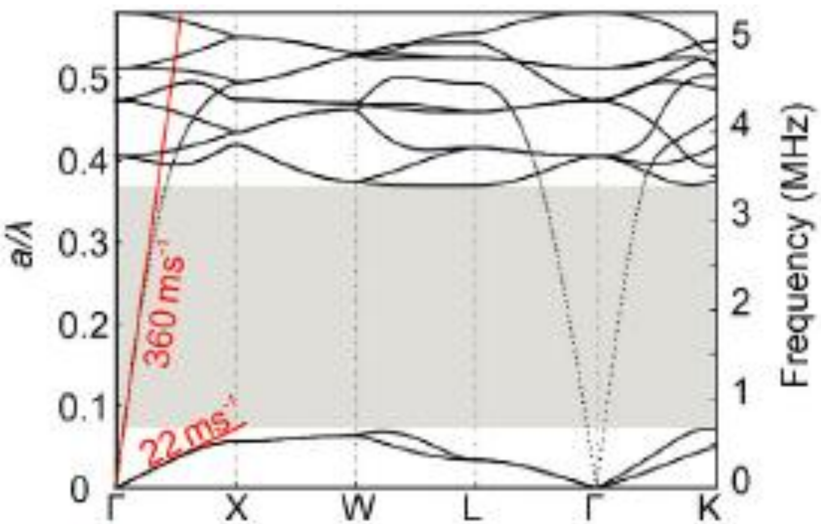
T. Brunet et al, Science, 342, 323 (2013)

Pentamode metamaterials

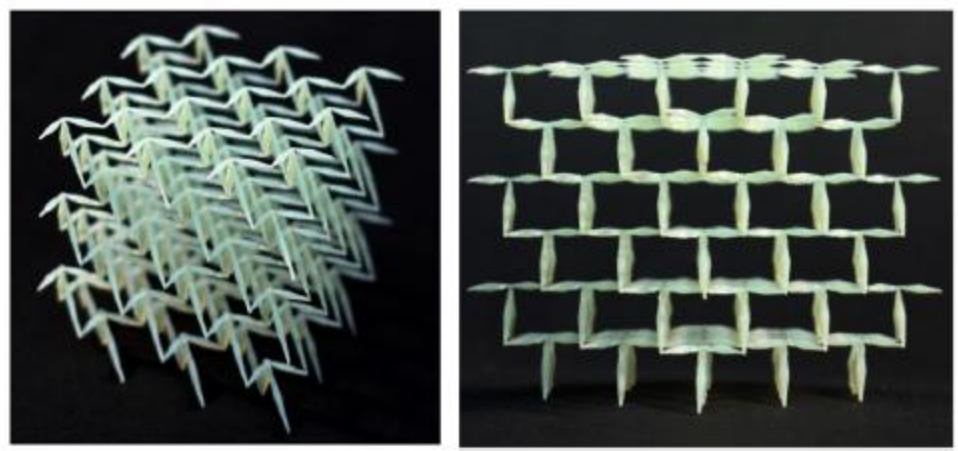




Polymer based pentamode metamaterial



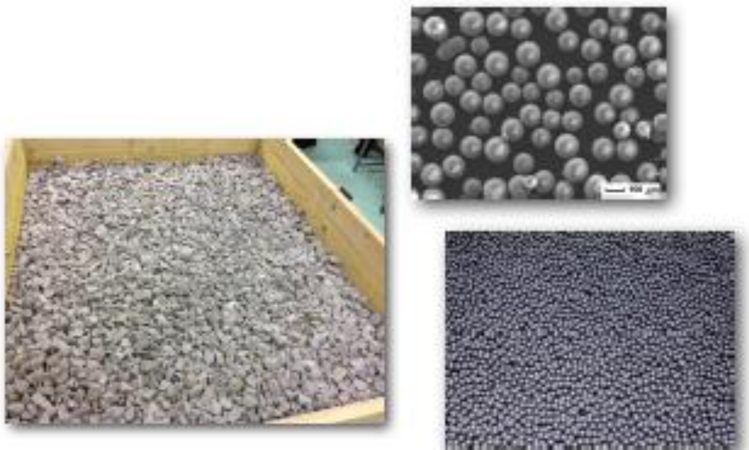
M. Kadic, T. Buckmann, N. Stenger, M. Thiel and M. Wegener, Appl. Phys. Lett. 100, 191901(2012)

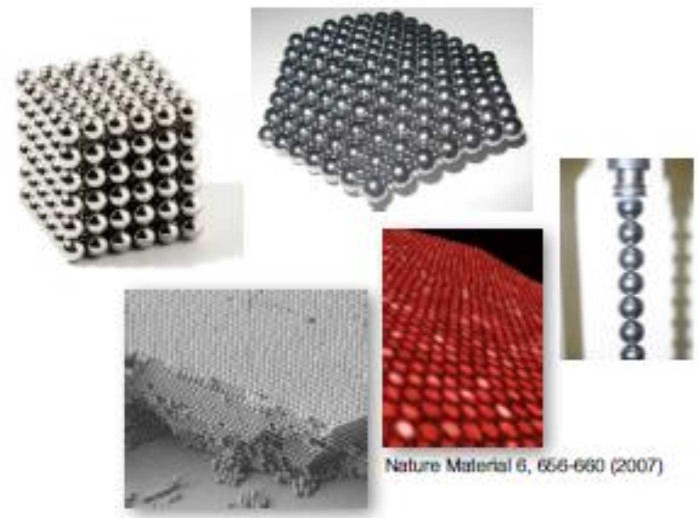


Anisotropic version of pentamode metamaterial

M. Kadic, T. Buckmann, R. Schittny and M. Wegener, Rep. Prog. Phys. 76,126501 (2013)

Elastic wave propagation in granular materials





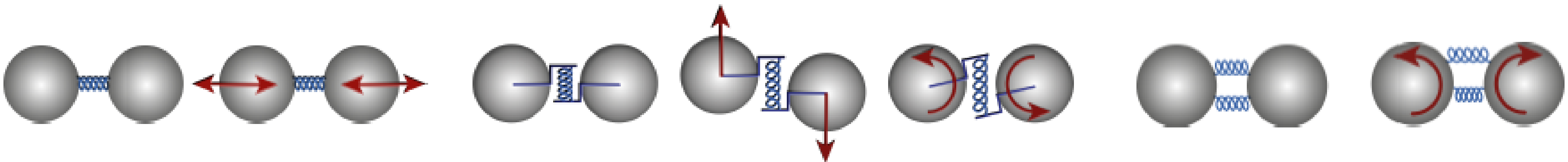
Nature Material 6, 656-660 (2007)

- ➪ Granular avalanches, packing stability
- ➪ Evaluation of powders, ballast
- ➪ Energy transport in amorphous solids

- ➪ Fundamental wave processes
- ➪ Model configurations for experiments
- ➪ Wave filtering, control, devices

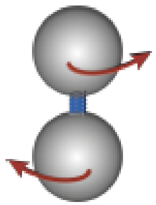
Courtesy of Vincent Tournat

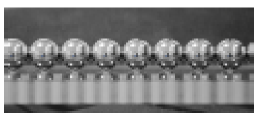
Non central forces and contact nonlinearity

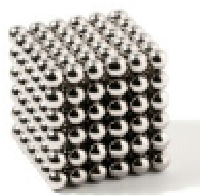


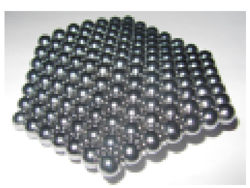
Longitudinal interaction | Shear interaction | Bending interaction (cohesive media)

Torsional interaction





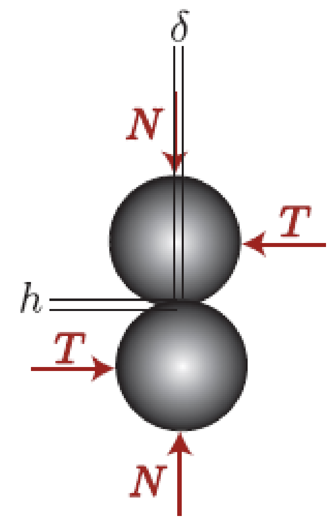




Nonlinear Hertz-Mindlin contact laws

Normal interaction $$h \propto N^{2/3}$$

Shear interaction $$\delta \propto N^{2/3}\left[1-(1-\frac{T}{\mu_f N})^{2/3}\right]$$

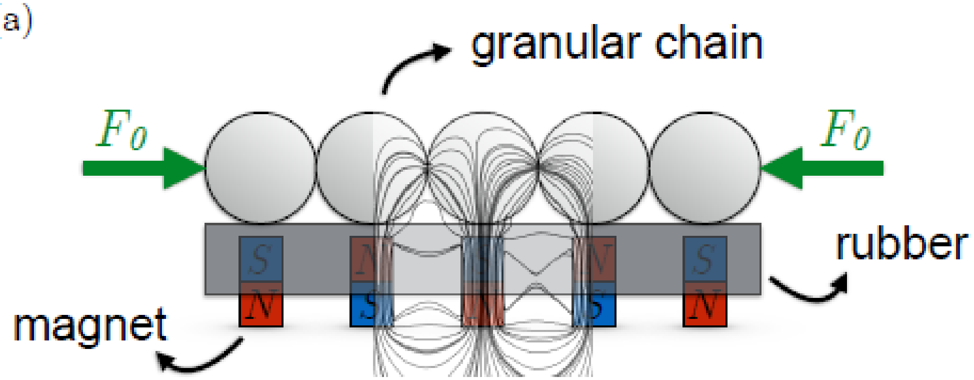


➪ Can be linearized for a large static force compared to the dynamic one

Courtesy of Vincent Tournat

Example of the 1D (free) granular chain

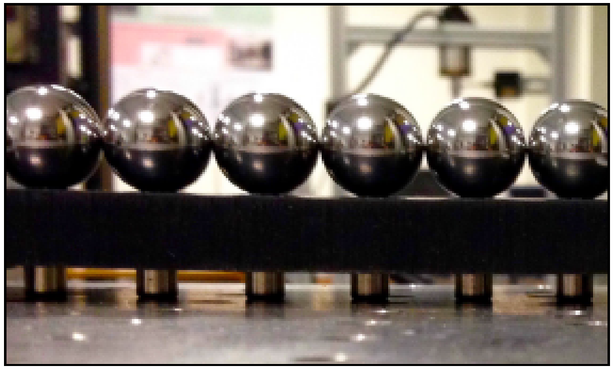
(a)



$$F_0 = 1\ N$$

Stainless steel beads

(b)

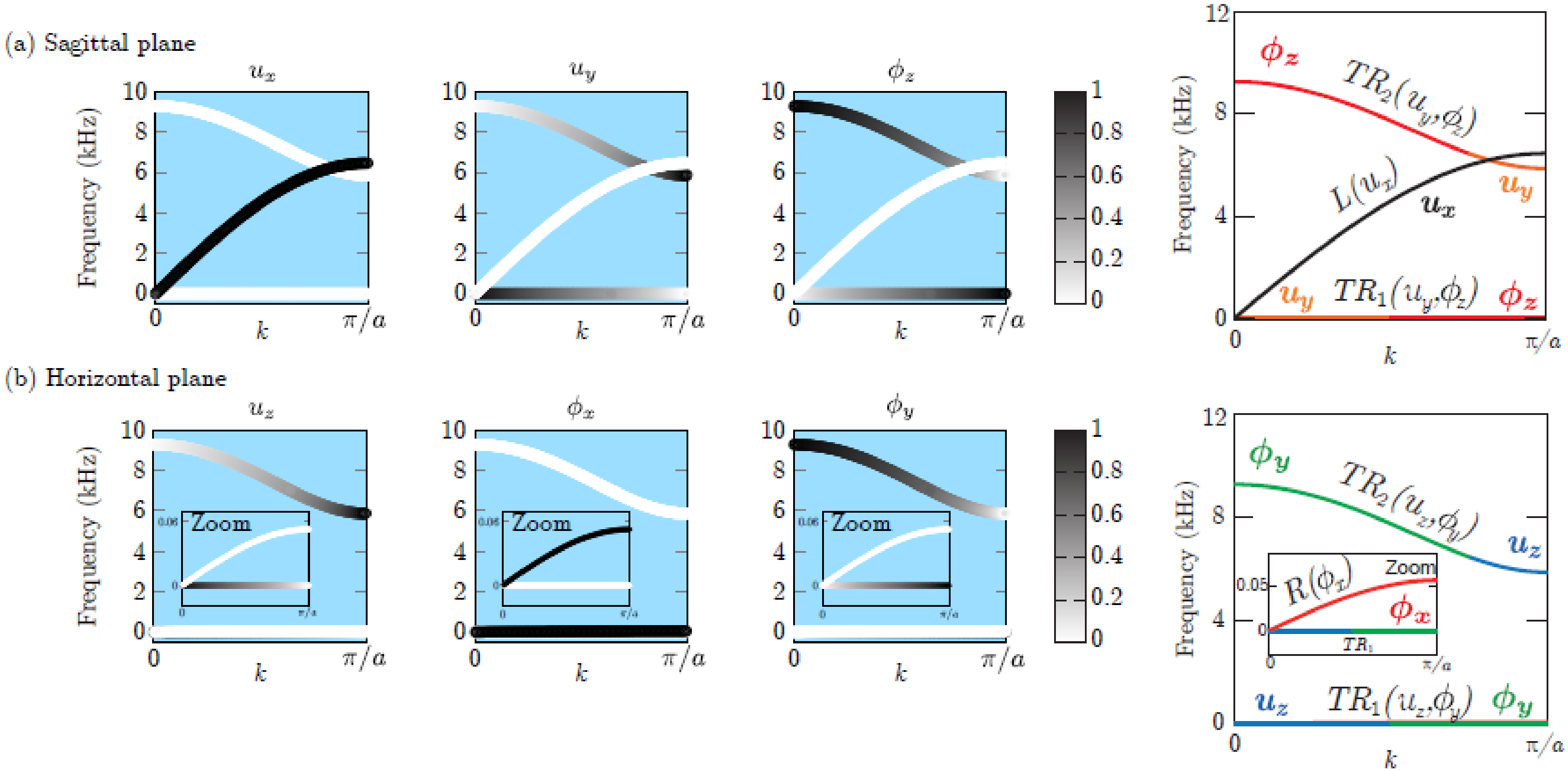


$$d = 16\ mm$$

F. Allein, V. Tournat, V. Gusev, G. Theocharis, **Transversal-rotational and zero group velocity modes in tunable magneto-granular phononic crystals**, Extreme Mechanics Letters 12, 65-70 (2017).

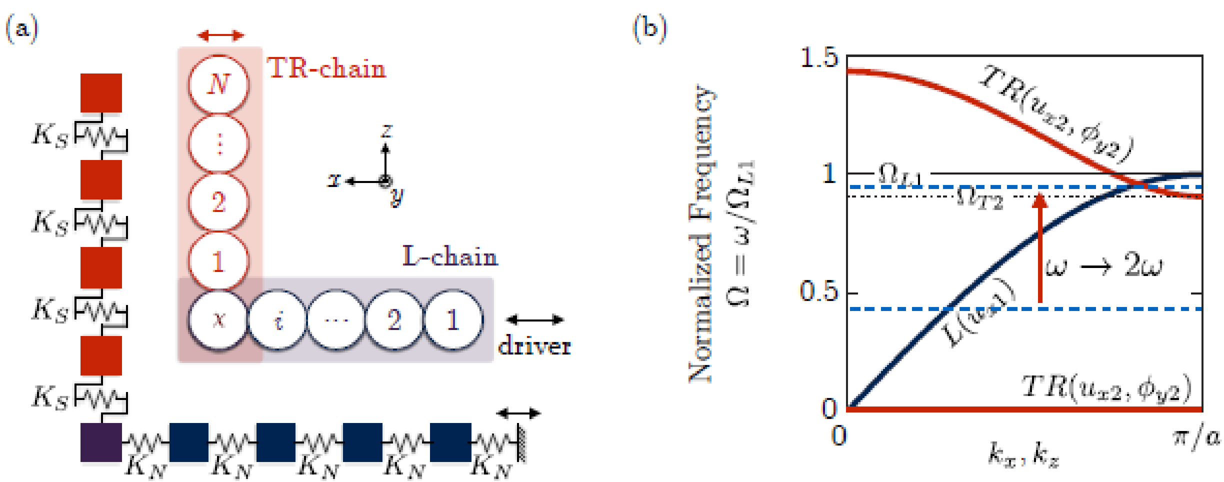
Courtesy of Vincent Tournat

Dispersion in the 1D free granular chain



Courtesy of Vincent Tournat

Frequency conversion via a L-shape granular device

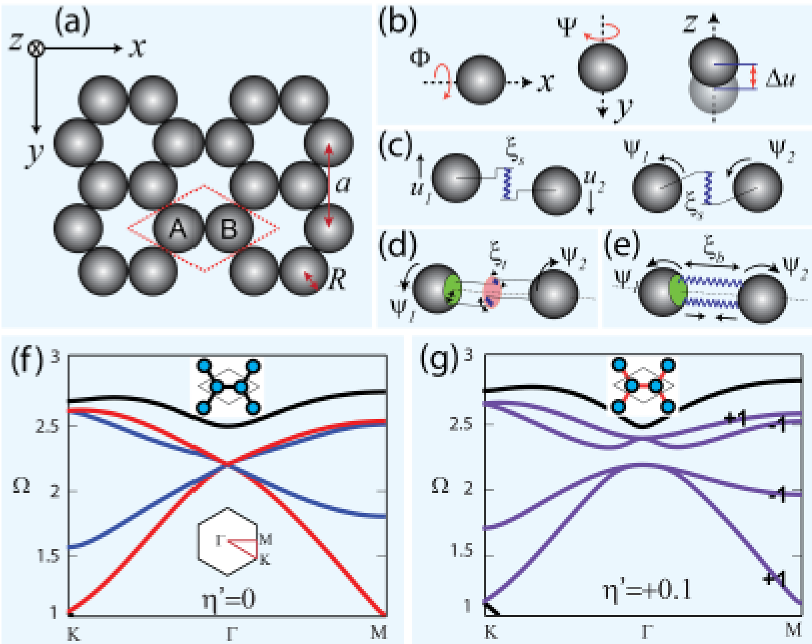


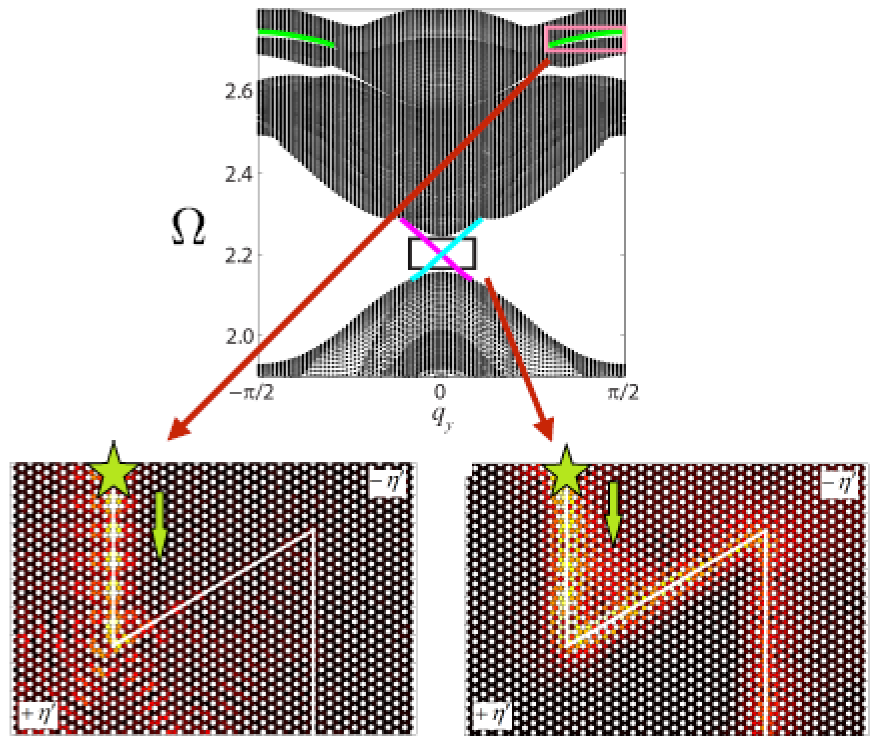
3 kHz → 6 kHz
(second harmonic generation)

F. Allein, G. Theocharis, V. Tournat, V. Gusev, in preparation (2017).

Courtesy of Vincent Tournat

One-way topologically protected rotational edge waves





topological trivial

protected topological non trivial

➜ Kane-Mele Hamiltonian (Z2 topological insulator)

➜ Effective spin-orbit coupling

L. Zheng, G. Theocharis, V. Tournat, V. Gusev, Topological rotational waves in granular graphene, submitted (2017).

Courtesy of Vincent Tournat

Outline

1.Simple analytical models to introduce basic notions

- Band gaps and localized modes associated to defects
- Zeros of transmission and Fano resonances

2. One-dimensional (1D) multilayer structures

- Theoretical methods
- Dispersion curves, band gaps and localized modes
- Transmission coefficient: tunnelling (fast)transmission and resonant (slow) transmission

3. Two-dimensional (2D) Phononic crystals

- Theoretical methods
- Dispersion curves and complete band gaps (Bragg gaps and hybridization gaps)
- Local resonances and low frequency gaps
- Waveguide and cavity modes

4. Phononic crystal slabs and nanobeams

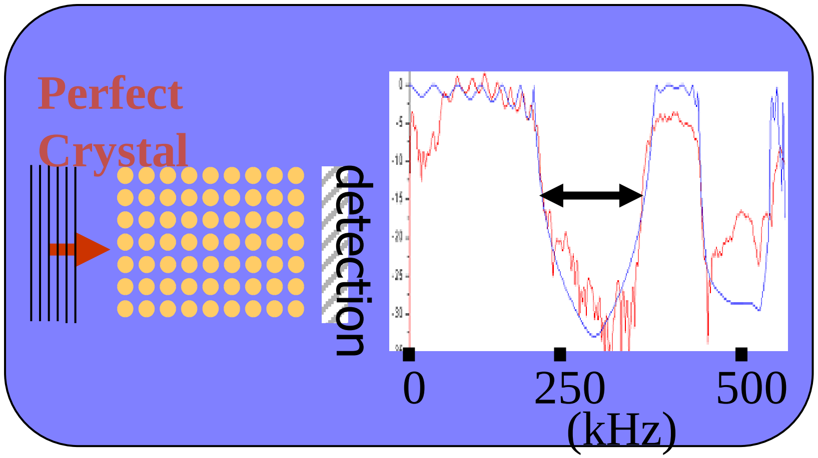
- Array of holes in a Si membrane
- Array of pillars on a thin membrane
- Surface waves in semi-infinite phononic crystals
- Nanobeam waveguides

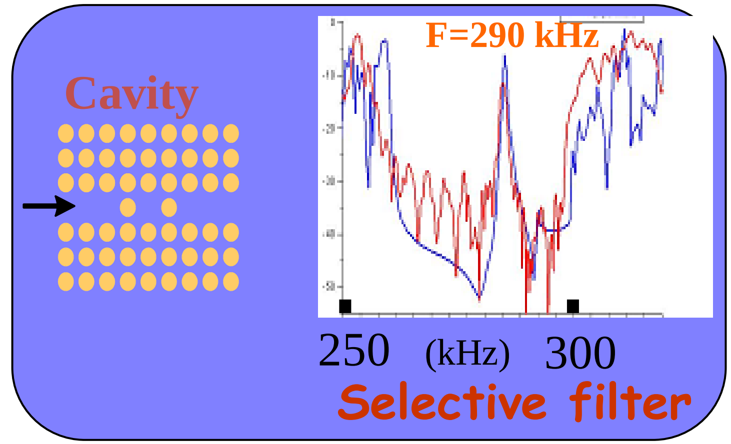
Defects in Phononic Crsytals

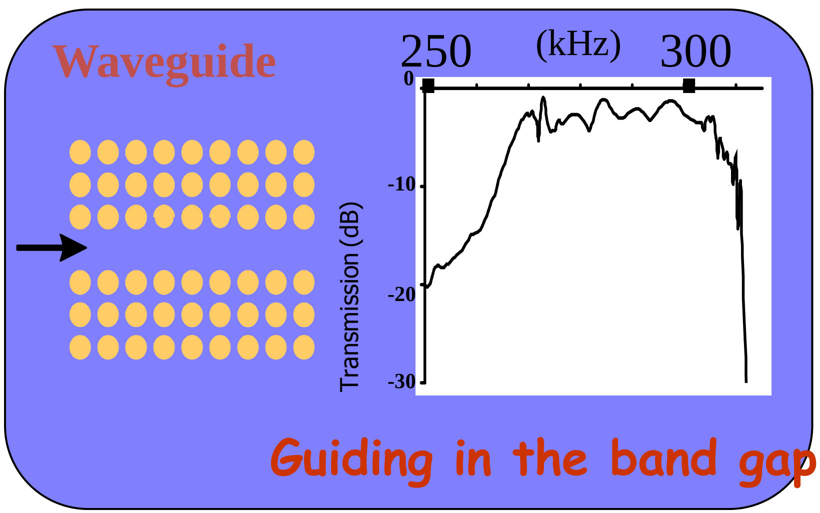
Examples of steel rods in water

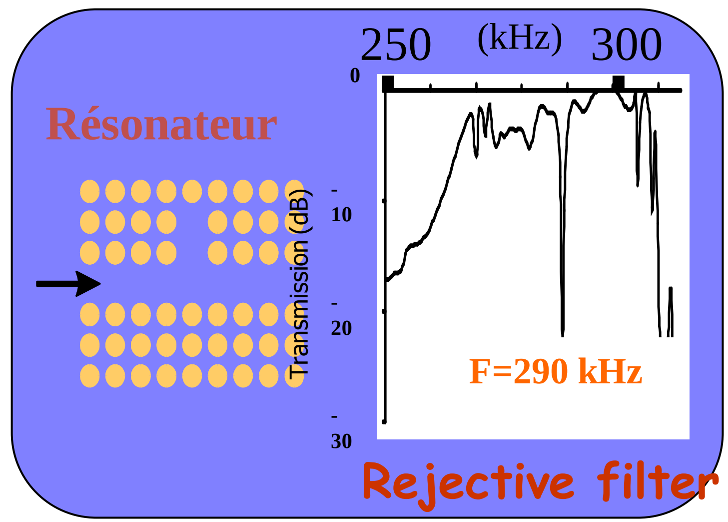
GUIDING

FILTERING









-Y. Pennec et al., Phys. Rev. E 69, 046608 (2004)

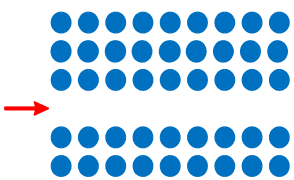
-J.O. Vasseur et al., Zeitschrift Für Kristallographie 220, 824 (2005)

-Y. Pennec et al, Appl. Phys. Lett. 87, 261012 (2005)

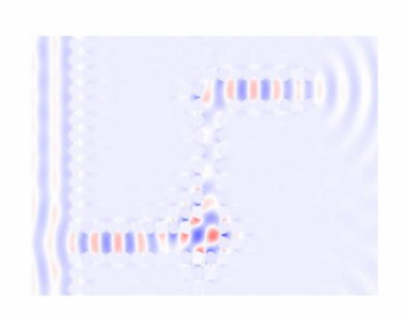
Summary on Defects in Phononic Crystals

Object: Controlling and manipulating the sound

Guiding

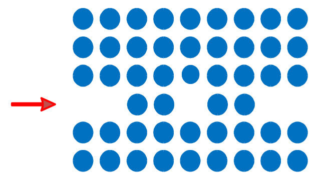


Straight waveguide

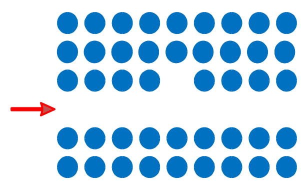


Bent waveguide

Filtering

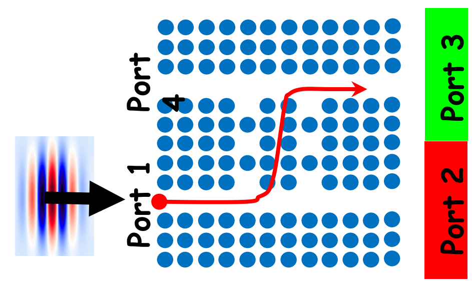


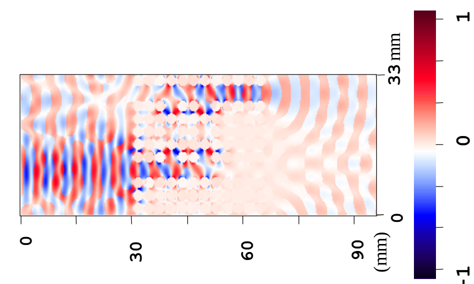
Selective



Rejective

Demultiplexing





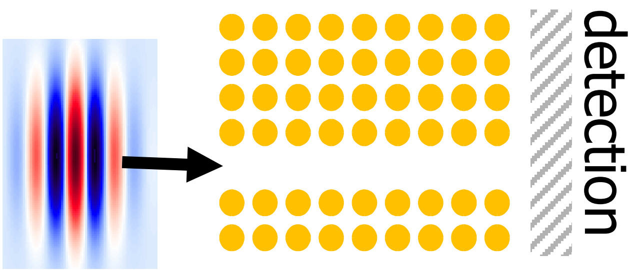
At f_0=290kHz, the incident field is transferred from port 1 to port 3

-*Y. Pennec et al., Phys. Rev. E 69, 046608 (2004)*
-*J.O. Vasseur et al., Zeitschrift Für Kristallographie 220, 824 (2005)*
-*Y. Pennec et al, Appl. Phys. Lett. 87, 261012 (2005)*

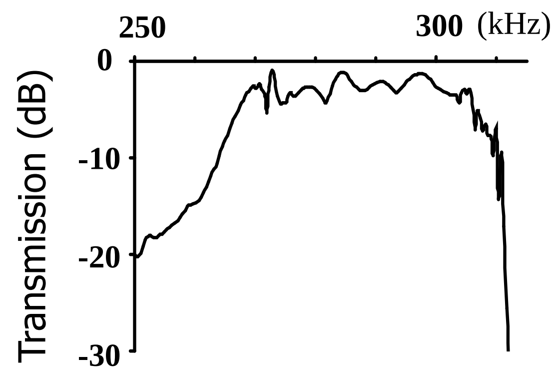
Guided Modes

Square array of steel cylinders in water

a = 3 mm; D = 2.5 mm

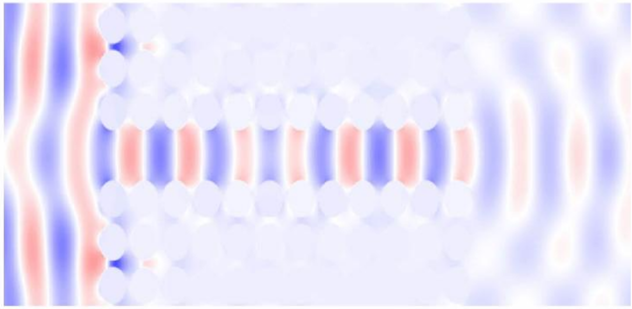


Band Gap = [250 310]kHz



Exemple : F=290 kHz

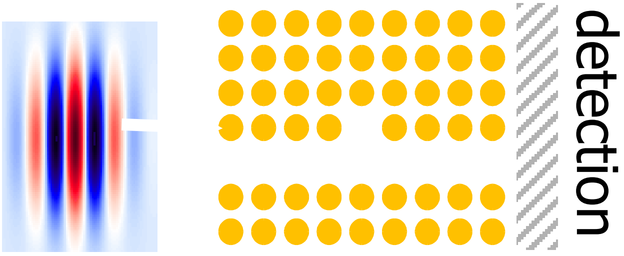
monochromatic Source



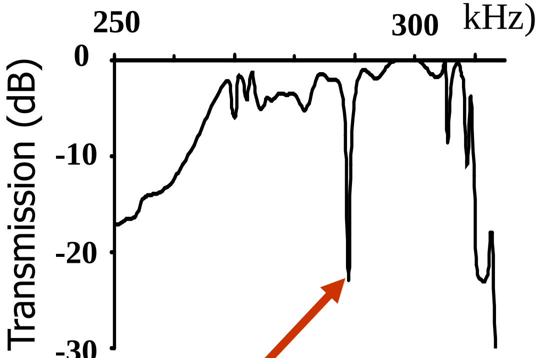
Waveguide coupled to a lateral stub

Square array of steel cylinders in water

a = 3 mm; D = 2.5 mm

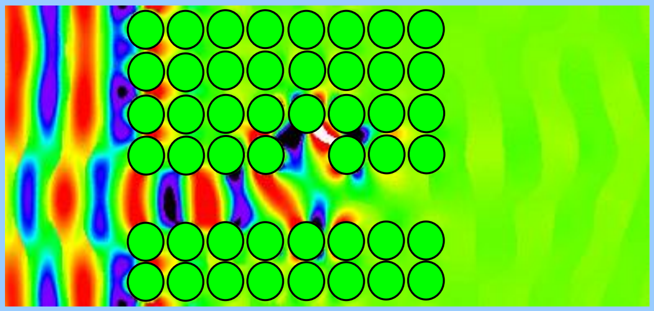


Band Gap = [250 310]kHz



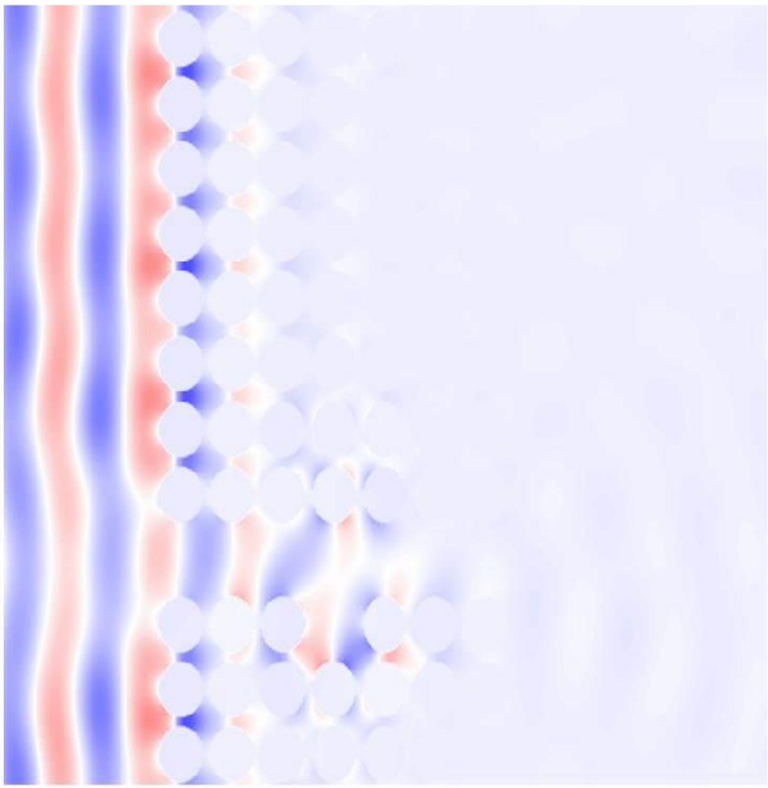
F=290 kHz : Zero of Transmission

Monochromatic Source at F=290kHz



→ **Rejection Filter**

Waveguiding and Filtering



Phase of the transmission coefficient

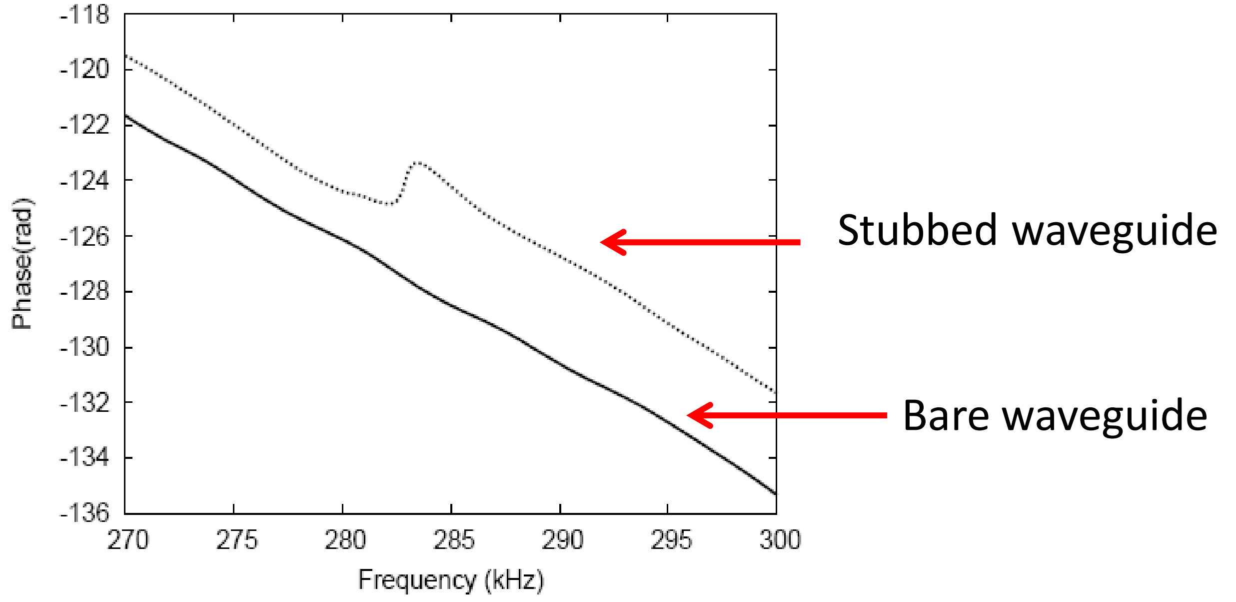
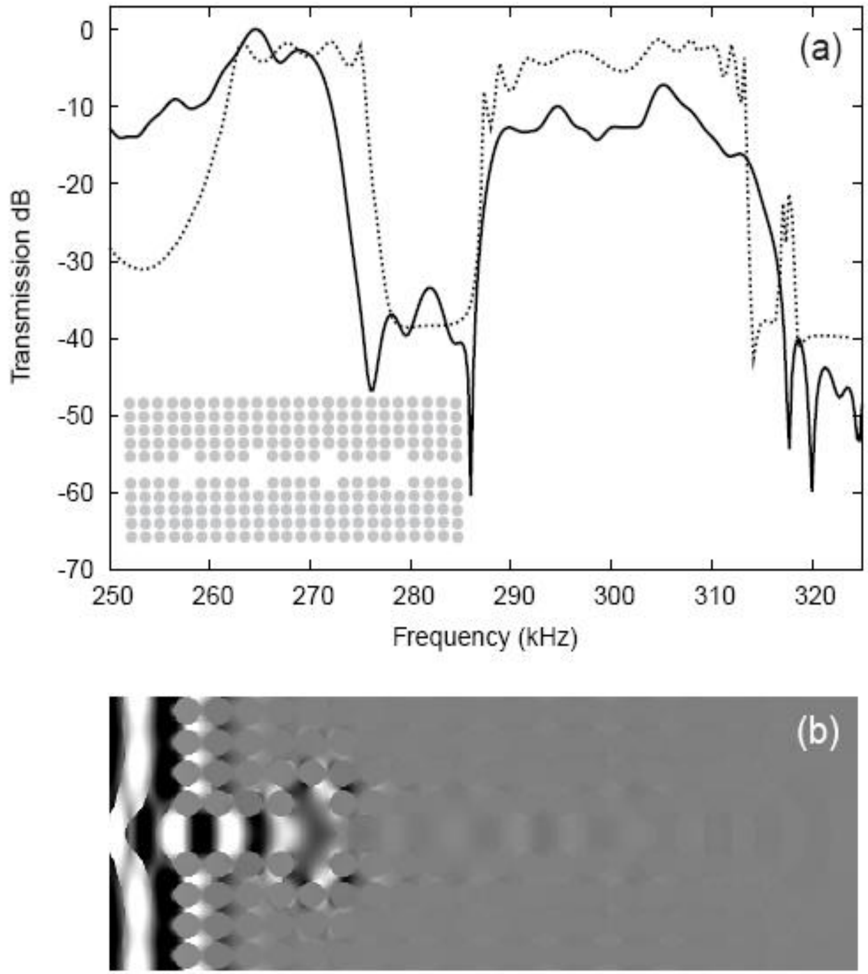


Fig. 3 – Experimental phase *vs.* frequency measurements for the bare waveguide (solid line) and for the guide with a grafted symmetrical stub (dashed line).

S. Benchabane, A. Khelif, A. Choujaa, B. Djafari Rouhnai, V. Laude, Europhysics Letters,71, 570 (2005)

Waveguide coupled to a set of lateral stubs



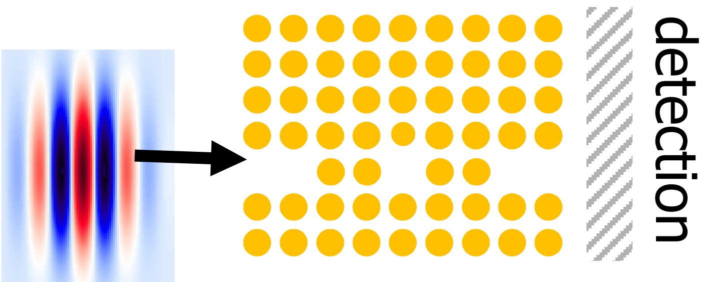
Band Gap of the crystal: [250 310]kHz

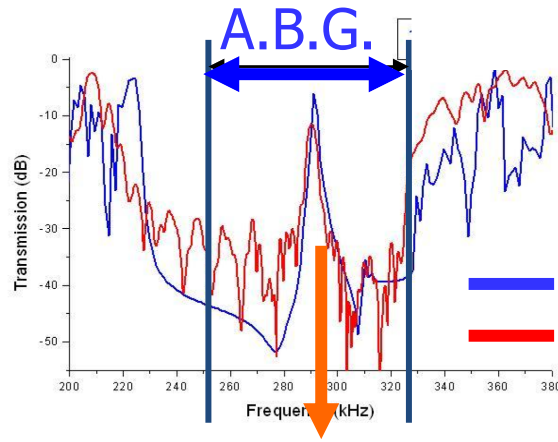
S. Benchabane, A. Khelif, A. Choujaa, B. Djafari Rouhnai, V. Laude, Europhysics Letters,71, 570 (2005)

A cavity inside a straight guide

Square array of steel cylinders in water

a = 3 mm; D = 2.5 mm

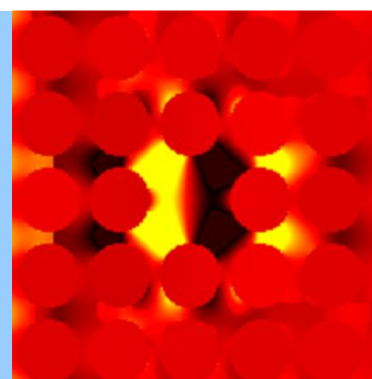




F=290 kHz : Selective Transmission

Selective Filter

F=290 kHz



Map of the displacement field

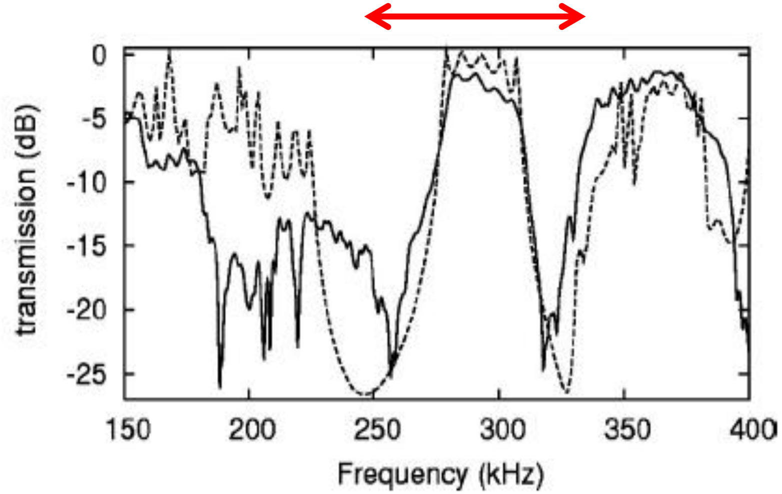
A. Khelif, A. Choujaa, B. Djafari Rouhani, M. Wilm, S. Ballandras, V. Laude, Phys. Rev. B68, 214301 (2003)

Waveguiding and Filtering

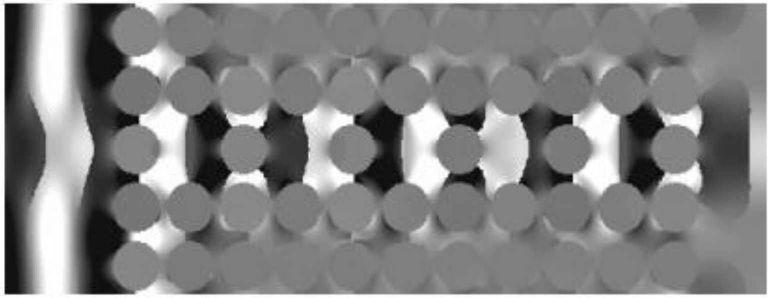
Waveguiding based on the evanescent coupling of defect modes

Square array of steel cylinders in water
a=3mm   r=1.25mm

Gap of the perfect crystal



(a)



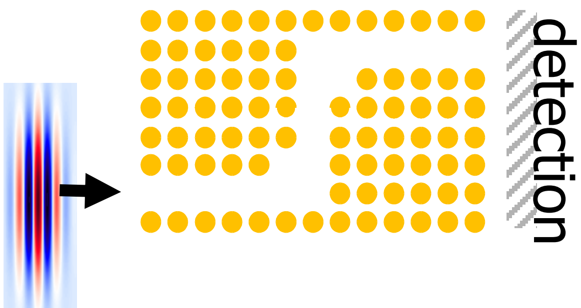
(b)

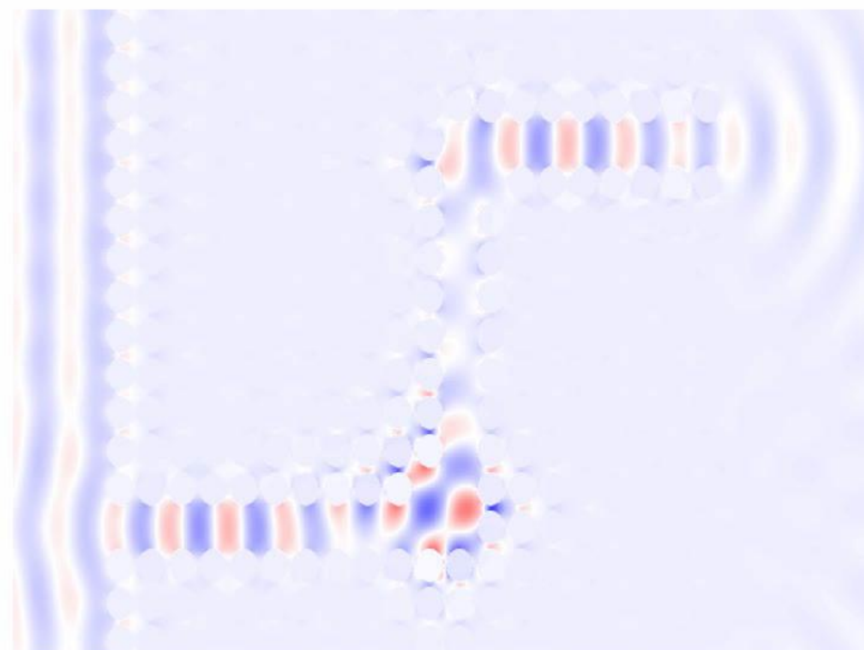
A. Khelif, A. Choujaa, B. Djafari Rouhani, M. Wilm, S. Ballandras, V. Laude, Phys. Rev. B68, 214301 (2003)

Bent Guide

a =3 mm; D =2.5 mm

Square array of steel cylinders in water

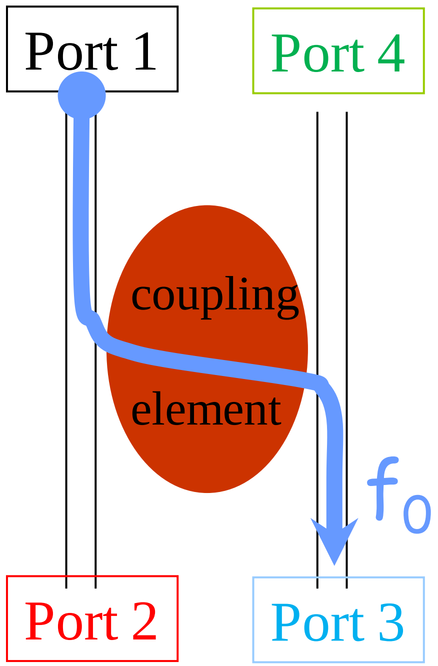




A. Khelif, A. Choujaa, B. Djafari Rouhani, M. Wilm, S. Ballandras, V. Laude, Phys. Rev. B68, 214301 (2003)

Demultiplexer

Principle

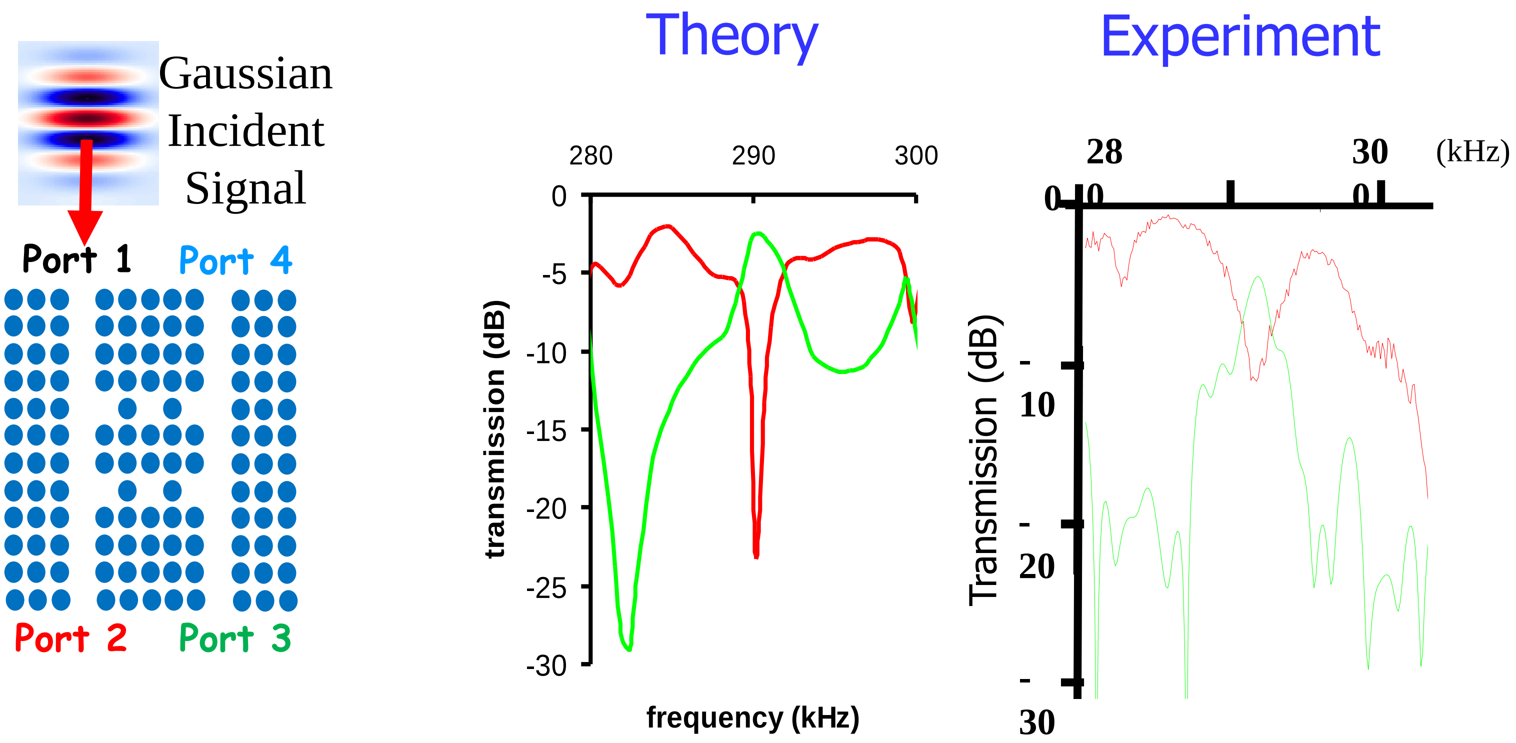


Two waveguides interacting through an appropriate coupling element

Transfer of one frequency f_0 one guide to the other, leaving all neighboring frequencies unaffected

Demultiplexer

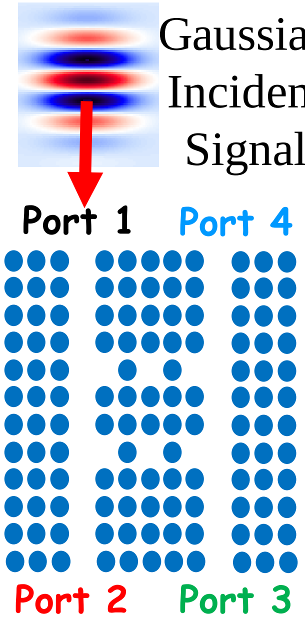
Demultiplexing



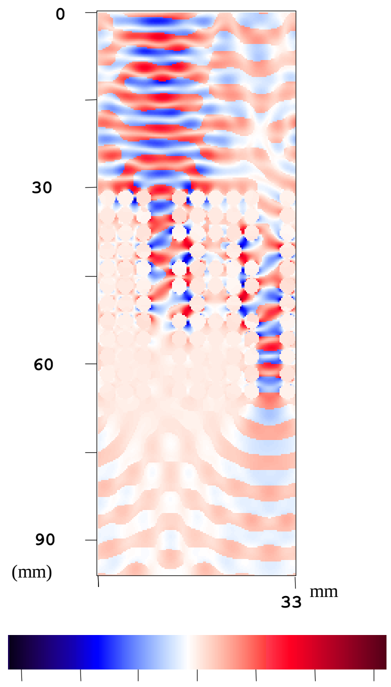
Y. Pennec ,B. Djafari Rouhani, J. Vasseur et al, Appl. Phys. Lett. 87, 261912 (2005)

Demultiplexer

Demultiplexing



Displacement field

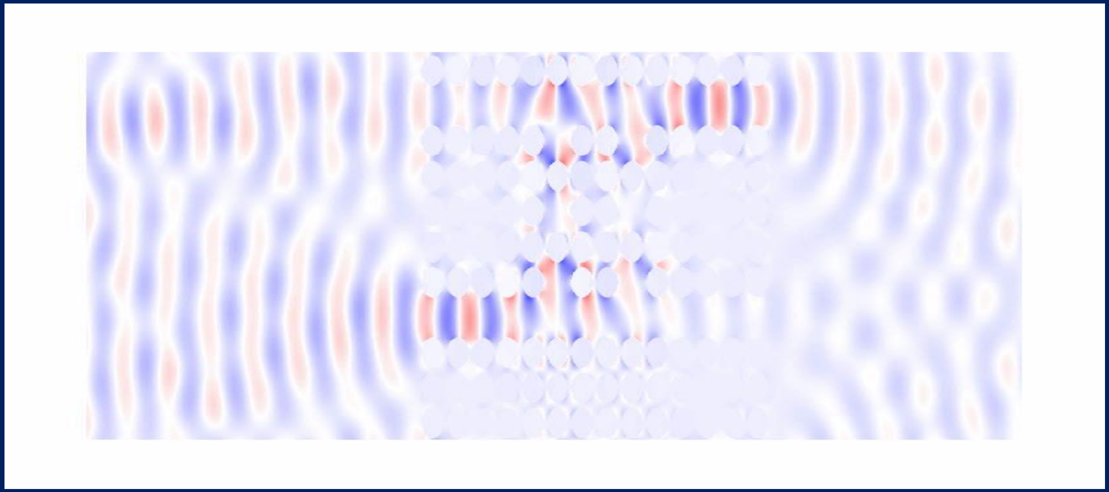


At f_0=290kHz, the incident field is transferred from port 1 to port 3

f_0=290kHz corresponds to both the resonant mode of the stub and the cavity

Y. Pennec ,B. Djafari Rouhani, J. Vasseur et al, Appl. Phys. Lett. 87, 261912 (2005)

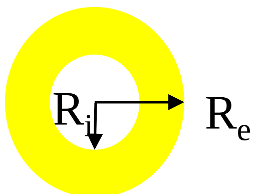
Demultiplexing



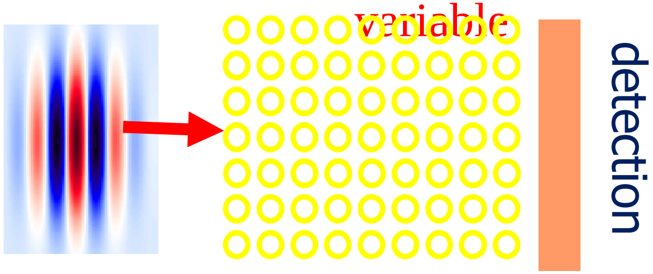
Demultiplexing

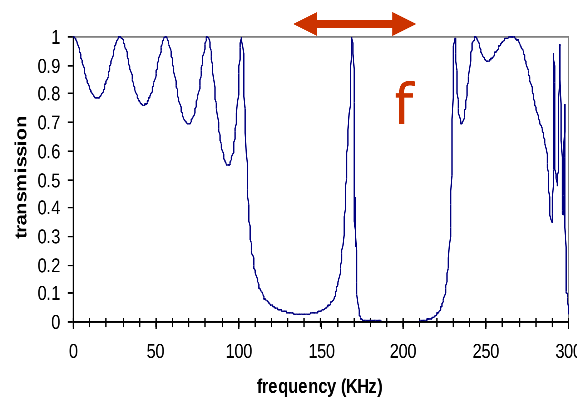
Frequency filtering with hollow scatterers

Square array of steel cylinders in water



a = 5 mm
R_e = 2 mm
R_i variable





The frequency f is a function of the internal radius Ri and the nature of the fluid inside and outside the cylinders

→ Tunable frequency filter

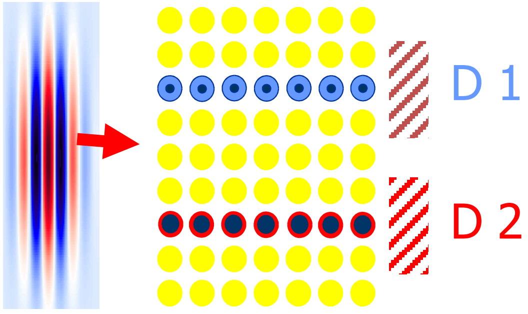
Y. Pennec, B. Djafari Rouhani, J. Vasseur, A. Khelif, P. Deymier, Phys. Rev. E69, 046608 (2004)

Demultiplexer

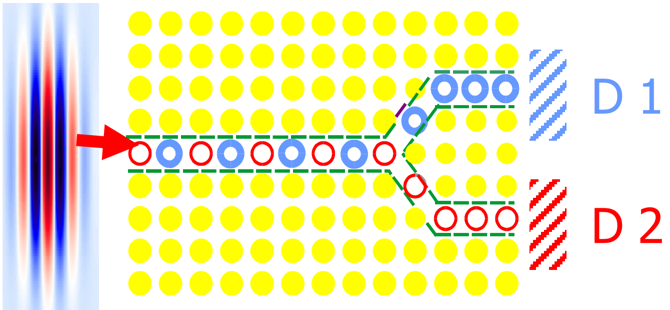
Demultiplexing

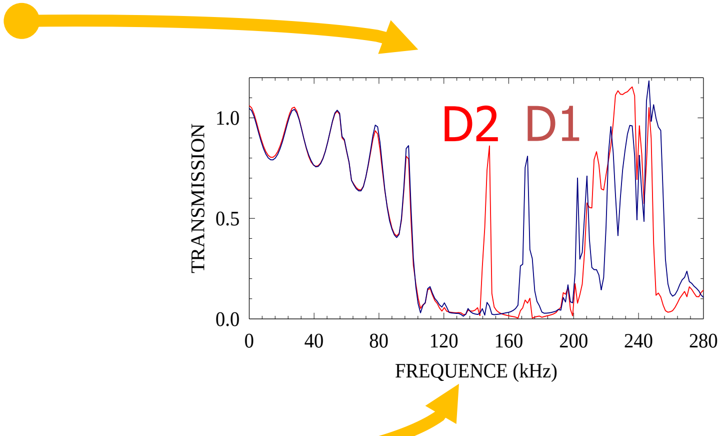
1) Two rows of cylinders with different internal radii

R_e = 2.3 mm,
a = 5.0 mm,
R_i = 1.0 mm
R_i = 1.2 mm



2) 'Y' shape waveguides





→ Selection of two different frequecies f_0 and f_1 at the detectors

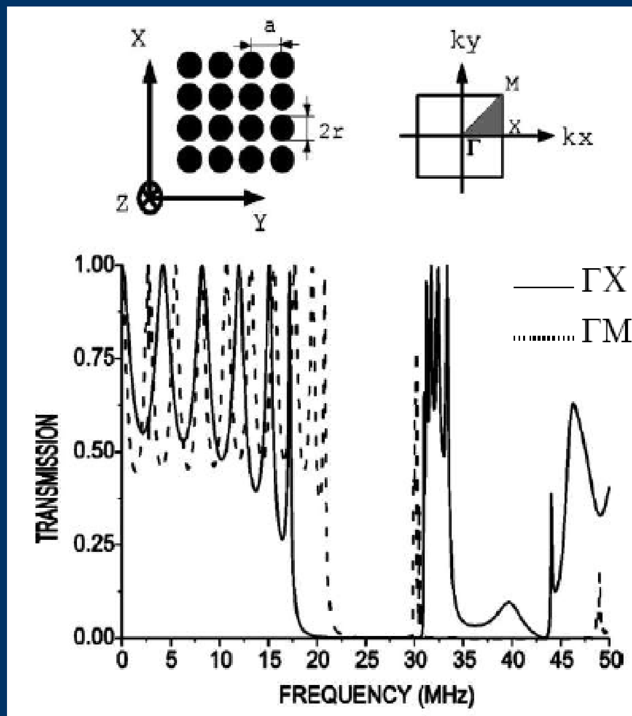
Y. Pennec, B. Djafari Rouhani, J. Vasseur, A. Khelif, P. Deymier, Phys. Rev. E69, 046608 (2004)

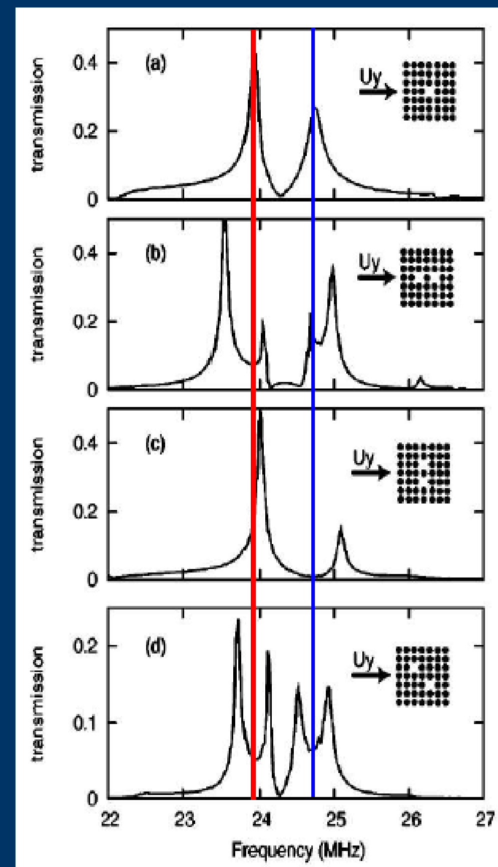
Cavity modes

Square array of air cylinders in silica

a=85µm
r=36µm

Localized defect elastic modes





2D phononic crystal
Computation by 2D-FDTD including shear and longitudinal waves

A. Khelif, B. Djafari Rouhani, V. Laude, M. Solal, J. Appl. Phys. 94, 7944 (2003)